LIQUIDITY PREMIA, PRICE-RENT DYNAMICS, AND BUSINESS CYCLES

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Abstract. In the U.S. economy over the past twenty five years, house prices exhibit fluctuations considerably larger than house rents and these large fluctuations tend to move together with business cycles. We build a simple theoretical model to characterize these observations by showing the tight connection between price-rent fluctuation and the liquidity constraint faced by productive firms. After developing economic intuition for this result, we estimate a medium-scale dynamic general equilibrium model to assess the empirical importance of the role the price-rent fluctuation plays in the business cycle. According to our estimation, a shock that drives most of the price-rent fluctuation explains 30% of output fluctuation over a six-year horizon.

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I. Introduction

In the U.S. economy we observe that house prices fluctuate more than house rents and that price-rent fluctuations tend to move with business cycles. Figure 1 shows that, over the past twenty five years, the time series of house price-rent ratios not only display a large volatility, but also tend to move together with the time series of output. Nothing illustrates such empirical evidence better than the impulse responses of output, house price, and house rent from an estimated Bayesian vector autoregression (BVAR) model with the recursive ordering suggested by Sims (1980) and Christiano, Eichenbaum, and Evans (2005). The responses, displayed as a 3×3 matrix of graphs in Figure 2, evince three important facts. First, output, house price, and house rent all have large hump-shaped responses (the three graphs along the diagonal of the graph matrix).[1] Second, the house price tends to comove with output (the first two graphs in the second column). Third, the house price fluctuates more than not just output (comparing the second graph in the second column to the first two graphs in the first row) but also house rent (comparing the last two graphs in the graph matrix). How to account for these salient observations in a tractable real business cycle (RBC) model has been a central but challenging issue in macroeconomics.

In recent papers Iacoviello (2005) and Iacoviello and Neri (2010) explain co-movements between house prices and consumption expenditures and Liu, Wang, and Zha (2013) explore co-movements between land prices and investment. As in much of the asset-pricing literature, the dynamic general equilibrium models studied by these authors imply that the house price is the discounted present value of future rents and thus both price and rent move in comparable magnitude. This implication does not square with the key fact in the housing market: the house price is much more volatile than the house rent.

In this paper we argue that this fact is a key to understanding the dynamic interactions between house prices and real business cycles. We build this argument in a model that is based on the primitive assumption of limited commitment by a productive firm to finance its working capital. We begin with a simple model without capital in which there is a continuum of heterogeneous firms with idiosyncratic productivity shocks. Firms trade housing units; their assets are in the form of real estate. A productive firm borrows from households to finance its working capital in the form of trade credit with a promise to repay the loan after the production takes place. Because the firm may choose to renege on its payment promise, an incentive compatibility constraint is imposed to resolve the limited enforcement problem. The optimal contract results in a liquidity constraint on how much of working capital the firm is able to finance. We show that this endogenously-derived constraint is directly influenced

[1] The response of output in the first column of Figure 2 will eventually come down, so its hump shape is even larger than the graph shows.

by the difference between the house price and the discounted present value of house rents. We call this difference the "liquidity premium."

A rise in the liquidity premium relaxes the firm's liquidity constraint and thus facilitates firm's production. The liquidity constraint is not always binding. A novel feature of our model is: whether a particular firm's liquidity constraint binds depends on both the nature of the shock and the realization of a firm's individual productivity. A shock that raises the liquidity premium simultaneously raises the threshold of the productivity level above which firms choose to produce until their liquidity constraint binds. A rise in such a cutoff level, in turn, weeds out unproductive firms and induces highly productive firms to function. In the aggregate it raises the total factor productivity (TFP). Such a dynamic interaction between the liquidity premium and endogenous TFP is the key crux of our paper.

To test the implications of our theory, we extend it to a medium-scale dynamic general equilibrium model that is fit to the U.S. time series. We find that traditional business-cycle shocks, such as shocks to technology, housing demand, and labor supply, cannot explain price-rent fluctuations in magnitude comparable to the observed time series. A shock to the liquidity premium, by contrast, accounts for the three observed facts delineated at the beginning of the introduction section: 1) the hump-shaped responses of output and the house price; 2) the comovement of output and the house price; and 3) the large volatility of the house price relative to both output and the house rent. Our estimation indicates that a liquidity premium shock explains not only most of the price-rent fluctuation but also 30% of the aggregate output fluctuation over a six-year forecast horizon.

There are two important strands of literature relevant to our analysis. One strand focuses on the housing market by analyzing the rise and fall of house prices relative to house rents (Campbell, Davis, Gallin, and Martin, 2009; Piazzesi and Schneider, 2009; Caplin and Leahy, 2011; Burnside, Eichenbaum, and Rebelo, 2011; Pintus and Wen, 2013). Another strand of literature analyzes the impact of financial frictions on the measured TFP (Jermann and Quadrini, 2007; Jeong and Townsend, 2007; Amaral and Quintin, 2010; Buera, Kaboski, and Shin, 2011; Miao and Wang, 2012; Gilchrist, Sim, and Zakrajšek, 2013; Buera and Shin, 2013; Liu and Wang, 2014; Midrigan and Xu, 2014; Moll, Forthcoming). This strand of literature is too large for us to list every relevant paper. Restuccia and Rogerson (2013) have an excellent review of the literature.[2] A general view is that financial frictions can cause resource misallocation and therefore TFP losses. Many important papers in this literature focus on a steady state analysis and on the implications for growth and development.

Our paper is more closely related to Buera and Moll (2013), who study the role of shocks to collateral constraints (or credit crunch) in business cycles.[3] They show that a credit

[2]See other papers in the special issue of the *Review of Economic Dynamics*, volume 16, issue 1, 2013.

[3]Jermann and Quadrini (2012) also study the impact of this shock on business cycles.

crunch results in a decrease of the cutoff productivity level above which firms are active. The implication of this result is that there is an entry of unproductive firms, causing a drop in TFP in recessions. This result is consistent with the evidence provided by Kehrig (2011), who documents that the dispersion of productivity in U.S. durable manufacturing firms is greater in recessions than in booms, implying a relatively higher share of unproductive firms in recessions.

Our paper places a different emphasis on the role of endogenous TFP dynamics. We focus on understanding the business-cycle properties of observed large price-rent fluctuations in the housing market. Unlike many papers in the literature on financial frictions, the liquidity constraint in our paper is derived from the optimal contract with the primitive assumption of limited commitment (Kehoe and Levine, 1993; Alvarez and Jermann, 2000; Albuquerque and Hopenhayn, 2004). A shock that moves the liquidity premium affects the liquidity constraint and provides the main source of endogenous TFP fluctuations.

By contrast, a housing demand shock emphasized in the previous literature cannot explain the observed price-rent dynamics because it moves both the house rent and the house price in similar magnitude. Once the house rent is explicitly taken into account in estimation, a housing demand shock plays almost no role in generating business cycles. Our new theoretical framework offers key intuition for how a shock that moves the liquidity premium can be transmitted to the real economy through endogenous TFP.

The paper is organized as follows. In Section II, we construct a simple theoretical framework that can be easily understood. This framework lays a foundation for our medium-scale empirical model. In Section III, we develop key intuition for the link between price-rent dynamics and aggregate fluctuations. In Section IV, we extends the simple model to a medium-scale dynamic general equilibrium model that aims to fit to the U.S. time series. In Section V, we discuss the empirical results from the estimated model. In Section VI, we discuss the propagation mechanism that is present in the medium-scale model but is lacking in the simple model. Section VII concludes the paper.

II. A Simple Model Without Capital

In this section we present a simple model without capital to obtain a closed-form solution up to first-order approximation. The closed-form results, discussed in Section III, enable us to illustrate the key mechanism that drives the link between output fluctuations and price-rent dynamics. Proofs of all the propositions in this section are provided in Appendix A.

II.1. **The Economy.** The economy is populated by the representative household and a continuum of firms.

Households. The representative household maximizes the lifetime utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t \Theta_t \left(\log C_t + \xi_t \left(h_{rt} + h_{ot} \right) - \frac{N_t^{1+\nu}}{1+\nu} \right),$$

where C_t represents consumption, N_t represents labor supply, h_{rt} represents rented housing units, and h_{ot} represents purchased housing units. The parameters $\beta \in (0,1)$ and $1/\nu > 0$ represent the subjective discount factor and the Frisch elasticity of labor supply, respectively.

Following Smets and Wouters (2007), Primiceri, Schaumburg, and Tambalotti (2006), and Albuquerque, Eichenbaum, and Rebelo (2012), we introduce an intertemporal shock, Θ_t, that influences the discount factor. We follow Iacoviello and Neri (2010) and Liu, Wang, and Zha (2013) and introduce an intratemporal shock, ξ_t, that influences the demand for housing. Let $\theta_{t+1} = \Theta_{t+1}/\Theta_t$. Both θ_t and ξ_t are assumed to follow an AR(1) process with

$$\log \theta_{t+1} = \rho_\theta \log \theta_t + \sigma_\theta \varepsilon_{\theta t+1}, \tag{1}$$

where $\sigma_\theta > 0$, $|\rho_\theta| < 1$, and $\varepsilon_{\theta t+1}$ is an i.i.d. normal random variable, and

$$\log \xi_{t+1} = (1-\rho_\xi) \log \bar{\xi} + \rho_\xi \log \xi_t + \sigma_\xi \varepsilon_{\xi t+1}, \tag{2}$$

where $\sigma_\xi > 0$, $\left|\rho_\xi\right| < 1$, and $\varepsilon_{\xi t+1}$ is an i.i.d. normal random variable.

The household's intertemporal budget constraint is given by

$$C_t + r_{ht} h_{rt} + p_t \left(h_{ot+1} - h_{ot} \right) = w_t N_t + D_t, \ t \geq 0,$$

where r_{ht} represents the house rent, p_t is the house price, w_t is the wage rate, and D_t is the dividend income. We assume that the household does not initially own any housing unit (i.e., $h_{ot} = 0$ when $t = 0$) and faces the short-sales constraint $h_{ot+1} \geq 0$ for all t. Assume that houses do not depreciate.

We obtain the following first-order conditions:

$$r_{ht} = \frac{\Theta_t \xi_t}{\Lambda_t}, \tag{3}$$

$$\frac{\Theta_t N_t^\nu}{\Lambda_t} = w_t, \tag{4}$$

and

$$p_t = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \left(p_{t+1} + r_{ht} \right) + \frac{\pi_t}{\Lambda_t}, \tag{5}$$

where

$$\Lambda_t = \frac{\Theta_t}{C_t} \tag{6}$$

is the marginal utility of consumption, and $\pi_t \geq 0$ is the Lagrange multiplier associated with the short-sales constraint $h_{ot+1} \geq 0$ with the complementary slackness condition $\pi_t h_{ot+1} = 0$.

Equation (3) indicates that the house rent is equal to the marginal rate of substitution between housing services and consumption. Equation (4) states that the wage rate is equal to the marginal rate of substitution between leisure and consumption. Equation (5) is the asset-pricing equation for housing.

Firms. Each firm $i \in [0,1]$ owns a constant-returns-to-scale technology that produces output y_t^i using labor input n_t^i according to

$$y_t^i = a_t^i A_t n_t^i,$$

where a_t^i represents an idiosyncratic productivity shock drawn independently and identically from a fixed distribution with pdf f and cdf F on $(0,\infty)$, and A_t represents an aggregate technology shock that follows the AR(1) process

$$\log A_{t+1} = \rho_a \ln A_t + \sigma_a \varepsilon_{at+1},$$

where $\sigma_a > 0$, $|\rho_a| < 1$, and ε_{at+1} is an i.i.d. normal random variable. Firm i maximizes its expected discounted present value of dividends

$$\max E_0 \sum_{t=0}^{\infty} \frac{\beta^t \Lambda_t}{\Lambda_0} d_t^i, \tag{7}$$

where d_t^i denotes dividends and $\beta^t \Lambda_t / \Lambda_0$ is the household's stochastic discount factor.

Firm i hires labor and trades and leases housing units. In each period t, prior to the sales of output and housing units, firm i must borrow to finance working capital of wage bills. Households extend trade credit to the firm in the beginning of period t and allows it to pay wage bills at the end of the period using revenues from sales of output and housing units. The firm's flow-of-funds constraint is given by

$$d_t^i + p_t(h_{t+1}^i - h_t^i) = a_t^i A_t n_t^i - w_t n_t^i + r_{ht} h_t^i,\ t \geq 0,\ \text{with } h_0^i \text{ given.} \tag{8}$$

Firms are not allowed to short-sell houses so that $h_{t+1}^i \geq 0$ for all t.

A key assumption of our model is that contract enforcement is imperfect. The firm has limited commitment and may choose not to pay wage bills. In such a default state, the firm would retain its production revenues $a_t^i A_t n_t^i$ as well as its house holdings h_t^i. But the firm would be denied access to financial markets in the future. In particular, it would be barred from selling any asset holdings for profit and from obtaining loans for working capital.[4]

In the default state, since the firm would have no access to working capital, it would be unable to produce. In short, the firm would be in autarky. Let $V_{t+1}^a(h_t^i)$ denote the continuation value for firm i that chooses to default in period t with house holdings h_t^i. Let $V_t(h_t^i, a_t^i)$ denote firm i's value function.[5] The firm has no incentive to default on the trade

[4]To focus on the role of working capital and make our economic mechanism transparent, we abstract from intertemporal loan markets. An introduction of such intertemporal elements would complicate the model a great deal without changing our key analytical and empirical results in this paper.

[5]The value function depends on aggregate state variables as well. We omit these state variables to keep notation simple.

credit if and only if the following incentive compatibility constraint holds:

$$V_t\left(h_t^i, a_t^i\right) \geq a_t^i A_t n_t^i + r_{ht} h_t^i + \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} V_{t+1}^a(h_t^i), \tag{9}$$

where the left-hand side of the inequality is the no-default value and the right-hand side gives the default value. Since $V_{t+1}^a(h_t^i)$ is equal to the sum of the rental value in period $t+1$ and the expected discounted present value of future rents, we have

$$\beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} V_{t+1}^a(h_t^i) = p_t^a h_t^i, \tag{10}$$

where p_t^a denotes the expected discounted present value of future rents (per housing unit)

$$p_t^a \equiv E_t \sum_{\tau=1}^{\infty} \beta^\tau \frac{\Lambda_{t+\tau}}{\Lambda_t} r_{ht+\tau} = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t}\left(p_{t+1}^a + r_{ht+1}\right). \tag{11}$$

Firm i's problem is to solve the Bellman equation

$$V_t(h_t^i, a_t^i) = \max_{n_t^i, h_{t+1}^i \geq 0} \; d_t^i + \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} V_{t+1}(h_{t+1}^i, a_{t+1}^i), \tag{12}$$

subject to (8) and (9).

II.2. **Liquidity Constraint and Asset Pricing.** One significant feature of our model is that the incentive constraint (9) gives rise to an endogenous liquidity constraint that depends on the liquidity premium for housing, as stated as follows.

Proposition 1. The value function takes the form $V_t(h_t^i, a_t^i) = v_t(a_t^i) h_t^i$, where $v_t(a_t^i)$ satisfies

$$p_t = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} v_{t+1}\left(a_{t+1}^i\right). \tag{13}$$

The incentive compatibility constraint (9) is equivalent to

$$w_t n_t^i \leq \left(p_t - p_t^a\right) h_t^i \equiv b_t h_t^i, \tag{14}$$

where we define the liquidity premium b_t as

$$b_t \equiv p_t - p_t^a \geq 0.$$

The linear form of the value function in Proposition 1 follows directly from the constant-returns-to-scale technology. Equation (13) is an equilibrium restriction on the house price. If $p_t > \beta E_t\left[v_{t+1}\left(a_{t+1}^i\right)\Lambda_{t+1}/\Lambda_t\right]$, firm i would prefer to sell all housing units, $h_{t+1}^i = 0$. All other firms would not hold housing units because the preceding inequality holds for any i as a_t^i is i.i.d. This would violate the market-clearing condition for the housing market. If $p_t < \beta E_t\left[v_{t+1}\left(a_{t+1}^i\right)\Lambda_{t+1}/\Lambda_t\right]$, all firms would prefer to own housing as much as possible, which again violates the market-clearing condition.

The pricing restriction (13) is essential to achieving the interpretive form (14) of the liquidity constraint. Using the Bellman equation (12), we can rewrite the incentive constraint (9) as

$$d_t^i + \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} V_{t+1}(h_{t+1}^i, a_{t+1}^i) \geq a_t^i A_t n_t^i + r_{ht} h_t^i + \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} V_{t+1}^a(h_t^i).$$

Given the value function and equations (8), (10), and (13), we can rewrite this constraint as

$$a_t^i A_t n_t^i - w_t n_t^i + r_{ht} h_t^i + p_t h_t^i \geq a_t^i A_t n_t^i + (r_{ht} + p_t^a) h_t^i.$$

Simplifying the proceeding inequality yields the constraint (14).[6] The left-hand side of (14) is the cost of working capital (wage bills); the right-hand side is the liquidity value. Housing provides liquidity for firms to finance working capital and thus commands a liquidity premium.

The key idea of this paper is that the liquidity premium provided by housing facilitates production.[7] The higher the premium, the more relaxed the liquidity constraint. A credit expansion allows firms to finance more working capital, hire more workers, and produce higher output. Relevant questions are: what factors influence the liquidity premium? And how quantitatively important are such premia in business cycles? As will be discussed in Section III, the shock process governing θ_t not only is a principal force that drives the fluctuation of liquidity premium but also plays a significant role in shaping business cycles. We call θ_t a "liquidity premium shock."

Proposition 1 enables us to solve the firm's decision problem and obtain asset-pricing equations for determining house prices.

Proposition 2. Firm i's optimal labor choice is given by

$$n_t^i = \begin{cases} \frac{b_t h_t^i}{w_t} & \text{if } a_t^i \geq a_t^* \\ 0 & \text{otherwise} \end{cases}, \tag{15}$$

where $a_t^* \equiv w_t / A_t$. The house price is determined by the two asset-pricing equations

$$p_t = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \left[r_{ht+1} + p_{t+1} + b_{t+1} \int_{a_{t+1}^*}^{\infty} \frac{a - a_{t+1}^*}{a_{t+1}^*} f(a) da \right], \tag{16}$$

and

$$b_t = \beta E_t b_{t+1} \frac{\Lambda_{t+1}}{\Lambda_t} \left[1 + \int_{a_{t+1}^*}^{\infty} \frac{a - a_{t+1}^*}{a_{t+1}^*} f(a) da \right]. \tag{17}$$

[6]The constraint (14) can be interpreted as an endogenous credit constraint of the Kiyotaki and Moore (1997) type, such that $w_t n_t^i \leq \lambda_t p_t h_t^i$ where $\lambda_t = b_t / p_t$ is endogenously determined.

[7]He, Wright, and Zhu (2013) and Miao, Wang, and Zhou (2014) study the role of the liquidity premium in the house price in theoretical models with multiple equilibria. This is not the focus of our paper.

Due to constant-returns-to-scale technology, only firms with $a_t^i \geq a_t^*$ employ labor and produce output. This property implies that the liquidity constraint (14) is *not* always binding. It binds for only productive firms that borrow to finance their wage bills. The cutoff productivity level a_t^* for determining the binding liquidity constraint varies with the house price, delivering an essential role of liquidity premia in business cycles.

Equations (16) and (17) show that the house price is positively influenced by not only the expected discounted present value of rents but also the liquidity premium. This premium in turn depends on the next-period credit yield for all productive firms:

$$\int_{a_{t+1}^*}^{\infty} \frac{a - a_{t+1}^*}{a_{t+1}^*} f(a) da. \tag{18}$$

It follows from (15) that one-dollar liquidity provided by one housing unit in the next period allows firm i to hire $1/w_{t+1}$ units of labor when $a_{t+1}^i \geq a_{t+1}^*$. This generates the average profit of $\left(a_{t+1}^i A_{t+1}/w_{t+1} - 1\right) = \left(a_{t+1}^i/a_{t+1}^* - 1\right)$ dollars when $a_{t+1}^i \geq a_{t+1}^*$. The credit yield in (18) reflects the average profit generated by one-dollar liquidity.

II.3. **Equilibrium.** We consider the interior equilibrium in which production takes place, labor supply N_t is positive, and the house price premium b_t is positive.[8]

Proposition 3. For the interior equilibrium, the household's optimal choice is not to own housing units, i.e., $h_{ot+1} = 0$ for all t.

It follows from equations (5) and (16) that the Lagrange multiplier π_t is positive and reflects the liquidity premium when $b_t > 0$ for all t. By the complementary slackness condition, we deduce that $h_{ot+1} = 0$ for all t. We normalize the house supply to unity. In equilibrium, all markets clear such that

$$\int n_t^i di = N_t, h_{ot} = 0, \int h_t^i di = h_{rt} = 1, \quad \int y_t^i di = Y_t = C_t.$$

The household' dividend income is $D_t = \int_0^1 d_t^i di$. The following proposition summarizes the equilibrium dynamics of our model.

Proposition 4. The equilibrium system is given by nine equations (3), (4), (11), (16), (17), $a_t^* = w_t/A_t$, $Y_t = C_t$,

$$Y_t = A_t N_t \frac{\int_{a_t^*}^{\infty} a f(a) da}{1 - F(a_t^*)}, \tag{19}$$

$$w_t N_t = (1 - F(a_t^*)) b_t, \tag{20}$$

for nine variables $\{r_{ht}\}$, $\{w_t\}$, $\{N_t\}$, $\{Y_t\}$, $\{C_t\}$, $\{a_t^*\}$, $\{p_t^a\}$, $\{p_t\}$, and $\{b_t\}$.

[8]There is a trivial equilibrium such that $b_t = 0$ for all t. In this trivial case, no production would take place. The equilibrium with $b_t > 0$ for all t is unique.

We need only to show how to derive (19) and (20). Using a law of large numbers, we obtain (20) by aggregating (15). To derive (19), we first aggregate individual firm production functions by using (15) in Proposition 2. By a law of large numbers we have

$$Y_t = A_t \int_0^1 a_t^i n_t^i di = \frac{A_t b_t}{w_t} \int_{a_t^*}^{\infty} a f(a) da.$$

We obtain equation (19) by using equation (20) to eliminate w_t from the preceding equation.

III. Economic Mechanism: An Illustration

What is the economic mechanism that links the financial sector to the real sector in our model? To answer this key question, we need both a shock that triggers a change in the cutoff productivity level a_t^* and an economic mechanism linking the liquidity premium (the difference between the market price of house and the discounted present value of rents, i.e., $b_t = p_t - p_t^a$) to real aggregate variables such as output and hours. It turns out that a shock to the liquidity premium, θ_t, is the primary shock driving the fluctuation of the cutoff productivity level a_t^*. In Section III.1 we focus exclusively on the mechanism that transmits this shock to both the financial sector and the real sector. In Section III.2 we assess the importance of a liquidity premium shock in comparison to other shocks.

III.1. **Intuition.**

A novel feature of our model, relative to the empirical literature on stochastic dynamic general equilibrium (DSGE) modeling, is that the cutoff productivity level a_t^* is endogenous and plays a crucial role in accounting for the dynamic links between the house price, the house rent, and aggregate real variables. We first demonstrate that a_t^* affects the real sector through TFP and labor reallocation. Equation (19) shows that our model generates endogenous TFP defined as

$$TFP_t = \frac{\int_{a_t^*}^{\infty} a f(a) da}{1 - F(a_t^*)}. \tag{21}$$

A rise in a_t^* discourages less efficient firms from production and induces more efficient firms to produce. As a result, the TFP increases with the cutoff productivity level a_t^*.

Dividing by $w_t N_t$ on the two sides of equation (19) and using $a_t^* = A_t / w_t$, we derive

$$Y_t = \frac{\int_{a_t^*}^{\infty} \frac{a}{a_t^*} f(a) da}{1 - F(a_t^*)} w_t N_t. \tag{22}$$

This equation shows that aggregate output exceeds the factor income because firms make positive profits due to financial frictions. Labor is reallocated to more productive firms and the marginal product of labor for each firm is not equal to the wage rate.

Eliminating w_t from equations (4) and (22) with $C_t = Y_t$ and using (6), we derive the labor-market equilibrium condition

$$N_t^{1+\nu} = \frac{1 - F(a_t^*)}{\int_{a_t^*}^{\infty} \frac{a}{a_t^*} f(a) da}. \tag{23}$$

An increase in a_t^* has three effects on N_t. First, it raises endogenous TFP, which increases the profit markup over the labor cost by (22). Firms demand less labor, *ceteris paribus*. Second, if we hold endogenous TFP fixed, it follows from (22) that the higher the cutoff productivity level, the less the profit markup. This selection effect increases demand for labor. Third, labor supply is reduced due to the wealth effect, as in the standard RBC model. The net effect on equilibrium labor hours N_t is ambiguous. When we use the estimated parameter values from our medium-scale empirical model developed in Section IV, labor hours decrease for the simple model but increase for the medium-scale model.

We use the top panel of Figure 3 to illustrate how a rise of the cutoff productivity level a_t^* affects output and hours in equilibrium. The production line, representing the aggregate production function (19), is positively sloped on the N_t-Y_t plane. The vertical line on the plane represents equation (23). These two lines determine equilibrium output and hours for a given cutoff productivity level a_t^*. In plotting these labor-output lines, we treat other factors, such as a_t^* and a liquidity premium shock, as potential shifters. We assume that the initial equilibrium (Point A) is at the steady state.

Consider a liquidity premium shock that raises the cutoff productivity level a_t^*. A rise in a_t^* induces firms whose productivity is higher than a_t^* to produce. As a consequence, endogenous TFP increases and the production line shifts upward. At the same time, the labor-market line also shifts. In Figure 3 we assume that the labor-market line shifts to the left (we show how this can happen in Section III.2). As long as the effect of endogenous TFP is sufficiently strong, the shift in the production line dominates the shift in the labor-market line. As a result, output rises while hours fall (from Point A to Point B in Figure 3).

The mechanism illustrated in the top panel of Figure 3 for the real sector is only one side of the story in our model. The other is the essential role of liquidity premia in facilitating production. Firms would be unable to produce if they failed to acquire liquidity for financing working capital. It is clear from the liquidity constraint (14) that the finance of working capital depends on the liquidity premium b_t.

A key result of our model is that a rise in the cutoff productivity level a_t^* raises the liquidity premium b_t that is necessary for production. The bottom panel of Figure 3 illustrates the mechanism for understanding this result. The asset-pricing curve on the a_t^*-b_t plane represents the asset-pricing equation (17) for the liquidity premium. In Section III.2, we show that a liquidity premium shock that raises the current cutoff productivity level a_t^* also raises both the liquidity premium b_t and the future cutoff productivity level a_{t+1}^*. According to (18), the future credit yield falls as a_{t+1}^* rises. Thus the asset-pricing curve describing (17) is downward sloping.

Eliminating N_t from (19) and (20) and using $a_t^* = w_t/A_t$, we can derive

$$b_t \int_{a_t^*} \frac{a}{a_t^*} f(a)da = Y_t. \tag{24}$$

The curve that describes the relationship between a_t^* and b_t in (24) is upward sloping. Since equation (24) is derived from the liquidity constraints, we call this upward-sloping curve the "liquidity-constraint curve." The two curves in the the bottom panel of Figure 3 determine a_t^* and b_t jointly. Assume that Point A is at the steady state.

Now consider a liquidity premium shock that raises a_t^*. The shock shifts the asset-pricing curve outward. A rise in a_t^* raises the TFP and consequently aggregate output (the top panel of Figure 3). An increase in aggregate output shifts the liquidity-constraint curve upward. The equilibrium moves from Point A to Point B (the bottom panel of Figure 3) with the resultant increase of the liquidity premium b_t higher than the increase of a_t^*. The large increase of b_t relaxes the liquidity constraint that is necessary to facilitate the output increase from productive firms.

In summary, our theoretical framework is capable of generating not only the comovement of asset prices and output but also the stronger response of asset prices than the response of output (as we observe in Figure 1).

III.2. **Assessing the Importance of a Liquidity Premium Shock.** There are three shocks in this simple economy: θ_t, A_t, and ξ_t. The key to understanding how these shocks influence price-rent dynamics and their impact on the aggregate economy is to analyze how these shocks affect the cutoff productivity level a_t^*. For this model, we are able to obtain a closed-form solution to the log-linearized equilibrium system around the deterministic steady state. We use the closed-form solution to show that 1) a shock to the liquidity premium, θ_t, is the only shock that drives the fluctuation of cutoff productivity a_t^* and 2) the other two shocks cannot generate the magnitude of price-rent dynamics as observed in the data. We then use the closed-form solution to verify the intuition developed in the preceding section.

Denote $\hat{x}_t = \log(x_t) - \log(x)$, where x_t is any variable of study and x is the corresponding deterministic steady state of x_t. The log-linearized expression for (21) is

$$\widehat{TFP}_t = \frac{\eta\mu}{1+\mu}\hat{a}_t^*, \tag{25}$$

where

$$\eta \equiv \frac{a^* f(a^*)}{1 - F(a^*)} > 0$$

denotes the steady-state hazard rate and

$$\mu = \frac{\int_{a^*}^{\infty} \frac{a}{a^*} f(a)\, da}{1 - F(a^*)} - 1 > 0.$$

Hence the log-linearized equations for (19) and (23) are

$$\hat{Y}_t = \hat{N}_t + \hat{A}_t + \widehat{TFP}_t, \tag{26}$$

$$\hat{N}_t = -\frac{1}{1+\nu} \frac{\mu\eta - (1+\mu)}{1+\mu} \hat{a}_t^*. \tag{27}$$

These two equations give the log-linearized version of the production line and the labor-market line in Figure 3. Whenever $\mu\eta > (1+\mu)$,[9] an increase in a_t^* shifts the labor-market line to the left up to the first-order approximation.

From (24) we derive the log-linearized equation

$$\hat{b}_t = \hat{Y}_t + \frac{\eta + 1 + \mu}{1+\mu} \hat{a}_t^*. \tag{28}$$

The log-linearized equation for (17) is

$$\hat{b}_t - \hat{Y}_t = E_t \left(\hat{b}_{t+1} - \hat{Y}_{t+1} + \hat{\theta}_{t+1} \right) - \frac{(1-\beta)(1+\mu)}{\mu} E_t \hat{a}_{t+1}^*. \tag{29}$$

The preceding two equations give the log-linearized version of the liquidity-constraint curve and the asset-pricing curve in Figure 3. Using (28) and (29) to eliminate $\hat{b}_t - \hat{Y}_t$ and $\hat{b}_{t+1} - \hat{Y}_{t+1}$, we obtain

$$\hat{a}_t^* = \rho_\theta \frac{1+\mu}{\eta+1+\mu} \hat{\theta}_t + \left[1 - (1-\beta) \frac{1+\mu}{\mu} \frac{1+\mu}{\eta+1+\mu} \right] E_t \hat{a}_{t+1}^*.$$

Solving this equation leads to

$$\hat{a}_t^* = \rho_\theta \frac{1+\mu}{\eta+1+\mu} \frac{1}{1-\rho_\theta \kappa} \hat{\theta}_t, \tag{30}$$

where

$$\kappa = 1 - (1-\beta) \frac{1+\mu}{\mu} \frac{1+\mu}{\eta+1+\mu} < 1.$$

From equations (25), (26), and (27) we deduce

$$\hat{Y}_t = \hat{A}_t + \frac{1}{1+\nu} \left(1 + \frac{\nu\eta\mu}{1+\mu} \right) \hat{a}_t^*. \tag{31}$$

This equation indicates that, even though hours N_t may decrease with a_t^*, output Y_t always increases with a_t^* up to the first-order approximation because the upward shift of the production line dominates the leftward shift of the labor-market line due to a large increase in endogenous TFP.

One can see from equation (30) that both the aggregate technology shock A_t and the housing demand shock ξ_t play no role in influencing the cutoff productivity level a_t^*. To

[9]This condition is implied by the estimated values for our medium-scale model in Section V.

gauge the magnitude of how these shocks are transmitted to asset prices and real aggregate variables, we log-linearize equations (3), (11), and $p_t = p_t^a + b_t$ as

$$\begin{aligned} \hat{r}_{ht} &= \hat{Y}_t + \hat{\xi}_t, & (32)\\ \hat{p}_t^a &= E_t\left[\hat{\theta}_{t+1} + \hat{Y}_t - \hat{Y}_{t+1} + (1-\beta)\hat{r}_{ht+1} + \beta\hat{p}_{t+1}^a\right], & (33)\\ \hat{p}_t &= \frac{p^a}{p}\hat{p}_t^a + \left(1 - \frac{p^a}{p}\right)\hat{b}_t, & (34)\end{aligned}$$

where we use the steady-state equilibrium conditions to derive

$$\frac{p^a}{p} = \frac{\bar{\xi}(1+\mu)}{\bar{\xi}(1+\mu)+\mu}.$$

Substituting (32) into (33) and solving $\hat{p}_t^a - \hat{Y}_t$ forward, we obtain

$$\hat{p}_t^a = \hat{Y}_t + \frac{\rho_\theta}{1-\beta\rho_\theta}\hat{\theta}_t + \frac{(1-\beta)\rho_\xi}{1-\beta\rho_\xi}\hat{\xi}_t. \tag{35}$$

From equations (25), (27), and (30), one can see that the aggregate technology shock A_t does *not* exert any influence on $\widehat{TFP}_t$, $\hat{a}_t^*$, and $\hat{N}_t$. Thus the A_t shock would have the same one-for-one effect on output Y_t [equation (31)], the liquidity premium b_t [equation (28)], the house rent r_{ht} [equation (32)], the expected discounted present value of rents $\hat{p}_t^a$ [equation (35)], and the house price $\hat{p}_t$ [equation (34)]. Because the house price is much more volatile than the house rent and output in the data, the aggregate technology shock in our model cannot be the main source for generating the link between price-rent dynamics and output fluctuations.

As in Liu, Wang, and Zha (2013), the housing demand shock ξ_t influences the house rent through equation (32) and in turn the house price through equation (34). But Liu, Wang, and Zha (2013) abstract from the central and challenging issue addressed in this paper: the fluctuations of house prices relative to those of house rents over business cycles. In our model, since the housing demand shock does *not* affect the liquidity premium, it has no influence on hours and output. Moreover, a one percent increase in the housing demand shock ξ_t raises the house rent by one percent, but raises the house price by less than one percent because

$$\frac{(1-\beta)\rho_\xi}{1-\beta\rho_\xi}\frac{p^a}{p} < 1.$$

Thus the housing demand shock is unable to generate price-rent dynamics observed in the data (Figure 1).

By contrast, it follows from (30) that the liquidity premium shock $\hat{\theta}_t$ is the only shock that influences cutoff productivity and therefore the TFP [equation (30)]. A positive liquidity premium shock raises the cutoff productivity level $\hat{a}_t^*$. The increase of the cutoff productivity level $\hat{a}_t^*$ raises endogenous TFP, causing aggregate output to rise [equations (26)]. In equilibrium, the increase of the liquidity premium $\hat{b}_t$ is greater than the increase of both output and cutoff productivity, as shown in equation (28).

Figure 4 illustrates the quantitative importance of the dynamic impact of a liquidity premium shock with the following parameterization:

$$\nu = 1.023, \eta = 9.313, \mu = 0.148, \bar{\xi} = 0.135, \beta = 0.994, \rho_\theta = 0.95, \sigma_\theta = 0.001.$$

Except for the values of ρ_θ and σ_θ, all other parameter values are taken from the estimates presented in Section V. The values of ρ_θ and σ_θ are selected for the best visual effect without altering the model's implications. The top panel of Figure 4 shows that, in log value, the response of the house price (the star line) is about ten times the response of the house rent (the circle line) as well as the response of cutoff productivity (the dashed line). The movement in the house price is mostly driven by the liquidity premium (the solid line). The bottom panel of Figure 4 shows that the responses of output (the circle line) is most driven by the response of endogenous TFP (the solid line).

These calibrated results are broadly consistent with the dynamics we observe in the data. The model, however, is unable to generate hump-shaped responses, which are prominent features in macroeconomic time series. To overcome this important shortcoming, we introduce capital into our model in the next section in order to fit the actual data. The economic mechanism explained in this section, however, remains the key to understanding the empirical results estimated from a more complicated structural model.

IV. A Tractable Medium-Scale Structural Model

In this section we build up a medium-scale dynamic general equilibrium model that aims to fit the house price-rent data and other macroeconomic data in the U.S. economy. By introducing capital, this medium-scale model is an expansion of the basic model developed in Section II. Although the dynamics and equilibrium conditions are much more complicated, all the intuition and insights discussed in Section II carry over to this medium-scale model.

We consider an economy populated by a continuum of identical households, a continuum of competitive intermediate goods producers of measure unity, and a continuum of heterogeneous competitive firms of measure unity. The representative household rents out capital and supplies labor to intermediate-goods producers. Firms use intermediate goods as input to produce final consumption good. Financial frictions occur in the final-good sector.

IV.1. **Households.** The representative household maximizes the expected lifetime utility

$$E_0 \sum_{t=0}^{\infty} \Theta_t \beta^t \left[\log \left(C_t - \gamma C_{t-1} \right) + \xi_t \log H_t - \psi_t \frac{N_t^{1+\nu}}{1+\nu} \right], \tag{36}$$

where C_t represents aggregate consumption, N_t is the household's total labor supply, and H_t denotes housing services. The parameters $\beta \in (0,1)$ and $\gamma \in (0,1)$ represent the subjective discount factor and habit formation. The variables $\theta_t \equiv \Theta_t/\Theta_{t-1}$, ξ_t, and ψ_t are exogenous shocks to liqudity premium, housing demand, and labor supply that follow AR(1) processes (1), (2), and

$$\log \psi_t = (1-\rho_\psi)\log \bar{\psi} + \rho_\psi \log \psi_{t-1} + \sigma_\psi \varepsilon_{\psi,t},$$

where $\sigma_\psi > 0$, $\left|\rho_\psi\right| < 1$, and $\varepsilon_{\psi,t}$ is an i.i.d. standard normal random variable.

The household chooses consumption C_t, investment I_t, housing services H_t, capital utilization rate u_t, and bonds B_{t+1}, subject to the intertemporal budget constraint

$$C_t + \frac{I_t}{Z_t} + \frac{B_{t+1}}{R_{ft}} + r_{ht}H_t \leq w_t N_t + u_t r_{kt} K_t + D_t + B_t, \tag{37}$$

where K_t, w_t, D_t, r_{kt}, r_{ht}, and R_{ft} represent capital, wage, dividend income, the rental rate of capital, the house rent, and the risk-free interest rate.[10] The variable Z_t represents an aggregate investment-specific technology shock that has both permanent and transitory components (Greenwood, Hercowitz, and Krusell, 1997; Krusell, Ohanian, Ríos-Rull, and Violante, 2000; Justiniano and Primiceri, 2008):

$$Z_t = Z_t^p v_{zt},\ Z_t^p = Z_{t-1}^p g_{zt},$$

$$\log g_{zt} = (1-\rho_z)\log \bar{g}_z + \rho_z \log(g_{z,t-1}) + \sigma_z \varepsilon_{zt}, \tag{38}$$

$$\log v_{zt} = \rho_{v_z} \log v_{z,t-1} + \sigma_{v_z}\varepsilon_{v_z,t}, \tag{39}$$

where $|\rho_z| < 1$, $\left|\rho_{v_a}\right| < 1$, $\sigma_z > 0, \sigma_{v_a} > 0$, and $\varepsilon_{z,t}$ and $\varepsilon_{v_z,t}$ are i.i.d. standard normal random variables.

Investment is subject to quadratic adjustment costs (Christiano, Eichenbaum, and Evans, 2005). Capital evolves according to the law of motion

$$K_{t+1} = (1-\delta(u_t))K_t + \left[1 - \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - \bar{g}_I\right)^2\right] I_t, \tag{40}$$

where $\delta_t \equiv \delta(u_t)$ is the capital deprecation rate in period t, $\bar{g}_I$ denotes the long-run growth rate of investment, and Ω is the investment adjustment cost parameter.

[10] If we allow households to trade housing units, their holdings will be zero given the short-sales constraint shown in Section II. For notational simplicity, we set the household's holdings of housing units to zero.

IV.2. **Intermediate-Goods Producers.** There is a continuum of intermediate goods. Each intermediate good $j \in [0,1]$ is produced by a continuum of identical competitive producers of measure unity. The representative producer owns a constant-returns-to-scale technology to produce good j by hiring labor $N_t(j)$ and renting capital $K_t(j)$ from households. The producer's decision problem is

$$\max_{N_t(j),\, K_t(j)} \; P_{Xt}(j)X_t(j) - w_t N_t(j) - r_{kt}K_t(j), \tag{41}$$

where $X_t(j) \equiv A_t K_t(j)^{\alpha} N_t(j)^{1-\alpha}$ and $P_{Xt}(j)$ represents the competitive price of good j. The aggregate technology shock A_t consists of permanent and transitory components (Aguiar and Gopinath, 2007)

$$A_t = A_t^p \nu_{a,t}, \;\; A_t^p = A_{t-1}^p g_{at},$$

$$\log g_{at} = (1-\rho_a)\log \bar{g}_a + \rho_a \log(g_{a,t-1}) + \sigma_a \varepsilon_{at},$$

$$\log \nu_{a,t} = \rho_{v_a} \log \nu_{a,t-1} + \sigma_{v_a} \varepsilon_{v_a,t},$$

where $|\rho_a| < 1$, $\left|\rho_{\nu_a}\right| < 1$, $\sigma_a > 0, \sigma_{\nu_a} > 0$, and ε_{at} and $\varepsilon_{v_a,t}$ are i.i.d. standard normal random variables.

IV.3. **Final-Good Firms.** There is a continuum of heterogeneous competitive firms. Each firm $i \in [0,1]$ combines intermediate goods $x_t^i(j)$ to produce the final consumption good with the aggregate production technology

$$y_t^i = a_t^i \exp\left(\int_0^1 \log x_t^i(j)dj\right), \tag{42}$$

where a_t^i represents an idiosyncratic productivity shock. Firm i purchases intermediate good j at the price $P_{Xt}(j)$. The total spending on working capital is $\int_0^1 P_{Xt}(j)x_t^i(j)dj$. The firm finances working capital in the form of trade credit prior to the realization of its revenues y_t^i.

Firm i buys and sells housing units as well as rents them out to households. The firm's income comes from profits and rents. Its flow-of-funds constraint is given by

$$d_t^i + p_t(h_{t+1}^i - h_t^i) = y_t^i - \int_0^1 P_{Xt}(j)x_t^i(j)dj + r_{ht}h_t^i, \; t \geq 0, \text{with } h_0^i \text{ given.} \tag{43}$$

The firm's objective (7) is to maximize the discounted present value of dividends.

In each period t, prior to sales of output and housing, firm i must borrow to finance its input costs. Intermediate-goods producers extend trade credit to the firm at the beginning of period t and allows it to pay input costs at the end of the period using revenues from sales of output and housing. The firm has limited commitment and may default on the trade credit. In the event of default, the firm would retain its production income y_t^i as well as its house holdings h_t^i. But the firm would be denied access to financial markets in the future. In particular, it would be barred from selling any asset holdings for profit and from obtaining loans for working capital. The following incentive compatibility constraint is imposed on the firm's optimization problem to make the contract self-enforceable:

$$V_t(h_t^i, a_t^i) \geq \left(y_t^i + r_{ht}h_t^i\right) + \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} V_{t+1}^a\left(h_t^i\right), \text{ all } t, \tag{44}$$

where $V_t(h_t^i, a_t^i)$ denotes the firm's value without default and $V_t^a(h_t^i)$ denotes the firm's value in the default state. As discussed in Section II, equation (10) still holds.

IV.4. **Equilibrium.** The markets clear for the housing sector and the intermediate-goods sector:

$$\int h_t^i di = H_t = 1, \quad \int x_t^i(j) di = X_t\left(j\right) = A_t K_t\left(j\right)^\alpha N_t\left(j\right)^{1-\alpha}.$$

Since the equilibrium is symmetric for intermediate-goods producers, we have

$$P_{Xt}(j) = P_{Xt},\ N_t(j) = N_t,\ K_t(j) = u_t K_t,\ X_t\left(j\right) = X_t = A_t\left(u_t K_t\right)^\alpha N_t{}^{1-\alpha},$$

for all j. The household's dividend income is $D_t = \int_0^1 d_t^i di$.

A competitive equilibrium consists of price sequences $\{w_t, r_{ht}, r_{kt}, p_t, b_t, R_{ft}, P_{Xt}\}_{t=0}^\infty$, allocation sequences $\{C_t, I_t, u_t, N_t, Y_t, B_{t+1}, K_{t+1}, X_t\}_{t=0}^\infty$ and a cutoff productivity sequence $\{a_t^*\}_{t=0}^\infty$, such that (1) given the prices, the allocations and cutoff productivity solve the optimizing problems for the households, intermediate-goods producers, and final-good firms; and (2) all the markets clear. Appendices B–D present all the details of characterizing and solving the equilibrium.

V. Empirical Analysis

The purpose of building the medium-scale model in the preceding section is to explain and understand, through the lenses of the structural model, house price-rent fluctuations over U.S. business cycles. To this end, we take the Bayesian approach and fit the log-linearized model to the six key U.S. time series over the period from 1987:Q1 to 2013:Q4: the price of house, the rent of house, the quality-adjusted relative price of investment, real per capita consumption, real per capita investment (in consumption units), and per capita hours worked. Appendix E presents the detailed description of the data and Appendix F provides the details of the estimation method.

V.1. **Parameter Estimates.** Our structural model fits the data remarkably well and is competitive against the Minnesota-prior BVAR. The model's marginal data density is $2,082$ in log value, while the BVAR's marginal data density is $2,078$ in log value. Following Smets and Wouters (2007), the empirical DSGE literature has used the Minnesota prior as the benchmark for the BVAR model.[11] Along with 90% probability bounds, Table 1 reports the estimates of key structural parameters and Table 2 reports the estimates of exogenous shock processes.

According to Table 1, the estimated inverse Frisch elasticity of labor supply is about 1.0, consistent with ranges of values discussed in the literature (Keane and Rogerson, 2011). The estimated hazard rate η is high, implying both a significant heterogeneity in firms' productivities and the importance of endogenous TFP. This large value, along with the estimated value $\mu = 0.148$ through steady state relationships, implies that the condition $\mu\eta > 1+\mu$ is satisfied. The steady-state elasticity of capacity utilization δ''/δ' is 4.0 (greater than the value discussed in the literature (Jaimovich and Rebelo, 2009)), suggesting that the effect of capacity utilization on output fluctuations is small and that our model does not have to rely on variable capacity utilization to fit the data. In a similar way, the estimated habit formation γ and capital-adjustment cost Ω are very small in magnitude. These factors are not a driving force for the dynamics of consumption and investment. The posterior probability intervals reported in Table 1 indicate that all these structural parameters are tightly estimated.

Table 2 reports the estimated persistence and standard-deviation parameters of exogenous shock processes. Among all shocks, the liquidity premium shock is the most persistent process. Other persistent shocks include the technology shock, the housing demand shock, and the labor supply shock. But the estimated standard deviation for the liquidity premium shock process is substantially smaller than those for all other shock processes. Indeed, the unconditional standard deviation for the liquidity premium shock process is only 0.0058. By contrast, the unconditional standard deviation is 0.0198 for housing demand, 0.0175 for stationary aggregate technology, and 0.0770 for labor supply. According to the 90% error bounds, the differences are both economically and statistically significant. The error bounds for the estimated standard deviation of the liquidity premium shock process are particularly tight. Such a small standard deviation implies that any large effects on asset prices and real aggregate variables must come from the model's internal propagation mechanism, which will be discussed in Section VI.

V.2. **Dynamic Impact.** In this subsection we discuss the dynamic impact on key financial and real variables of four most relevant shocks: a liquidity premium shock, a housing demand shock, a stationary technology shock, and a labor supply shock. The primary

[11] Sims and Zha (1998) propose a comprehensive prior that takes into account the feature of unit roots and cointegration inherent in the data. Our medium-scale model does not fit to the data as well as the BVAR with the Sims and Zha (1998) prior. Model comparison, however, is not the main purpose of our exercise as we can always improve the fit by making the exogenous processes more complicated than the simple AR(1) processes (see Smets and Wouters (2007)).

empirical finding is as follows. Although the estimated volatility of a shock to the liquidity premium is many times in magnitude less than the estimated volatilities of shocks to housing demand, technology, and labor supply, it accounts for most of the interaction between price-rent dynamics and real aggregate fluctuations. By comparison, shocks to housing demand, technology, and labor demand are all unable to generate large price-rent fluctuations.

Table 3 reports variance decompositions by the contributions from these four shocks for key financial and real variables (in log level) over the 24-quarter forecast horizon.[12] The stationary technology shock explains a majority of output fluctuations on impact (64.77%), but over the longer horizon the liquidity premium shock dominates the technology shock in explaining output fluctuations (reaching more than 30% at the end of the sixth-year horizon). The labor supply shock explains most of the hours fluctuation but not much of the output fluctuation. The housing demand shock affects only the house rent; and its contribution to rent fluctuations declines steadily over time from 59% on impact to 20% at the end of the forecast horizon. In Liu, Wang, and Zha (2013), the housing demand shock is important in explaining fluctuations of real variables. Once one takes into account the observation that the house price is more volatile than the house rent, a shock to housing demand no longer plays a role in real business cycles.

Figure 5-8 report the impulse responses (in log level) to all four shocks. The estimated dynamic response of the house rent to a housing demand shock is substantially higher than the corresponding response of the house price, making the fluctuations in the house price in relation to the rent inconsistent with the data (Figure 2 versus Figure 5). Moreover, since the housing demand shock has no impact on the other variables in the model, we do not display them in Figure 5. The intuition for this result has been explained in Section III.2.

Shocks to the labor supply and technology also fail to generate the price-rent fluctuation in magnitude comparable to the data. As shown in Figures 6 and 7, a labor supply shock produces simultaneous responses of rent and price almost one for one, while a technology shock generates exactly one-for-one responses. A labor supply shock has a much stronger impact on hours than a technology shock, but its dynamic impact on all other real variables is weaker. The response of output to a labor supply shock comes mostly from the response of hours, while a technology shock has a direct impact on output. Both shocks generate a much weaker response of endogenous TFP than the output response.

By contrast, a shock to the liquidity premium drives most fluctuations in both endogenous TFP and the house price without a significant effect on the rent fluctuation (Figure 8). Thus, this shock is capable of generating a majority of price-rent fluctuations. These results are

[12] We do not report the error bounds on variance decompositions for reasons articulated in Sims and Zha (1999). The error bands are best reported for the corresponding impulse responses.

remarkable given the very small standard deviation of this shock process as compared with other shock processes.

As explained in Section III, endogenous TFP is a primary transmission channel for the significant effect of the liquidity premium on aggregate output to take place. A more important factor is the strong propagation effect generated by a liquidity premium shock, as shown in Figure 8. Despite our assumption that the liquidity premium shock process is AR(1), the house price rises on impact and continues to rise over time in response to the shock. This large hump-shaped response[13] is generated entirely by the model's internal mechanism, which will be discussed in Section VI. The rent response is much smaller by comparison. As a result, a shock to the liquidity premium generates large price-rent dynamics. The response of endogenous TFP is strong on impact and stays elevated, while the response of aggregate output exhibits a large hump shape. Unlike the calibrated simple model in Section III, the response of hours here is positive. We will discuss the intuition behind this result in Section VI.

In summary, a technology shock has a direct and significant effect on output, but it causes endogenous TFP to fall (Figure 7). We will explain the latter result further in Section VI. Even though there is a hump-shaped response of consumption, the output response rises on impact and declines steadily (no hump shape). In comparison with the effect of a liquidity premium shock, the investment response to a technology shock rises more significantly on impact but declines more rapidly in subsequent periods (Figures 7 and 8). Labor supply and housing demand shocks have even less impact on consumption, investment, and output (Figures 5 and 6). Unlike a shock to the liquidity premium, these three shocks play almost no role in the price-rent fluctuation over the business cycle.

V.3. **Persistence and Volatility.** The Introduction provides empirical evidence through the lenses of the BVAR model. To be sure, the BVAR model we use does not identify any fundamental economic shock but rather provides a formal way of summarizing reduced-form empirical facts about volatility and persistence. One must therefore study all the columns of Figure 2, as there are three distinct facts evinced by this figure. First, output, house price, and house rent all have large hump-shaped responses, as shown by the diagonal of the 3×3 matrix of graphs in Figure 2. For our structural model, such large hump-shaped responses (especially the response of output) are identified as those to a liquidity premium shock (Figure 8). A shock to aggregate technology leads to a hump-shaped response of the house price, but the magnitude of volatility is too small compared to the price response to a liquidity premium shock (Figure 7 versus Figure 8).

[13]It is hump-shaped because the response is near the peak at the end of the forecast horizon and will eventually fall.

The second fact is the comovement between the house price and output, as shown in the first two graphs of the second column of Figure 2. Such a comovement can be generated by our structural model and is indeed captured by the dynamic responses to a liquidity premium shock (Figure 8). The third observation is that the house price is more volatile than not only output but also house rent. This observation is explained by the dynamic responses to a liquidity premium shock. The large response of the liquidity premium, as shown in Figure 8, is a driving force behind the large response of the house price *relative to the small response of the house rent.* Taking account of this salient fact has a profound implication on how we identify both a transmission channel and a propagation mechanism that are fundamentally different from what the previous literature that focuses on housing demand shocks has shown.

One of our important findings is that the estimated standard deviation for the liquidity premium shock process is considerably smaller than the estimated standard deviations for all other shock processes. A natural question is how much of the observed volatility is attributed to the liquidity premium shock. Table 4 reports the observed and model-generated volatilities of output, the house price, and the house rent. Using the estimates of model parameters, we simulate a sample of 112 periods (the same sample length as the data sample length) with only liquidity premium shocks. We repeat the simulation 100,000 times and compute the median volatility of output, the house price, and the house rent, along with 90% probability bounds. According to the median simulation, liquidity premium shocks alone can account for 56% of the observed output volatility, 23% of the observed house-rent volatility, and about 100% of the observed house-price volatility. These results offer a different perspective than the variance decomposition approach in gauging the significance of liquidity premium shocks and how it matters for the observed volatility. They affirm one of our key empirical findings: a liquidity premium shock is capable of generating a large house-price fluctuation without having a large effect on the house rent and the shock also exerts considerable influence on aggregate output fluctuation.

VI. Transmission Channel and Propagation Mechanism

Since all of our exogenous shocks are assumed to follow an AR(1) process, it is not surprising that we have the monotone responses in the simple model discussed in Section III. For our medium-scale structural model, therefore, it is all the more important to understand the inherent mechanism that generates hump-shaped impulse responses of both asset prices and real variables following a liquidity premium shock. With the presence of capital accumulation, households are now able to postpone their consumption by accumulating productive capital. This intertemporal substitution between current and future consumption contributes to the hump-shaped response of consumption even without habit (our estimate of habit is very small). Such a result is not new in the RBC literature.

What is new is that our medium-scale structural model identifies the source that accounts for the observed hump-shaped responses of the house price and output (Section V). Since our estimate of investment-adjustment costs is negligible, its contribution to the hump-shaped response of output is largely muted. Indeed, the dynamic response of output in response to an aggregate technology shock is monotone (Figure 7). By comparison, a monotone liquidity premium shock is capable of generating large hump-shaped responses of both asset prices and aggregate output. What is the transmission channel and what is the propagation mechanism?

To delve into intuitive answers, we begin with Figure 9. The figure plots the asset-pricing curve and the liquidity-constraint curve, which represent equations (17) and (24). These two equations continue to be the equilibrium conditions for our medium-scale structural model, except

$$\Lambda_t = \frac{\Theta_t}{C_t - \gamma C_{t-1}} - \beta\gamma E_t \frac{\Theta_{t+1}}{C_{t+1} - \gamma C_t}, \tag{45}$$

and the cutoff productivity level a_t^* is now determined in Appendix B. Now consider a positive stationary shock to aggregate technology. Point A in Figure 9 represents the initial equilibrium at the steady state. The technology shock increases aggregate output directly and hence shifts the liquidity-constraint curve upward. The rise of output has a positive wealth effect on consumption, shifting the asset-pricing curve upward as well. Since the direct effect of the aggregate technology shock on output is larger than the indirect effect on consumption, the cutoff productivity level declines and the equilibrium moves from Point A to Point B on impact.

In the subsequent period, an increase of consumption as a result of intertemporal substitution continues to shift the asset-pricing curve upward, but output drops (no hump shape) because the technology shock begins to decline. The direct output effect shifts the liquidity-constraint curve downward, resulting in a lower value of cutoff productivity and dampening the increase of the liquidity premium. The equilibrium moves from Point B to Point C in Figure 9. Over time, the direct output effect continues to dominate and the liquidity premium will begin to decline. Consequently, we see from Figure 7 the decline of cutoff productivity and no hump shape of the output response, even though the responses of consumption and the liquidity premium are hump-shaped.

By contrast, the dynamic impact of a positive liquidity premium shock presents a different picture. Figure 10, similar to Figure 3, has two panels. The top panel plots the production and labor-market curves. The bottom panel plots the asset-pricing and liquidity-constraint curves. We use these two panels to illustrate how the financial sector interacts with the real sector and how the interaction sheds light on the propagation mechanism that is lacking in

Section III. The production curve describes aggregate output

$$Y_t = (TFP_t)\, A_t\, (u_t K_t)^{\alpha}\, N_t^{1-\alpha}. \tag{46}$$

To derive the labor-market curve, we use the labor supply equation

$$\Lambda_t w_t = \Theta_t \psi_t N_t^{\nu} \tag{47}$$

and the labor demand equation

$$(1-\alpha) Y_t = \frac{\int_{a_t^*}^{\infty} \frac{a}{a_t^*} f(a) da}{1 - F(a_t^*)} w_t N_t \tag{48}$$

to eliminate w_t.[14] We then obtain the equation for the labor-market curve

$$N_t^{1+\nu} = \frac{1 - F(a_t^*)}{\int_{a_t^*}^{\infty} \frac{a}{a_t^*} f(a) da} \frac{(1-\alpha)\, Y_t \Lambda_t}{\Theta_t \psi_t}. \tag{49}$$

In contrast to Figure 3, the labor-market curve is upward sloping in Figure 10 because Y_t and Λ_t/Θ_t can no longer cancel each other out.

Suppose that the initial equilibrium is Point A at the steady state for both panels of Figure 10. According to equations (17) and (45), a positive shock delivers immediate impetus to the liquidity premium, shifting the asset-pricing curve upward and raising cutoff productivity. A rise in cutoff productivity increases aggregate output through endogenous TFP as the transmission channel. An increase in aggregate output causes the liquidity-constraint curve to shift upward [equation (24)]. The direct effect of the liquidity premium shock on asset prices dominates the indirect effect on aggregate output so that the net effect on cutoff productivity is positive (Figure 9 vs. the bottom panel of Figure 10). The equilibrium moves from Point A to Point B on impact, with an increase of both cutoff productivity and the liquidity premium.

As an increase of cutoff productivity raises aggregate output and thus shifts the production curve upward, it simultaneously shifts the labor-market curve upward so long as the term[15]

$$\frac{1}{1 - F(a_t^*)} \int_{a_t^*}^{\infty} \frac{a}{a_t^*} f(a)\, da$$

increases with a_t^* and the impact of Λ_t is relatively small. When the rise of the production curve dominates the rise of the labor-market curve, both output and hours increase with cutoff productivity a_t^* and the equilibrium moves from Point A to Point B on impact (the top panel).

In the simple model articulated in Section II, no matter how persistent the AR(1) process of the liquidity premium shock is, one cannot obtain a hump-shaped response of either

[14]The preceding three equations are derived in Appendix B.

[15]As shown in Section III.2, if $\mu\eta > (1+\mu)$, then this term increases with a_t^* up to the first-order approximation.

house price or aggregate output. With capital accumulation in our medium-scale model, it is optimal for households to postpone consumption for investment. Thus the hump-shaped response of consumption leads to a further upward shift of the asset-pricing curve in subsequent periods, pushing cutoff productivity higher. A higher cutoff productivity level, in turn, leads to higher endogenous TFP and higher aggregate output. As a result of higher aggregate output, the liquidity-constraint curve shifts further up, generating an even higher liquidity premium. As long as the liquidity premium shock is very persistent as is seen in our estimation, the effect on the asset-pricing curve is likely to continue to dominate the effect on the liquidity-constraint curve, moving the equilibrium from Point B to Point C (the bottom panel of Figure 10) with an increase in both liquidity premium and cutoff productivity.

At the same time, a higher cutoff productivity level shifts both the production curve and the labor-market curve further upward to support higher aggregate output while hours begin to decline, moving the equilibrium from Point B to Point C (the top panel of Figure 10). The propagation mechanism described here generates hump-shaped responses of both aggregate output and the liquidity premium. The real sector cannot be understood apart from the financial sector—both panels of Figure 10 are necessary for understanding the interaction between the two sectors.

VII. Conclusion

DSGE models studied in the previous literature have had difficulty in generating the result that the house price is much more volatile than the house rent, as is observed in the data. Overcoming this difficulty leads to a new economic mechanism that is fundamentally different from the previous literature on the dynamic links between house prices and real aggregate variables. The contribution of this paper is the development of such a mechanism that accounts for not only the observed price-rent dynamics in the housing market but also their impact on real business cycles. The mechanism is built on a dynamic general equilibrium model that confronts the U.S. time series. The estimated medium-scale structural model fits the data well, providing empirical support for the transmission channel and the propagation mechanism developed in this paper.

To make the findings and the mechanism transparent, our model abstracts from many other dimensions that merit further study in the future. One such dimension is the inclusion of mortgage markets and intertemporal loans in the model. Another dimension for expanding our model is the introduction of monetary and regulatory policies for studying potential roles of the government. We hope that the mechanism developed in the paper lays the groundwork for extending our model along these and other important dimensions.

Table 1. Posterior distributions of structural parameters

		Posterior estimates		
Parameter	Representation	Mode	Low	High
ν	Inverse Frisch	1.0229	0.6145	2.1178
η	Hazard rate	9.3134	8.2581	12.899
δ''/δ'	Capacity utilization	4.3031	1.6745	9.1139
γ	Habit formation	0.1079	0.0332	0.2724
Ω	Capital adjustment	0.0166	0.0040	0.0719

Note: "Low" and "high" denote the bounds of the 90% probability interval for each parameter.

Table 2. Posterior distributions of shock parameters

		Posterior estimates		
Parameter	Representation	Mode	Low	High
ρ_z	Permanent investment tech	0.1619	0.0880	0.2958
ρ_{ν_z}	Stationary investment tech	0.0168	0.0154	0.6733
ρ_a	Permanent neutral tech	0.9270	0.1803	0.9496
ρ_{ν_a}	Stationary neutral tech	0.9273	0.8359	0.9401
ρ_θ	Liquidity premium	0.9996	0.9973	0.9998
ρ_ξ	Housing demand	0.9380	0.8953	0.9730
ρ_ψ	Labor supply	0.9908	0.9758	0.9967
σ_z	Permanent investment tech	0.0054	0.0046	0.0060
σ_{ν_z}	Stationary investment tech	0.0001	0.0001	0.0018
σ_a	Permanent neutral tech	0.0005	0.0004	0.0040
σ_{ν_a}	Stationary neutral tech	0.0065	0.0058	0.0076
σ_θ	Liquidity premium	0.0002	0.0001	0.0002
σ_ξ	Housing demand	0.0069	0.0063	0.0079
σ_ψ	Labor supply	0.0104	0.0083	0.0170

Note: "Low" and "high" denote the bounds of the 90% probability interval for each parameter.

TABLE 3. Variance decompositions (%) of key financial and real variables

	Shock to			
Horizons (quarters)	Housing	Labor	Technology	Liquidity
	Price-rent ratio			
1	5.89	0.00	0.00	94.11
4	4.97	0.00	0.00	95.03
8	4.02	0.00	0.00	95.98
16	2.78	0.00	0.00	97.22
24	2.05	0.00	0.00	97.95
	Rent			
1	59.52	10.69	16.56	3.68
4	53.20	12.08	17.89	5.21
8	43.71	14.00	20.35	8.23
16	29.07	16.20	20.58	13.99
24	19.85	16.62	17.51	18.09
	Output			
1	0.00	15.84	64.77	19.19
4	0.00	16.96	59.68	22.04
8	0.00	17.99	52.47	25.32
16	0.00	18.48	38.82	29.69
24	0.00	17.68	28.36	31.65
	Cutoff productivity			
1	0.00	0.12	2.52	96.19
4	0.00	0.14	2.18	96.67
8	0.00	0.11	1.53	97.64
16	0.00	0.08	0.86	98.66
24	0.00	0.06	0.57	99.09
	Hours			
1	0.00	80.50	11.82	2.52
4	0.00	82.19	10.40	2.75
8	0.00	85.85	7.96	2.61
16	0.00	90.20	5.13	2.27
24	0.00	92.24	3.84	1.99

Note: Variance decompositions attributed to shocks to housing demand, labor supply, stationary aggregate technology, and liquidity premium.

TABLE 4. Key data volatilities explained by the liquidity premium shock (%)

		Liquidity premium shock		
Volatility	Data	Median	Low	High
$std\,(\Delta Y_t)$	0.744	0.428	0.382	0.478
$std\,(\Delta p_t)$	2.727	2.786	2.485	3.103
$std\,(\Delta r_{h,t})$	0.556	0.176	0.154	0.204

Note: "Low" and "high" denote the bounds of the 90% probability interval.

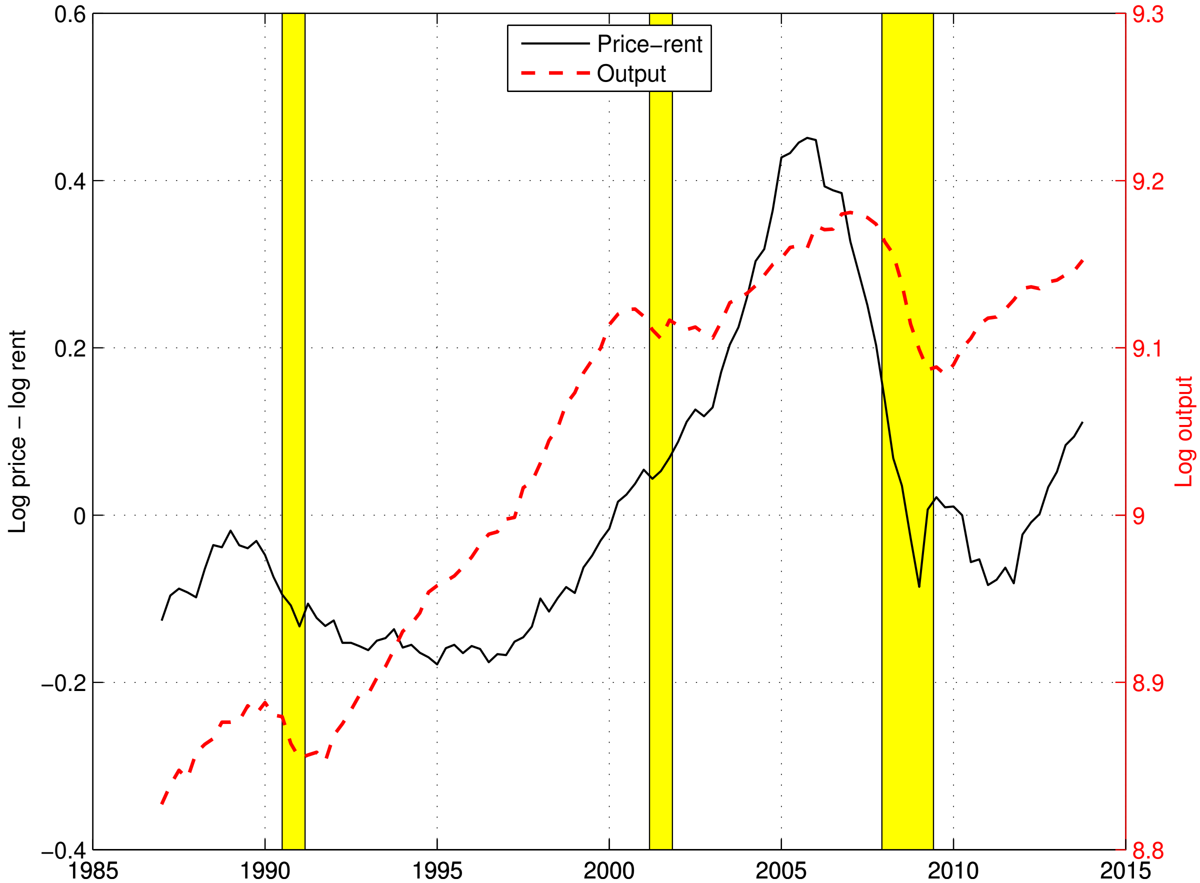

FIGURE 1. The time series of the log price-rent ratio in the U.S. housing sector (the left scale) and the time series of log output in the U.S. economy (the right scale).

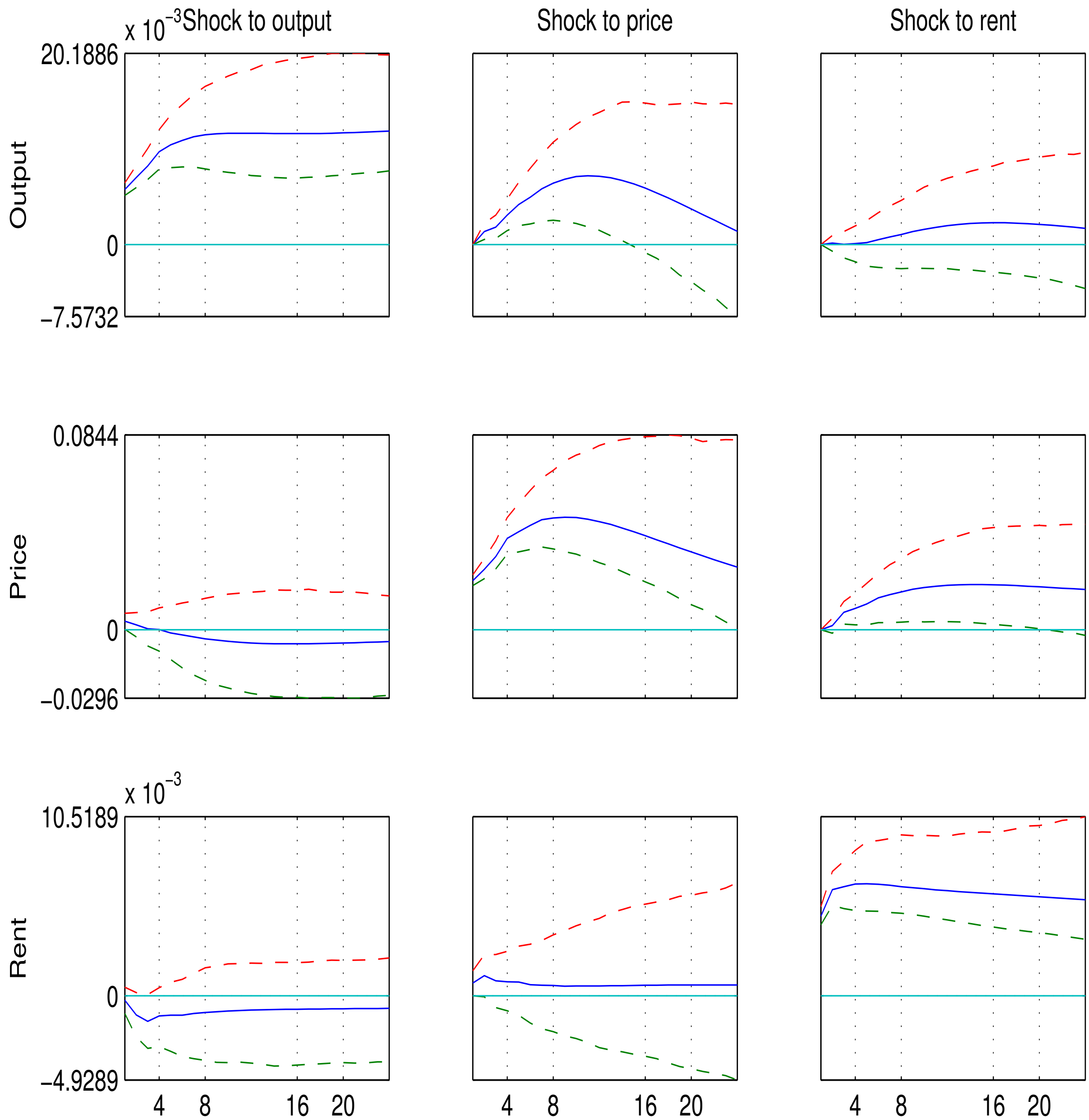

FIGURE 2. Impulse responses of output, house price, and house rent from an estimated BVAR model with Sims and Zha (1998)'s prior and with four lags. All the variables are expressed in log level. The shocks are orthogonalized with output ordered first, the house price second, and the house rent third. The solid lines represent the estimated results and the dashed lines demarcate the 90% error bands.

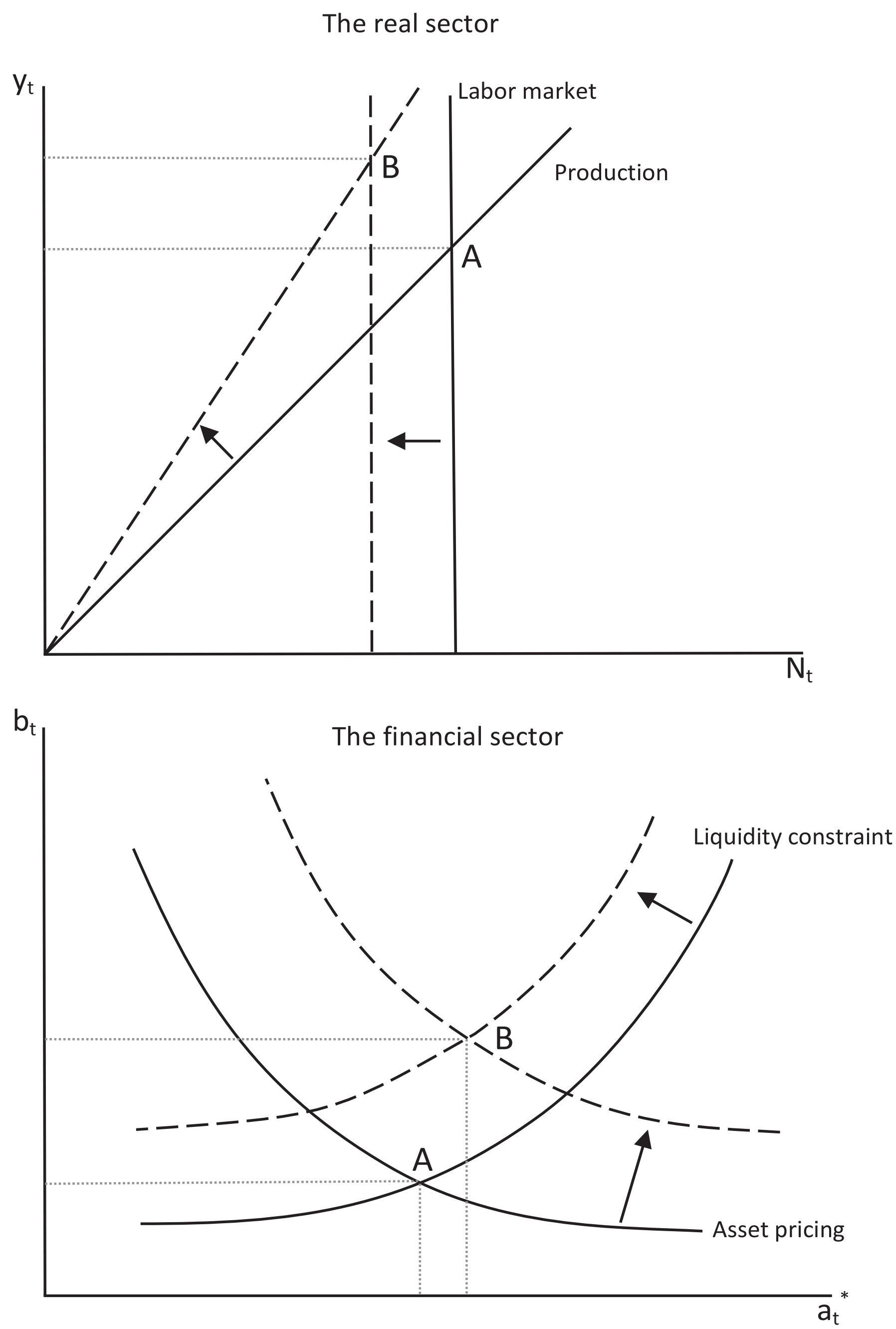

Figure 3. Impact of a positive liquidity premium shock: An illustration of the key economic mechanism. The production line represents equation (19) and the labor-market line represents equation (23). The asset-pricing and liquidity-constraint curves plot equations (17) and (24).

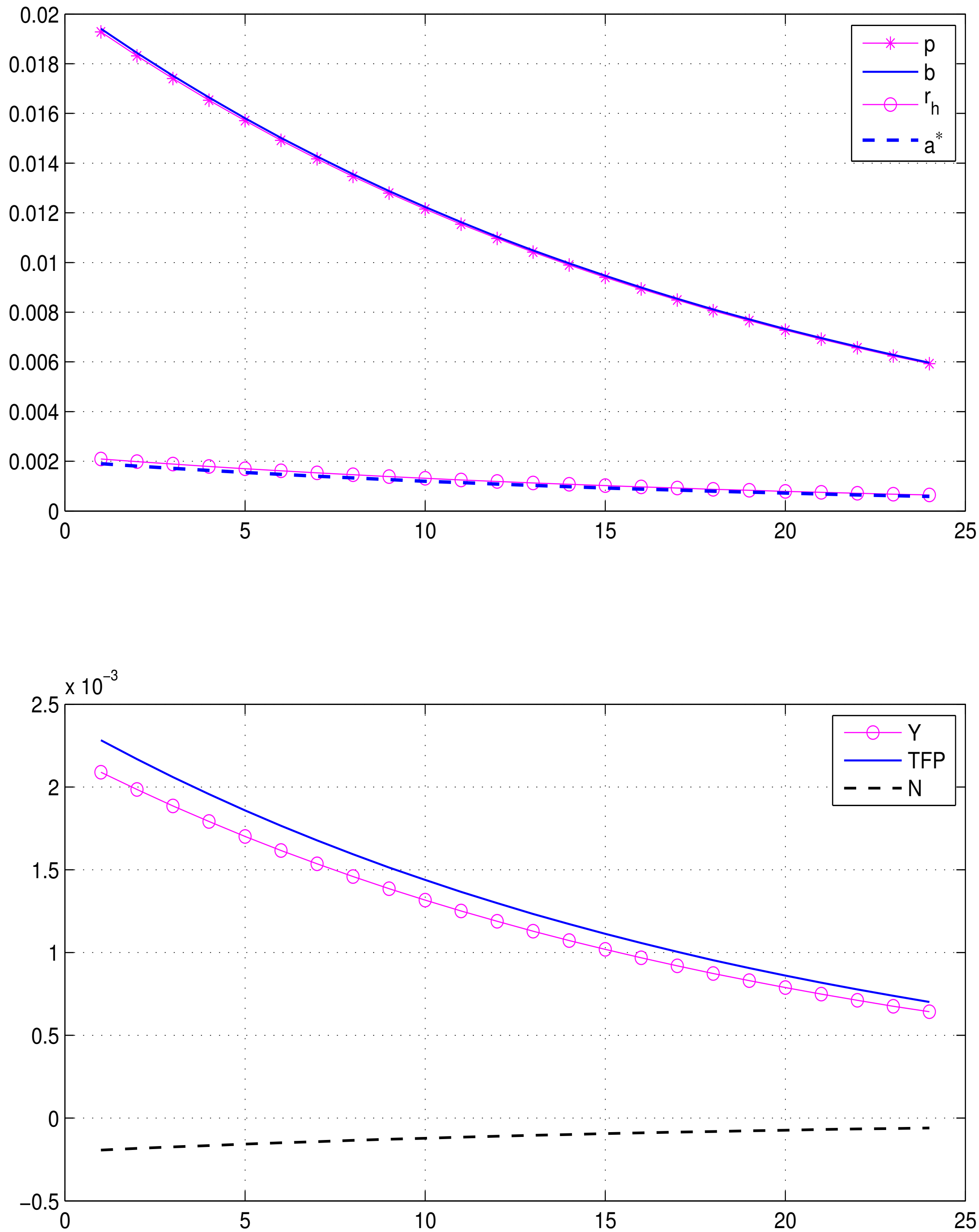

FIGURE 4. Calibrated impulse responses to a positive liquidity premium shock for the simple general equilibrium model without capital, where p is the house price, b is the liquidity premium, r_h is the house rent, a^* is the cutoff productivity level, Y is aggregate output, and N is labor hours.

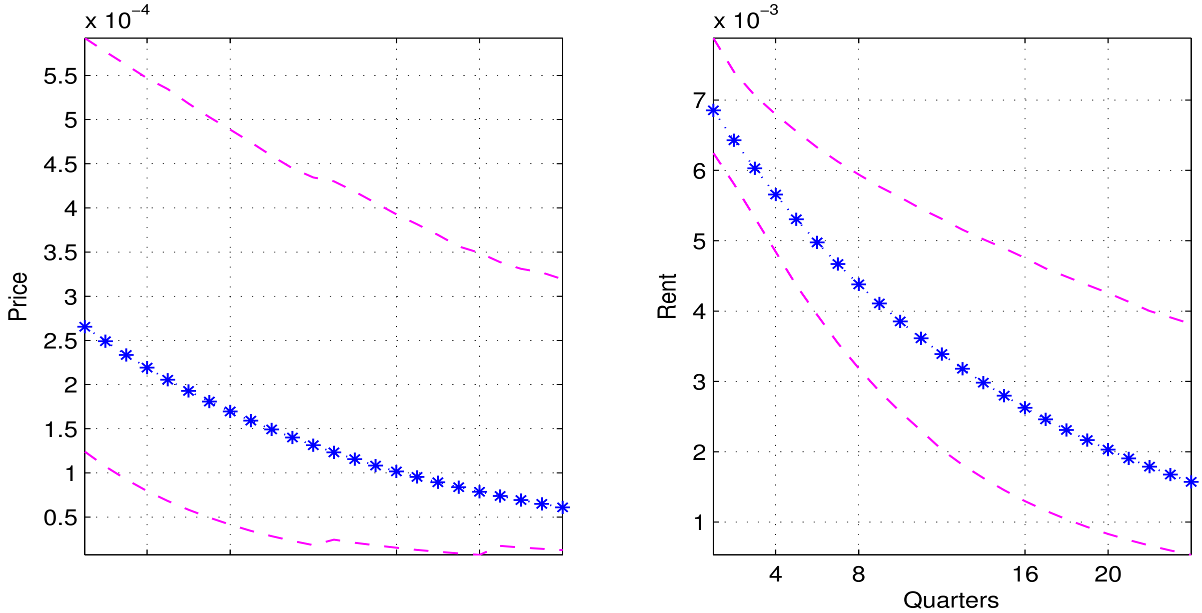

FIGURE 5. Impulse responses of key financial and real variables to a one-standard-deviation housing demand shock. The asterisk lines represent the estimated results and the dashed lines demarcate the 90% error bands.

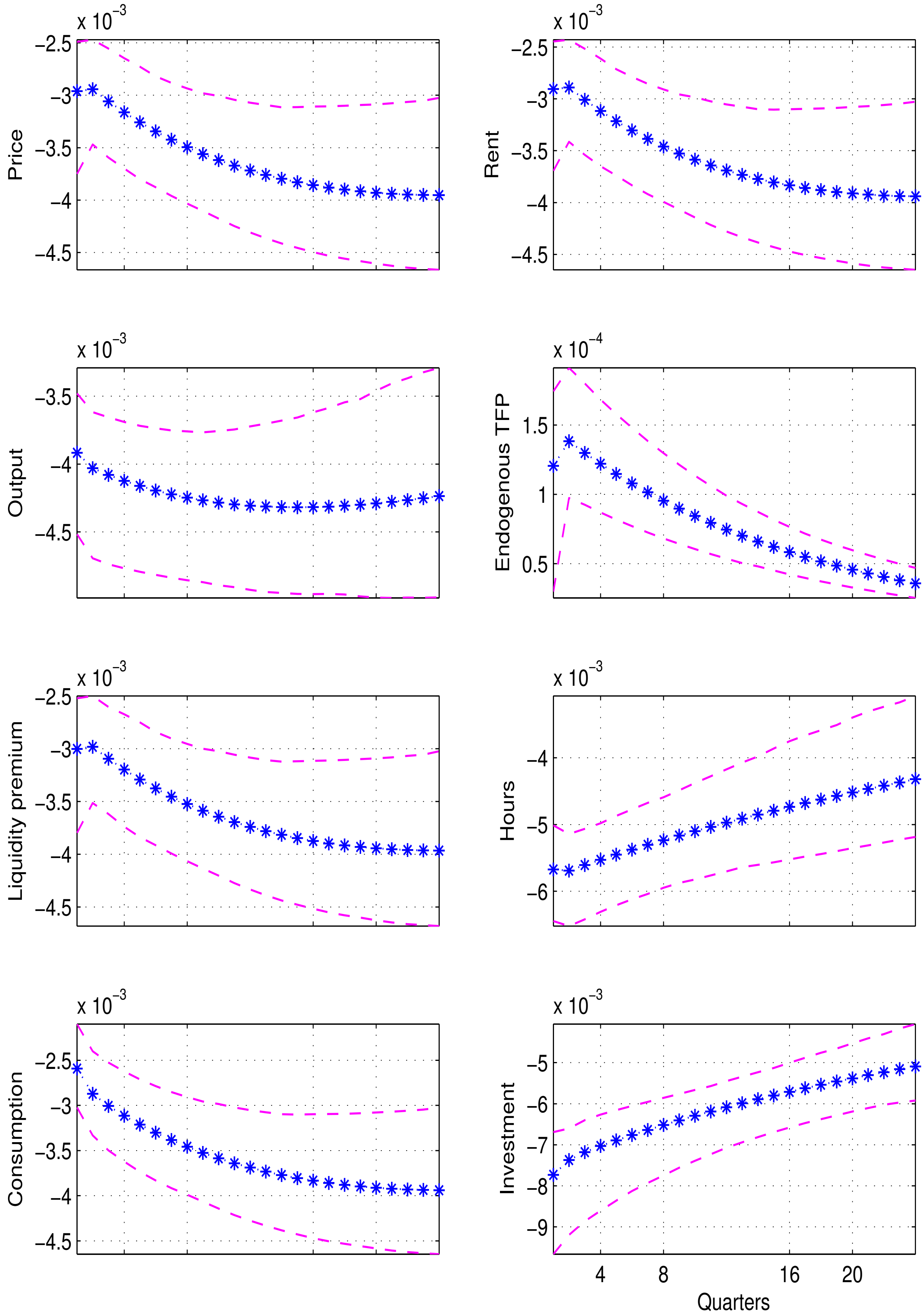

FIGURE 6. Impulse responses of key financial and real variables to a one-standard-deviation labor supply shock. The asterisk lines represent the estimated results and the dashed lines demarcate the 90% error bands.

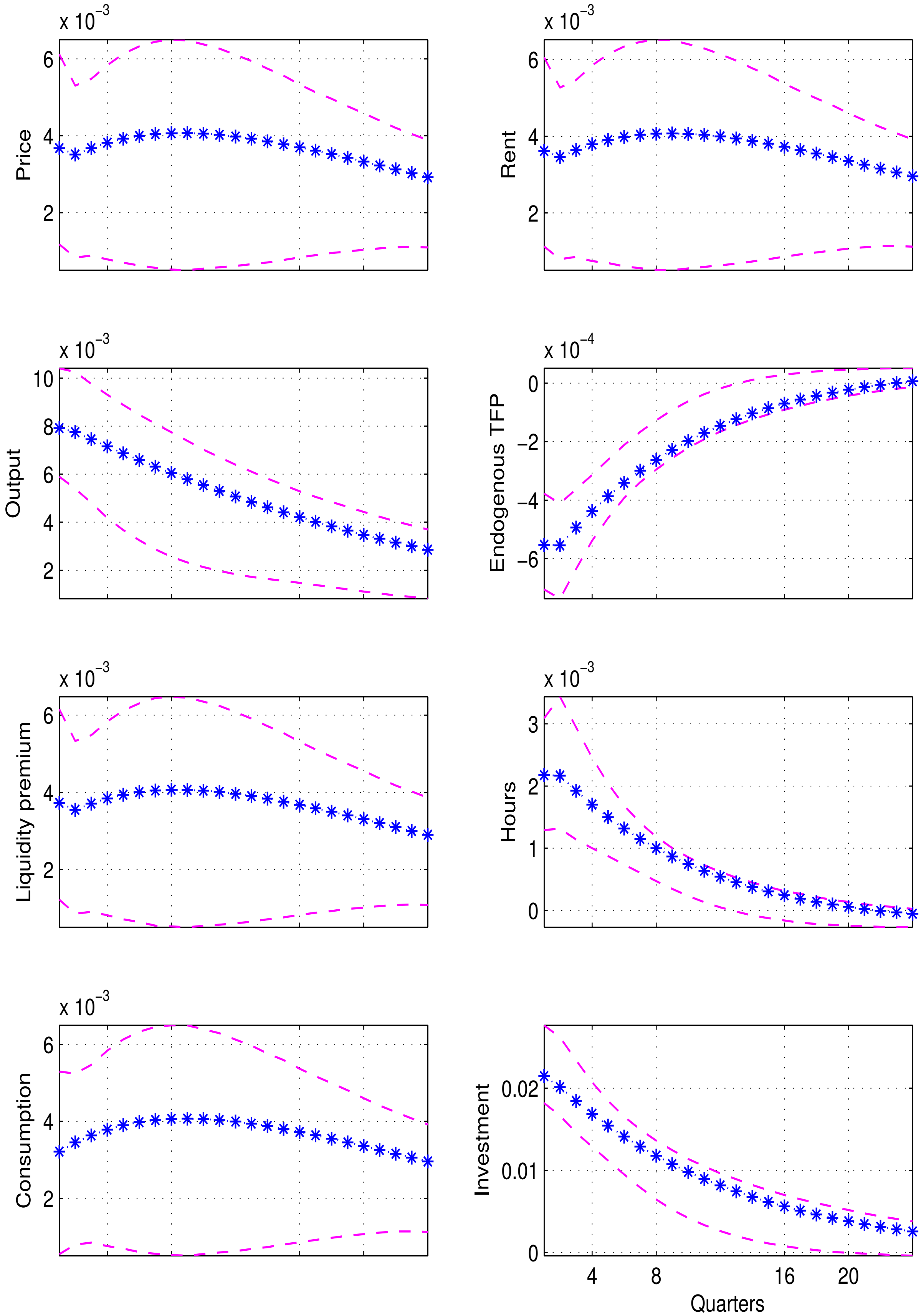

FIGURE 7. Impulse responses of key financial and real variables to a one-standard-deviation stationary technology shock. The asterisk lines represent the estimated results and the dashed lines demarcate the 90% error bands.

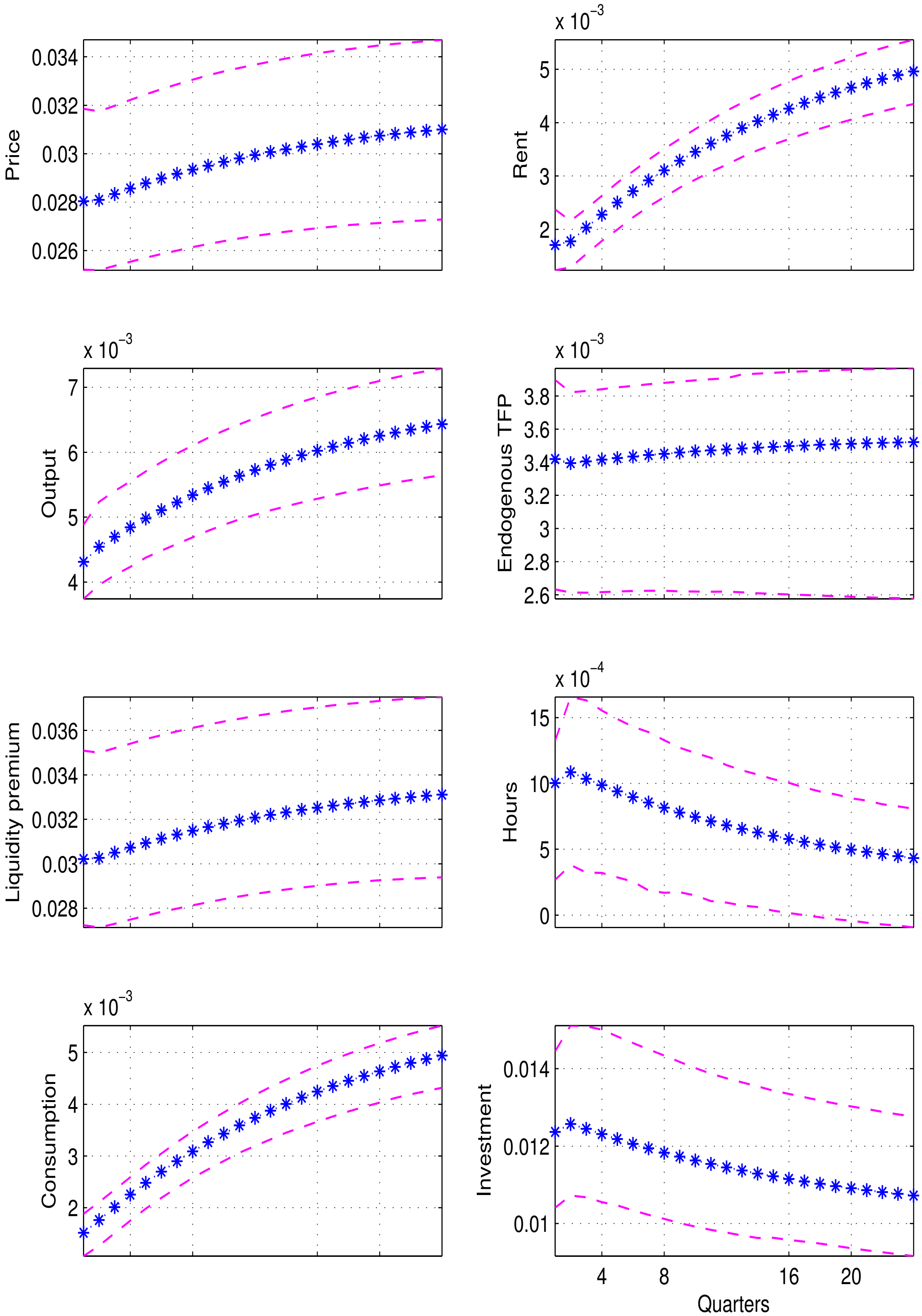

FIGURE 8. Impulse responses of key financial and real variables to a one-standard-deviation liquidity premium shock. The asterisk lines represent the estimated results and the dashed lines demarcate the 90% error bands.

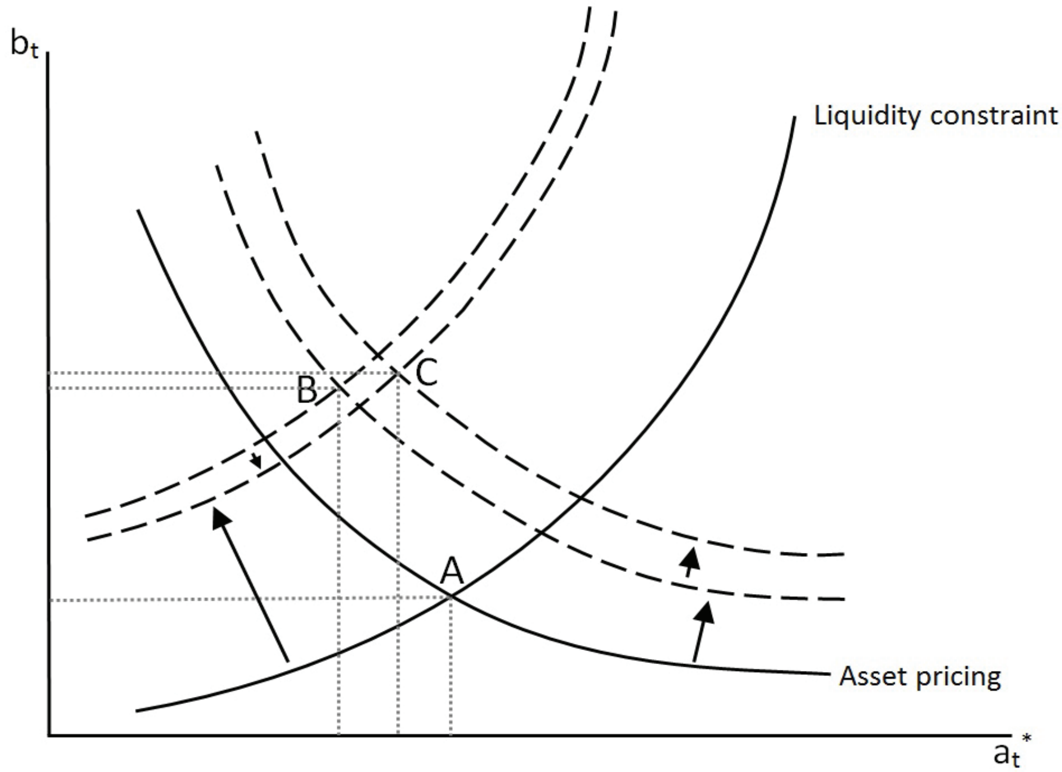

Figure 9. An illustration of the propagation mechanism that transmits a positive technology shock. The asset-pricing curve represents equation (17) and the liquidity-constraint curve represents equation (24).

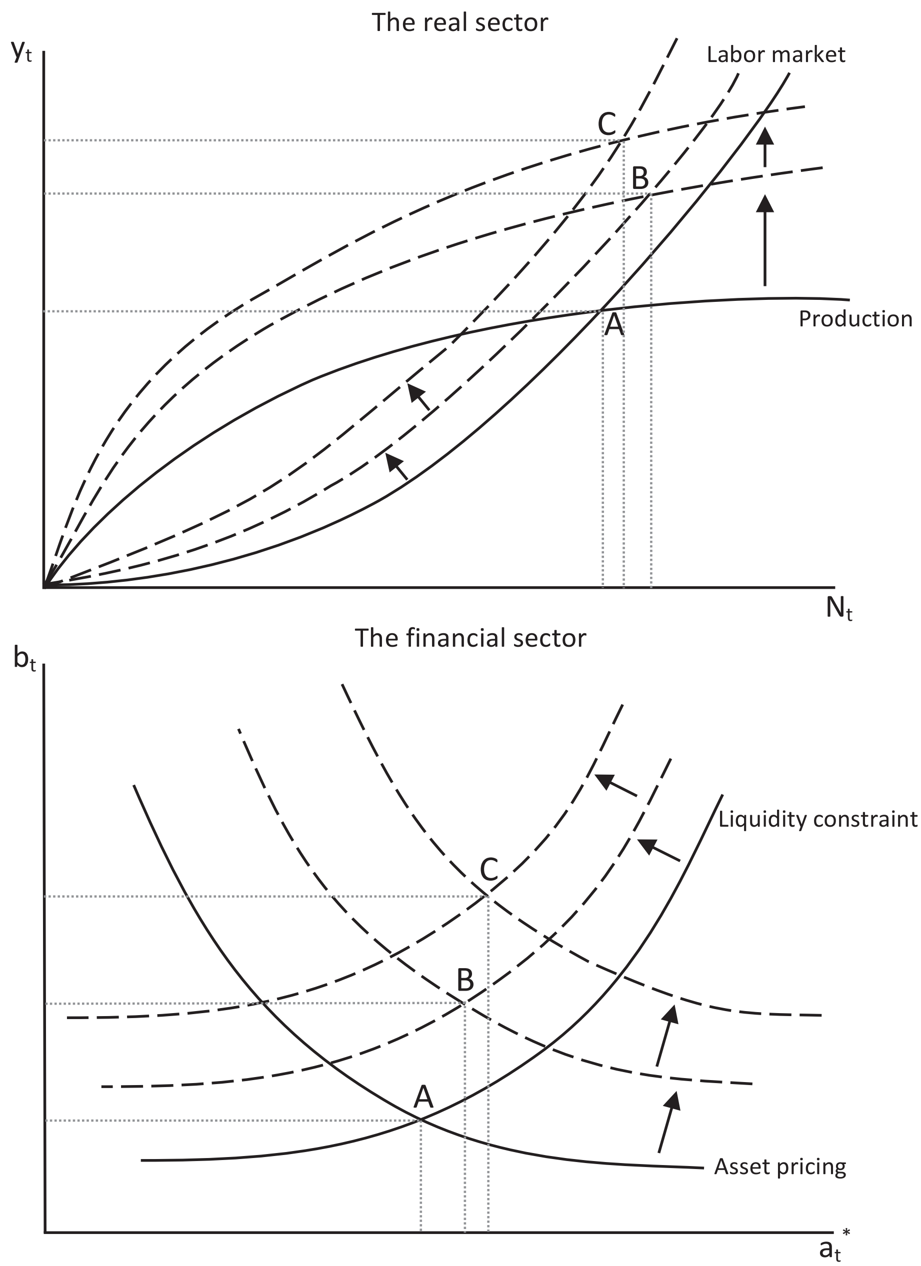

Figure 10. An illustration of the propagation mechanism that transmits a positive liquidity premium shock. The production and labor-market curves represent equations (46) and (49). The asset-pricing and liquidity-constraint curves represent equations (17) and (24).

Appendix A. Proofs of Propositions 1-3

We conjecture that the value function takes the form $V_t\left(h_t^i, a_t^i\right) = v_t\left(a_t^i\right) h_t^i$, where $v_t\left(a_t^i\right)$ satisfies (13). Using the Bellman equation (12), we can rewrite the incentive constraint (9) as follows

$$d_t^i + \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} V_{t+1}(h_{t+1}^i, a_{t+1}^i) \geq a_t^i A_t n_t^i + \left(r_{ht} + p_t^a\right) h_t^i.$$

Given the conjectured value function and equations (8), (10), and (13), we can rewrite this constraint as

$$a_t^i A_t n_t^i - w_t n_t^i + r_{ht} h_t^i + p_t h_t^i \geq a_t^i A_t n_t^i + \left(r_{ht} + p_t^a\right) h_t^i.$$

Simplifying the proceeding inequality yields the constraint (14).

Substituting the conjectured value function into the Bellman equation (12) yields

$$v_t\left(a_t^i\right) h_t^i = \max_{n_t^i, h_{t+1}^i} a_t^i A_t n_t^i - w_t n_t^i + r_{ht} h_t^i - p_t\left(h_{t+1}^i - h_t^i\right) + p_t h_{t+1}^i.$$

Simplifying yields

$$v_t\left(a_t^i\right) h_t^i = \max_{n_t^i} \; \left(a_t^i A_t - w_t\right) n_t^i + r_{ht} h_t^i + p_t h_t^i.$$

When $a_t^i \geq a_t^* = w_t/A_t$, the credit constraint (14) binds. Thus the preceding equation implies that

$$v_t\left(a_t^i\right) = \begin{cases} \left(a_t^i A_t - w_t\right) \frac{b_t}{w_t} + r_{ht} + p_t & \text{if } a_t^i \geq a_t^* \\ r_{ht} + p_t & \text{otherwise} \end{cases}. \tag{A1}$$

We also obtain the optimal labor choice in (15). Finally, we substitute (A1) into (13) and obtain (16). Using (11) and $b_t = p_t - p_t^a$, we obtain (17).

By equations (5), (16), and (17), we can derive that

$$\frac{\pi_t}{\Lambda_t} = \beta E_t b_{t+1} \frac{\Lambda_{t+1}}{\Lambda_t} \int_{a_{t+1}^*} \frac{a - a_{t+1}^*}{a_{t+1}^*} f(a) da.$$

If $b_t > 0$ for all t, then $\pi_t > 0$. It follows from the complementary slackness condition $\pi_t h_{ot+1} = 0$ that the household will not possess housing units, i.e., $h_{ot+1} = 0$ whenever $b_t > 0$ for all t.

Appendix B. Equilibrium System for the Medium-Scale Model

The representative household chooses consumption, labor supply, investment, capital, and capacity utilization in order to maximize (36). The first-order conditions are given by

$$\Lambda_t = \frac{\Theta_t}{C_t - \gamma C_{t-1}} - \beta\gamma E_t \frac{\Theta_{t+1}}{C_{t+1} - \gamma C_t}, \tag{A2}$$

$$r_{ht} = \frac{\Theta_t \xi_t}{\Lambda_t}, \tag{A3}$$

$$\Lambda_t w_t = \Theta_t \psi_t N_t^{\nu}, \tag{A4}$$

$$\begin{aligned}\frac{1}{Z_t} &= Q_{kt}\left[1-\frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}}-\bar{g}_I\right)^2-\Omega\left(\frac{I_t}{I_{t-1}}-\bar{g}_I\right)\frac{I_t}{I_{t-1}}\right] \\ &\quad +\beta E_t\frac{\Lambda_{t+1}}{\Lambda_t}Q_{kt+1}\Omega\left(\frac{I_{t+1}}{I_t}-\bar{g}_I\right)\frac{I_{t+1}^2}{I_t^2}, \end{aligned} \tag{A5}$$

$$Q_{kt}=\beta E_t\frac{\Lambda_{t+1}}{\Lambda_t}(u_{t+1}r_{kt+1}+(1-\delta)Q_{kt+1}), \tag{A6}$$

$$r_{kt}=\delta'(u_t)Q_{kt}, \tag{A7}$$

$$\frac{1}{R_{ft}}=\beta E_t\frac{\Lambda_{t+1}}{\Lambda_t}. \tag{A8}$$

These equations admit the usual interpretations. Note that we have imposed the market clearing condition $H_t = 1$ in (A3).

The first-order conditions for the intermediate goods producers are given by

$$\alpha P_{Xt}(j)A_tK_t\left(j\right)^{\alpha-1}N_t\left(j\right)^{1-\alpha}=r_{kt}, \tag{A9}$$

and

$$(1-\alpha)P_{Xt}(j)A_tK_t\left(j\right)^{\alpha}N_t\left(j\right)^{-\alpha}=w_t. \tag{A10}$$

Now we compute that

$$\begin{aligned}E_t\frac{\beta\Lambda_{t+1}}{\Lambda_t}V_{t+1}^a\left(h_t^i\right) &= E_t\frac{\beta\Lambda_{t+1}}{\Lambda_t}r_{ht+1}h_t^i+E_t\frac{\beta\Lambda_{t+2}}{\Lambda_t}r_{ht+2}h_t^i+... \\ &= p_t^a h_t^i,\end{aligned}$$

where p_t^a satisfies the recursive equation

$$p_t^a=\beta E_t\frac{\Lambda_{t+1}}{\Lambda_t}\left[r_{ht+1}+p_{t+1}^a\right]. \tag{A11}$$

We write firm i's decision problem by dynamic programming

$$V_t(h_t^i,a_t^i)=\max_{x_t^i(j),h_{t+1}^i\geq 0} d_t^i+\beta E_t\frac{\Lambda_{t+1}}{\Lambda_t}V_{t+1}(h_{t+1}^i,a_{t+1}^i), \tag{A12}$$

subject to (43) and (44).

To solve the entrepreneur's decision problem, we first derive the unit cost of production. Define the total cost of producing y_{it} as

$$\Phi(y_t^i,a_t^i)\equiv\min_{x_t^i(j)}\int P_{Xt}(j)x_t^i(j)dj, \tag{A13}$$

subject to $a_t^i\exp\left(\int\log x_t^i(j)dj\right)\geq y_t^i$. Cost-minimization implies that

$$\Phi(y_t^i,a_t^i)=y_t^i\frac{a_t^*}{a_t^i}, \tag{A14}$$

where the term a_t^* is given by

$$a_t^*\equiv\exp\left[\int\log P_{Xt}(j)dj\right], \tag{A15}$$

and the demand for each $x_t^i(j)$ satisfies

$$P_{Xt}(j)x_t^i(j) = a_t^* \exp\left(\int \log x_t^i(j)dj\right). \tag{A16}$$

Using the cost function in (A14), we can rewrite entrepreneur i's budget constraint as

$$d_t^i + p_t(h_{t+1}^i - h_t^i) \leq y_t^i - y_t^i \frac{a_t^*}{a_t^i} + r_{ht}h_t^i. \tag{A17}$$

Conjecture that

$$V_t\left(h_t^i, a_t^i\right) = v_t\left(a_t^i\right) h_t^i,$$

where $v_t\left(a_t^i\right)$ satisfies

$$\beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} v_{t+1}(a_{t+1}^i) = p_t. \tag{A18}$$

We can also rewrite the credit constraint (44) as

$$y_t^i \frac{a_t^*}{a_t^i} \leq b_t h_t^i, \tag{A19}$$

where $b_t = p_t - p_t^a$ represents a liquidity premium.

Substituting the preceding conjecture and (A17) into the Bellman equation (A12), we obtain

$$v_t\left(a_t^i\right) h_t^i = \max_{y_t^i} \; y_t^i \left(1 - \frac{a_t^*}{a_t^i}\right) + r_{ht}h_t^i - p_t(h_{t+1}^i - h_t^i) + p_t h_{t+1}^i,$$

subject to (A19). We then obtain the optimal output choice

$$y_t^i = \begin{cases} \frac{a_t^i}{a_t^*} b_t h_t^i & \text{if } a_t^i \geq a_t^* \\ 0 & \text{otherwise} \end{cases}. \tag{A20}$$

Substituting this decision rule back into the Bellman equation and matching coefficients, we obtain

$$v_t\left(a_t^i\right) = \begin{cases} \left(\frac{a_t^i}{a_t^*} - 1\right) b_t + r_{ht} + p_t & \text{if } a_t^i \geq a_t^* \\ r_{ht} + p_t & \text{otherwise} \end{cases}.$$

Substituting this expression into (A18) we obtain

$$p_t = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \left[r_{ht+1} + p_{t+1} + b_{t+1} \int_{a_{t+1}^*} \frac{a - a_{t+1}^*}{a_{t+1}^*} f(a)da \right]. \tag{A21}$$

By (A11) and (A21),

$$b_t = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} b_{t+1} \left[1 + \int_{a_{t+1}^*} \frac{a - a_{t+1}^*}{a_{t+1}^*} f(a)da \right]. \tag{A22}$$

The usual transversality conditions hold.

Equation (A15) implies

$$a_t^* = P_{Xt}. \tag{A23}$$

We hence have

$$\alpha P_{Xt}\frac{X_t}{u_t K_t} = r_{kt}, \tag{A24}$$

$$(1-\alpha)\, P_{Xt}\frac{X_t}{N_t} = w_t, \tag{A25}$$

and the resource constraint:

$$C_t + \frac{I_t}{Z_t} = Y_t. \tag{A26}$$

By the market-clearing conditions and (A20), aggregate output is given by

$$Y_t = \int y_t^i di = \int_{a_t^i \geq a_t^*} \frac{a_t^i}{a_t^*} b_t h_t^i di = \frac{b_t}{a_t^*}\int_{a_t^*}^{\infty} a f(a) da. \tag{A27}$$

By the market-clearing conditions, (42), (A16), and (44), the total production cost is given by

$$P_{Xt}X_t = \int P_{Xt} x_t^i(j) di dj = \int_{a_t^i \geq a_t^*} \frac{a_t^*}{a_t^i} y_t^i di = b_t \left[1 - F\left(a_t^*\right)\right]. \tag{A28}$$

Using the fact that $P_{Xt} = a_t^*$ and $X_t = A_t\left(u_t K_t\right)^{\alpha} N_t^{1-\alpha}$, we can derive that

$$b_t = \frac{a_t^* A_t \left(u_t K_t\right)^{\alpha} N_t^{1-\alpha}}{1 - F(a_t^*)}. \tag{A29}$$

Using this equation, we can rewrite aggregate output in (A27) as

$$Y_t = A_t\left(u_t K_t\right)^{\alpha} N_t^{1-\alpha} \frac{\int_{a_t^*}^{\infty} a f(a) da}{1 - F(a_t^*)}, \tag{A30}$$

where the last expectation is taken with respect to the density f and gives the endogenously determined TFP.

By (A24) and (A25),

$$r_{kt} u_t K_t = \alpha A_t a_t^* \left(u_t K_t\right)^{\alpha} N_t^{1-\alpha} = \frac{\alpha Y_t}{\frac{1}{1-F(a_t^*)}\int_{a_t^*}^{\infty} \frac{a}{a_t^*} f(a) da}, \tag{A31}$$

and

$$w_t N_t = (1-\alpha) A_t a_t^* \left(u_t K_t\right)^{\alpha} N_t^{1-\alpha} = \frac{(1-\alpha) Y_t}{\frac{1}{1-F(a_t^*)}\int_{a_t^*}^{\infty} \frac{a}{a_t^*} f(a) da}. \tag{A32}$$

Define

$$\mu_t + 1 \equiv \frac{1}{1-F(a_t^*)}\int_{a_t^*}^{\infty} \frac{a}{a_t^*} f(a) da > 1.$$

A competitive equilibrium consists of 15 stochastic processes for $\{K_t\}$, $\{\Lambda_t\}$, $\{N_t\}$, $\{I_t\}$, $\{Q_{kt}\}$, $\{u_t\}$, $\{p_t\}$, $\{b_t\}$, $\{C_t\}$, $\{a_t^*\}$, $\{Y_t\}$, $\{r_{kt}\}$, $\{r_{ht}\}$, $\{R_{ft}\}$, and $\{w_t\}$ such that a system of 15 equations hold: (40), (A2)-(A8), (A21), (A22), (A26), (A27), (A30), (A31), and (A32). Note that equation (A15) is implied by equations (A30), (A31), and (A32). The usual transversality conditions also hold.

Appendix C. Stationary Equilibrium

We make the following transformations of the variables:

$$\tilde{C}_t \equiv \frac{C_t}{\Gamma_t},\ \tilde{I}_t \equiv \frac{I_t}{Z_t\Gamma_t},\ \tilde{Y}_t \equiv \frac{Y_t}{\Gamma_t},\ \tilde{K}_t \equiv \frac{K_t}{\Gamma_{t-1}Z_{t-1}},$$
$$\tilde{w}_t \equiv \frac{w_t}{\Gamma_t},\ \tilde{r}_{ht} \equiv \frac{r_{ht}}{\Gamma_t},\ \tilde{p}_t \equiv \frac{p_t}{\Gamma_t},\ \tilde{b}_t \equiv \frac{b_t}{\Gamma_t},$$
$$\tilde{r}_{kt} \equiv r_{kt}Z_t,\ \tilde{Q}_{kt} \equiv Q_{kt}Z_t,\ \tilde{\Lambda}_t \equiv \frac{\Lambda_t}{\Theta_t}\Gamma_t,$$

where $\Gamma_t = Z_t^{\frac{\alpha}{1-\alpha}} A_t^{\frac{1}{1-\alpha}}$. The other variables are stationary and there is no need to scale them.

Let $G_{zt} = \frac{Z_t}{Z_{t-1}}$ and $G_{at} = \frac{A_t}{A_{t-1}}$. Then

$$\begin{aligned}\log G_{zt} &= \log g_{zt} + \log g_{\nu z,t},\\ \log G_{at} &= \log g_{at} + \log g_{\nu a,t},\end{aligned}$$

where

$$\begin{aligned}\log g_{\nu z,t} &= \log \nu_{z,t} - \log \nu_{z,t-1},\\ \log g_{\nu a,t} &= \log \nu_{a,t} - \log \nu_{a,t-1}.\end{aligned}$$

Denoting the gross growth rate of Γ_t by $g_{\gamma t} \equiv \Gamma_t/\Gamma_{t-1}$, we have

$$\log g_{\gamma t} = \frac{\alpha}{1-\alpha}\log G_{zt} + \frac{1}{1-\alpha}\log G_{at}.$$

Denoting the non-stochastic steady-state of $g_{\gamma t}$ by g_γ, we have

$$\log g_\gamma \equiv \frac{\alpha}{1-\alpha}\log g_z + \frac{1}{1-\alpha}\log g_a. \tag{A33}$$

On the nonstochastic balanced growth path, investment and capital grow at the rate of $g_I \equiv g_\gamma g_z$; consumption, output, wages, and the liquidity premium grow at the rate of g_γ; and the house rent, the rental rate of capital, Tobin's marginal Q, and the relative price of investment goods decrease at the rate g_z. We now display the equilibrium system for the stationary variables.

(1) Marginal utility of consumption,

$$\tilde{\Lambda}_t = \frac{1}{\tilde{C}_t - \gamma\tilde{C}_{t-1}/g_{\gamma t}} - \beta\gamma E_t\theta_{t+1}\frac{1}{\tilde{C}_{t+1}g_{\gamma t+1} - \gamma\tilde{C}_t}. \tag{A34}$$

(2) Labor supply,

$$\tilde{\Lambda}_t\tilde{w}_t = \psi_t N_t^\nu. \tag{A35}$$

(3) Rent of house,

$$\tilde{r}_{ht} = \frac{\xi_t}{\tilde{\Lambda}_t}. \tag{A36}$$

(4) Investment,

$$\begin{aligned} 1 &= \tilde{Q}_{kt}\left[1-\frac{\Omega}{2}\left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}}G_{zt}g_{\gamma t}-g_I\right)^2-\Omega\left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}}G_{zt}g_{\gamma t}-g_I\right)\frac{\tilde{I}_t}{\tilde{I}_{t-1}}G_{zt}g_{\gamma t}\right] \\ &\quad +\beta E_t\theta_{t+1}\frac{\tilde{\Lambda}_{t+1}}{\tilde{\Lambda}_t}\tilde{Q}_{kt+1}\Omega\left(\frac{\tilde{I}_{t+1}}{\tilde{I}_t}g_{\gamma t+1}G_{zt+1}-g_I\right)\frac{\tilde{I}_{t+1}^2}{\tilde{I}_t^2}g_{\gamma t+1}G_{zt+1}. \end{aligned} \tag{A37}$$

(5) Marginal Tobin's Q_k,

$$\tilde{Q}_{kt}=\beta E_t\theta_{t+1}\frac{\tilde{\Lambda}_{t+1}}{\tilde{\Lambda}_t}\frac{1}{g_{\gamma t+1}G_{zt+1}}[u_{t+1}\tilde{r}_{kt+1}+(1-\delta(u_{t+1}))\tilde{Q}_{kt+1}]. \tag{A38}$$

(6) Capital utilization,

$$\tilde{r}_{kt}=\delta'(u_t)\tilde{Q}_{kt}. \tag{A39}$$

(7) Liquidity premium,

$$\tilde{b}_t=\beta E_t\frac{\tilde{\Lambda}_{t+1}}{\tilde{\Lambda}_t}\theta_{t+1}\tilde{b}_{t+1}\left[1+\int_{a_{t+1}^*}(\frac{a}{a_{t+1}^*}-1)f(a)da\right]. \tag{A40}$$

(8) House price,

$$\tilde{p}_t=\beta E_t\frac{\tilde{\Lambda}_{t+1}}{\tilde{\Lambda}_t}\theta_{t+1}\left[\tilde{r}_{ht+1}+\tilde{p}_{t+1}+\tilde{b}_{t+1}\int_{a_{t+1}^*}(\frac{a}{a_{t+1}^*}-1)f(a)da\right]. \tag{A41}$$

(9) Rental rate of capital,

$$\tilde{r}_{kt}u_t\tilde{K}_t=\frac{\alpha G_{zt}g_{\gamma t}\tilde{Y}_t}{\frac{1}{1-F(a_t^*)}\int_{a_t^*}^{\infty}\frac{a}{a_t^*}f(a)da}. \tag{A42}$$

(10) Labor demand,

$$\tilde{w}_tN_t=\frac{(1-\alpha)\tilde{Y}_t}{\frac{1}{1-F(a_t^*)}\int_{a_t^*}^{\infty}\frac{a}{a_t^*}f(a)da}. \tag{A43}$$

(11) Aggregate output,

$$\tilde{Y}_t=\frac{1}{(G_{zt}G_{at})^{\frac{\alpha}{1-\alpha}}}\left(u_t\tilde{K}_t\right)^{\alpha}N_t^{1-\alpha}\frac{\int_{a_t^*}^{\infty}af(a)da}{1-F(a_t^*)}. \tag{A44}$$

(12) Liquidity constraint,

$$\frac{\tilde{b}_t}{a_t^*}\int_{a_t^*}^{\infty}af(a)da=\tilde{Y}_t. \tag{A45}$$

(13) Aggregate capital accumulation,

$$\tilde{K}_{t+1}=(1-\delta(u_t))\frac{\tilde{K}_t}{g_{zt}g_{\gamma t}}+\left[1-\frac{\Omega}{2}\left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}}g_{zt}g_{\gamma t}-g_I\right)^2\right]\tilde{I}_t. \tag{A46}$$

(14) Resource constraint,

$$\tilde{C}_t+\tilde{I}_t=\tilde{Y}_t. \tag{A47}$$

(15) Risk-free rate,

$$1 = \beta R_{ft} E_t \left[\frac{\tilde{\Lambda}_{t+1}\theta_{t+1}}{\tilde{\Lambda}_t} \frac{1}{g_{\gamma,t+1}} \right]. \tag{A48}$$

Appendix D. Log-Linearized System

We log-linearize the stationary model given in the preceding appendix around the deterministic steady state.

(1) Marginal utility of consumption,

$$\begin{aligned}\hat{\Lambda}_t \left(g_\gamma - \beta\gamma\right)\left(g_\gamma - \gamma\right) = &\left[-g_\gamma^2 \hat{C}_t + \gamma g_\gamma \left(\hat{C}_{t-1} - \hat{g}_{\gamma t}\right)\right] \\ &- \beta\gamma E_t \left[-g_\gamma \left(\hat{C}_{t+1} + \hat{g}_{\gamma t+1}\right) + \gamma \hat{C}_t + \hat{\theta}_{t+1}(g_\gamma - \gamma)\right].\end{aligned} \tag{A49}$$

(2) Labor supply,

$$\hat{\Lambda}_t + \hat{w}_t = \hat{\psi}_t + \nu \hat{N}_t. \tag{A50}$$

(3) House rent,

$$\hat{r}_{ht} = -\hat{\Lambda}_t + \hat{\xi}_t. \tag{A51}$$

(4) Investment,

$$\begin{aligned}0 = \hat{Q}_{kt} - \Omega\left(g_z g_\gamma\right)^2 &\left[\hat{I}_t - \hat{I}_{t-1} + \hat{g}_{zt} + \hat{g}_{vzt} + \hat{g}_{\gamma t}\right] \\ &+ \beta\Omega\left(g_z g_\gamma\right)^2 E_t \left(\hat{I}_{t+1} - \hat{I}_t + \hat{g}_{zt+1} + \hat{g}_{\gamma t+1} + \hat{g}_{vzt+1}\right).\end{aligned} \tag{A52}$$

(5) Marginal Tobin's Q_k,

$$\begin{aligned}\hat{Q}_{kt} + \hat{\Lambda}_t \quad = \quad & E_t\left[\hat{\theta}_{t+1} + \hat{\Lambda}_{t+1} - \hat{g}_{\gamma t+1} - \hat{g}_{zt+1} - \hat{g}_{vzt+1}\right] \\ & +(1-\beta(1-\delta))E_t\left(\hat{u}_{t+1} + \hat{r}_{kt+1}\right) \\ & +\beta(1-\delta)E_t\left[\hat{Q}_{kt+1} - \frac{\delta'(1)}{1-\delta}\hat{u}_{t+1}\right].\end{aligned} \tag{A53}$$

(6) Capital utilization,

$$\hat{r}_{kt} = \frac{\delta''(1)}{\delta'(1)}\hat{u}_t + \hat{Q}_{kt}. \tag{A54}$$

(7) Liquidity premium,

$$\hat{b}_t + \hat{\Lambda}_t = E_t(\hat{\theta}_{t+1} + \hat{\Lambda}_{t+1} + b_{t+1}) - [1-\beta]\,\frac{1+\mu}{\mu} E_t \hat{a}^*_{t+1}. \tag{A55}$$

(8) House price,

$$\hat{p}_t + \hat{\Lambda}_t = E_t\left(\hat{\theta}_{t+1} + \hat{\Lambda}_{t+1}\right) + \frac{\beta(\tilde{r}_h/\tilde{Y})}{\tilde{p}/\tilde{Y}} E_t \hat{r}_{ht+1} + \beta E_t \hat{p}_{t+1} + \frac{(1-\beta)(\tilde{b}/\tilde{Y})}{\tilde{p}/\tilde{Y}} E_t\left[\hat{b}_{t+1} - \frac{1+\mu}{\mu}\hat{a}^*_{t+1}\right].$$

(9) Rental rate of capital,

$$\hat{r}_{kt} + \hat{u}_t + \hat{K}_t = \hat{Y}_t + \hat{g}_{zt} + \hat{g}_{\gamma t} + \hat{g}_{vzt} + \left[1 - \frac{\eta\mu}{1+\mu}\right]\hat{a}^*_t. \tag{A56}$$

(10) Labor demand,

$$\hat{w}_t + \hat{N}_t = \hat{Y}_t + \left(1 - \frac{\eta\mu}{1+\mu}\right)\hat{a}^*_t. \tag{A57}$$

(11) Aggregate output,

$$\hat{Y}_t = \alpha(\hat{u}_t + \hat{K}_t) + (1-\alpha)\hat{N}_t + \frac{\eta\mu}{1+\mu}\hat{a}^*_t - \frac{\alpha}{1-\alpha}\left(\hat{g}_{zt} + \hat{g}_{vzt} + \hat{g}_{at} + \hat{g}_{vat}\right). \tag{A58}$$

(12) Liquidity constraint,

$$\hat{b}_t - \frac{1+\eta+\mu}{1+\mu}\hat{a}^*_t = \hat{Y}_t. \tag{A59}$$

(13) Aggregate capital accumulation,

$$\hat{K}_{t+1} = \frac{(1-\delta)}{g_z g_\gamma}\hat{K}_t + \left(1 - \frac{1-\delta}{g_z g_\gamma}\right)\hat{I}_t - \frac{\delta'(1)}{g_z g_\gamma}\hat{u}_t - (1-\delta)\left[\frac{\hat{g}_{zt} + \hat{g}_{vzt}}{g_z g_\gamma} + \frac{\hat{g}_{\gamma t}}{g_z g_\gamma}\right]. \tag{A60}$$

(14) Resource constraint,

$$\frac{\tilde{C}}{\tilde{Y}}\hat{C}_t + \frac{\tilde{I}}{\tilde{Y}}\hat{I}_t = \hat{Y}_t. \tag{A61}$$

(15) Risk-free rate,

$$\hat{\Lambda}_t = \hat{R}_{ft} + E_t(\hat{\Lambda}_{t+1} + \hat{\theta}_{t+1} - \hat{g}_{\gamma t+1}). \tag{A62}$$

We have 7 shocks.

(1) Permanent IST shock,

$$\hat{g}_{zt} = \rho_z \hat{g}_{zt-1} + \sigma_z \varepsilon_{zt}. \tag{A63}$$

(2) Temporary IST shock,

$$\hat{\nu}_{z,t} = \rho_{v_z}\hat{\nu}_{z,t-1} + \sigma_{v_z}\varepsilon_{v_z,t}. \tag{A64}$$

(3) Permanent technology shock,

$$\hat{g}_{at} = \rho_a \hat{g}_{at-1} + \sigma_a \varepsilon_{at}. \tag{A65}$$

(4) Temporary technology shock,

$$\hat{\nu}_{a,t} = \rho_{v_a}\hat{\nu}_{a,t-1} + \sigma_{v_a}\varepsilon_{v_a,t}. \tag{A66}$$

(5) Liquidity premium shock,

$$\hat{\theta}_t = \rho_\theta \hat{\theta}_{t-1} + \sigma_\theta \varepsilon_{\theta t}. \tag{A67}$$

(6) Housing demand shock,

$$\hat{\xi}_t = \rho_\xi \hat{\xi}_{t-1} + \sigma_\xi \varepsilon_{\xi t}.$$

(7) Labor supply shock,

$$\hat{\psi}_t = \rho_\psi \hat{\psi}_{t-1} + \sigma_\psi \varepsilon_{\psi t}. \tag{A68}$$

Appendix E. Data

All the data used in this paper was constructed by Patrick Higgins at the Federal Reserve Bank of Atlanta, some of which are collected directly from the Haver Analytics Database (Haver for short). In this section, we describe how the data was constructed in detail.

The model estimation is based on six U.S. aggregate time series: the real price of house (p_t^{Data}), the real rental price of house (r_{ht}^{Data}), the quality-adjusted relative price of investment ($(1/Z_t)^{\text{Data}}$), real per capita consumption (C_t^{Data}), real per capita investment (I_t^{Data}), and per capita total hours (H_t^{Data}). These series are constructed as follows:

- $p_t^{\text{Data}} = \frac{\text{LiqCoreLogic87}}{\text{PriceNonDurPlusServExHous}}$.
- $r_{ht}^{\text{Data}} = \frac{\text{PCERentOERPriceIndex}}{\text{PriceNonDurPlusServExHous}}$.
- $(1/Z_t)^{\text{Data}} = \frac{\text{GordonPriceCDplusES}}{\text{PriceNonDurPlusServExHous}}$.
- $C_t^{\text{Data}} = \frac{\text{(NomConsNHSplusND)/PriceNonDurPlusServExHous}}{\text{POPSMOOTH@USECON}}$.
- $I_t^{\text{Data}} = \frac{\text{(CD@USECON + FNE@USECON)/PriceNonDurPlusServExHous}}{\text{POPSMOOTH@USECON}}$.
- $H_t^{\text{Data}} = \frac{\text{TotalHours}}{\text{POPSMOOTH@USECON}}$.

Sources for the constructed data, along with the Haver keys (all capitalized letters) to the data, are described below.

LiqCoreLogic87: Liquidity-adjusted price index for housing. To construct this series, we first obtain Haver's seasonally adjusted CoreLogic home price index (USLPHPIS@USECON) from 1987Q1 to 2013Q4. We then adjust this home price index using the method of Quart and Quigley (1989, 1991) to take into account time-on-market uncertainty. The CoreLogic home price index series provided by the Core Logic Databases is similar to the Case-Shiller home price index but covers far more counties than the Case-Shiller series.

PCERentOERPriceIndex: Rental price index for housing. Constructed by using the Fisher chain-weighted aggregate of PCE OER [JCSRD_USNA] and PCE tenant rent [JCSHT_USNAqtr] price indices. Average of 2005 prices = 100. Haver Description for PCE OER [JCSRD_USNA] is "PCE: Imputed Rental of Owner-Occupied Nonfarm Housing Price Index (SA, 2005=100)." The key JCSHT_USNAqtr represents the PCE-based measure of home rental prices reported by Haver as JCSHT@USNA and described by Haver as "rental of tenant-occupied non-farm housing." This series is revised over time and is probably less subject to breaks due to improved methodology. It may still have a substantial break in 1977 and a smaller break in 1985 due to "non-response bias" (Crone, Nakamura, and Voith, 2010). Our sample starts in 1987Q1, so this potential problem is avoided.

PriceNonDurPlusServExHous: Consumer price index. Price deflator of non-durable consumption and non-housing services, constructed by Tornqvist aggregation of price deflator of non-durable consumption and non-housing related services (2009=100).

GordonPriceCDplusES: Price of investment goods. Quality-adjusted price index for consumer durable goods, equipment investment, and software investment. This is a weighted index from a number of individual price series within this category. For each individual price series from 1947 to 1983, we use Gordon (1990)'s quality-adjusted price index. Following Cummins and Violante (2002), we estimate an econometric model of Gordon's price series as a function of time trend and several macroeconomic indicators in the National Income and Product Account (NIPA), including the current and lagged values of the corresponding NIPA price series; the estimated coefficients are then used to extrapolate the quality-adjusted price index for each individual price series for the sample from 1984 to 2008. These constructed price series are annual. We use Denton (1971)'s method to interpolate these annual series at quarterly frequency. We then use the Tornqvist procedure to construct the quality-adjusted price index from the interpolated individual quarterly price series.

NomConsNHSplusND: Nominal personal consumption expenditures. Nominal non-durable goods and non-housing services (SAAR, billion of dollars). It is computed as

CN@USECON + CS@USECON - CSRU@USECON.

POPSMOOTH@USECON: Population. Smoothed civilian noninstitutional population with ages 16 years and over (thousands). This series is smoothed by eliminating breaks in population from 10-year censuses and post-2000 American Community Surveys using the "error of closure" method. This fairly simple method is used by the Census Bureau to get a smooth monthly population series and reduce the unusual influence of drastic demographic changes.[16]

CD@USECON: Consumer durable goods expenditures. Nominal personal consumption expenditures: durable goods (SAAR, billion of dollars).

FNE@USECON: Equipment and software expenditures. Nominal private nonresidential investment: equipment & software (SAAR, billion of dollars).

[16] The detailed explanation can be found at `http://www.census.gov/popest/archives/methodology/intercensal\_nat\_meth.html`.

TotalHours: Total hours in the non-farm business sector.

Appendix F. Estimation Procedure

We apply the Bayesian methodology to the estimation of the log-linearized medium-scale structural model, using our own C/C++ code. The advantage of using our own code instead of using Dynare is the flexibility and accuracy we have for finding the posterior mode. We generate over a half million draws from the prior as a starting point for our optimization routine and select the estimated parameters that give the highest posterior density. The optimization routine is a combination of NPSOL software package and the csminwel routine provided by Christopher A. Sims.

In estimation, we use the log-linearized equilibrium conditions, reported in Appendix D, to form the likelihood function fit to the six quarterly U.S. time series from 1987Q1 to 2013Q4: the house price, the house rent, the quality-adjusted relative price of investment, real per capita consumption, real per capita investment (in consumption units), and per capita hours worked. The data for the estimated sample begins with 1987Q1 for two reasons. First, the Case-Shiller house price time series begins in 1987. Second, the CoreLogic house price time series is similar to the Case–Shiller house price series, but covers far more counties than the Case-Shiller series. The CoreLogic house price series collected for the period before 1987 it does not have as much coverage as the series collected for the after-1986 period. The Case-Shiller house price time series exists only for the period after 1986, which we use to verify the quality of the CoreLogic house price series.

We fix the values of certain parameters as an effective way to sharpen the identification of other key parameters in the model. The capital share α is set at 0.33, consistent with the average labor income share of capital input. The growth rate of aggregate investment-specific technology, $g_z = 1.013$, is consistent with the average growth rate of the inverse relative price of investment goods. The growth rate of aggregate output, $g_\gamma = 1.0035$, is consistent with the average common growth rate of consumption and investment. The interest rate R_f is set at 1.01. The steady state capacity utilization u is set at 1. The steady-state labor supply as a fraction of the total time is normalized at $N = 0.3$. To solve the steady state, we impose three additional restrictions to be consistent with the data: 1) the capital-output ratio is 1.15 at annual frequency; 2) the investment-capital ratio is 0.2 at annual frequency; and 3) the rental-income-to-output ratio is 0.1.[17]

[17] Rental income of house is housing rental income of persons with capital consumption adjustment (SAAR, million dollars) from Table 7.4.5 in the National Income and Product Accounts. The output data used for our model is a sum of personal consumption expenditures and private domestic investment. Consumption is the private expenditures on nondurable goods and nonhousing services. Investment is the private expenditures on consumer durable goods and fixed investment in equipment and software. Accordingly, we measure capital stock using the annual stocks of equipment, software, and consumer durable goods.

We estimate five structural parameters as well as all the persistence and volatility parameters that govern exogenous shock processes. The five structural parameters are the inverse Frisch elasticity of labor supply ν, the hazard rate η, the elasticity of capacity utilization $\delta''(1)/\delta'(1)$, the habit formation γ, and the investment-adjustment cost Ω. The remaining parameters are then obtained from the steady state relationships that satisfy the aforementioned data restrictions. These parameters are: the capital depreciation rate ($\delta = 0.0404$), the subjective discount factor ($\beta = 0.9936$), the parameter related to cutoff productivity ($\mu = 0.1482$), the capacity utilization rate ($\delta'(1) = 0.0635$), the housing demand ($\bar{\xi} = 0.1348$), and the labor disutility ($\bar{\psi} = 8.9843$).

For the estimated parameters, we specify a prior that is agnostic enough to cover a wide range of values that are economically plausible (Table 5). The prior for ν, η, $\delta''(1)/\delta'(1)$, and Ω has a gamma distribution with the shape hyperparameter $a = 1$ and the rate hyperparameter $b = 0.5$. These hyperparameters allow a positive probability density at the zero value and the implied 90% prior probability bounds are from 0.1 to 6. The prior for γ has a beta distribution with the hyperparameters taking the values of 1 and 2. This particular specification allows a positive probability of no habit formation and at the same time permits a wide range of values considered in the literature (Boldrin, Christiano, and Fisher, 2001).

The prior for the persistence parameters of exogenous shock processes follows the beta distribution with the 90% probability interval between 0.026 and 0.776. The prior for the standard deviations of shock processes follows the inverse gamma distribution with the 90% probability interval between 0.0001 and 2. All these prior specifications are far more diffuse than those used in the literature.

TABLE 5. Prior distributions of structural and shock parameters

Parameter	Distribution	a	b	Low	High
ν	Gamma(a,b)	1.0	3.0	0.017	1.000
η	Gamma(a,b)	1.0	0.5	0.100	6.000
δ''/δ'	Gamma(a,b)	1.0	0.5	0.100	6.000
γ	Beta(a,b)	1.0	2.0	0.026	0.776
Ω	Gamma(a,b)	1.0	0.5	0.100	6.000
ρ_z	Beta(a,b)	1.0	2.0	0.026	0.776
ρ_{ν_z}	Beta(a,b)	1.0	2.0	0.026	0.776
ρ_a	Beta(a,b)	1.0	2.0	0.026	0.776
ρ_{ν_a}	Beta(a,b)	1.0	2.0	0.026	0.776
ρ_θ	Beta(a,b)	1.0	2.0	0.026	0.776
ρ_ξ	Beta(a,b)	1.0	2.0	0.026	0.776
ρ_ψ	Beta(a,b)	1.0	2.0	0.026	0.776
σ_z	Inv-Gam(a,b)	0.3261	1.45e04	0.0001	2.0000
σ_{ν_z}	Inv-Gam(a,b)	0.3261	1.45e04	0.0001	2.0000
σ_a	Inv-Gam(a,b)	0.3261	1.45e04	0.0001	2.0000
σ_{ν_a}	Inv-Gam(a,b)	0.3261	1.45e04	0.0001	2.0000
σ_θ	Inv-Gam(a,b)	0.3261	1.45e04	0.0001	2.0000
σ_ξ	Inv-Gam(a,b)	0.3261	1.45e04	0.0001	2.0000
σ_ψ	Inv-Gam(a,b)	0.3261	1.45e04	0.0001	2.0000

Note: "Low" and "high" denote the bounds of the 90% probability interval for each parameter.

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