Asset Bubbles and Foreign Interest Rate Shocks

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November 19, 2016

Abstract

We provide an infinite-horizon general equilibrium model of a small open economy with both domestic and international financial market frictions. Firms face credit constraints and use a bubble asset (land) as collateral to borrow. A land bubble can provide liquidity and relax credit constraints. Low foreign interest rates are conducive to bubble formation. A rise in foreign interest rate can cause the collapse of the asset bubble and a sudden stop. Asset bubbles provide an important amplification mechanism.

JEL Classification:
Keywords: Asset bubbles, small open economy, collateral, sudden stop, interest rate, liquidity

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1 Introduction

After the 2008 global financial crisis, strong expansionary monetary policies implemented by major advanced economies stimulated recovery in capital inflows into many emerging market economies. In 2013, following Bernanke's congressional testimony about the Federal Reserve's potential goal to bring the expansionary monetary policy to its normalcy, many emerging economies experienced remarkable capital flow reversals. From the experiences of several emerging markets in the past two decades and recent capital flow reversals,[1] we have learned the stylized facts of "Sudden Stops:" the reversal of international capital flows, the sudden increase in net exports and the corresponding increase from large current account deficits to smaller deficits or smaller surpluses, declines in production and consumption, real exchange rate depreciation, and a collapse in asset prices.[2] Furthermore, Uribe and Yue (2006) and Mackowiak (2007) document empirical evidence that U.S. interest rate shocks are an important driver of business cycles in emerging economies. There are also discussions that a low foreign interest rate can cause capital inflows and fuel asset bubbles in housing markets. An increase in the foreign interest rate can cause asset bubbles to burst and have a large adverse impact on the domestic economy.

The goal of our paper is to develop a theoretical model to understand the impact of foreign interest rate shocks on emerging economies. Unlike most papers in the existing literature surveyed later, our theory is based on the collateral channel of the asset bubble. Building on Miao and Wang (2012, 2015b) and Miao, Wang and Zhou (2015), we propose an infinite-horizon dynamic general equilibrium model of a small open economy in which there are frictions in both the domestic credit market and the international financial market. Domestic firms use capital, imported materials, and labor to produce output. Domestic and foreign goods can be exchanged at a real exchange rate (the price of foreign goods in terms of domestic goods). Following Aoki, Benigno, and Kiyotaki (2015) and Chang, Liu, and Spiegel (2015), we specify an exogenous function of foreign demand for the domestic country's exported goods, which is positively related to the real exchange rate. Firms face idiosyncratic investment efficiency shocks and credit constraints. They use their physical capital and a bubble asset as collateral to borrow. One may view the bubble asset as land, which is assumed to be intrinsically useless. Land is also an asset traded by firms and can provide liquidity.

Firms can borrow from domestic and international financial markets. International financial transactions are intermediated by financial institutions or banks subject to portfolio adjustment costs. Portfolio adjustment costs represent frictions in the international financial markets and cause

[1] Examples include southeast Asia and Russia in the late 1990s, South America in the early 2000s, and peripheral Europe in the late 2000s.

[2] See Mendoza (2010) and Korinek and Mendoza (2014) for a summary of this evidence.

the interest rate parity condition to fail. When the foreign interest rate is initially lower than the domestic interest rate, domestic firms finance their investment by selling bonds intermediated by banks to borrow from the foreign financial market. This can generate a low domestic interest rate and fuel a land bubble. If all agents believe that the intrinsically useless land has value, it can provide liquidity for firms to finance real investment. This belief can be self-fulfilling and results in a bubbly equilibrium. Since land is used as collateral, the existence of a land bubble can relax credit constraints. This allows efficient firms to borrow more and make more investment. This is the crowd-in effect. On the other hand, inefficient firms buy land from efficient firms and do not make real investment. This is the crowd-out effect. The net effect on aggregate investment and welfare is ambiguous. We show that there can also exist another equilibrium in which a land bubble does not exist. When no one believes that land has value, this belief can also be self-fulfilling and results in a bubbleless equilibrium.

When the foreign interest rate rises, capital begins to flow out of the domestic country. This generates a current account surplus or a shrink of the current account deficit. The increased supply of domestic goods in foreign markets leads to real depreciation, causing imports to decrease and exports to increase. Decreased imports lower output and hence investment and consumption. Meanwhile, investing in the international financial market crowds out resources for domestic firms to buy land. Moreover, payoffs from foreign bond holdings can also help provide liquidity to finance investments. Thus demand for asset bubbles is weakened by capital outflows. Asset bubbles begin to fall. When the foreign interest rate is sufficiently high, asset bubbles can burst immediately.

We show that the rise of the foreign interest rate has a much larger adverse impact on the domestic economy when there is an asset bubble than when there is no asset bubble. This is because an asset bubble can help relax credit constraints. The crash of the bubble tightens credit constraints, causing investment by efficient firms to drop more. When this crowd-in effect dominates, aggregate investment declines significantly. As a result, the impact on output, consumption, real depreciation, and capital flows is much larger in an economy with asset bubbles.

We make two contributions to the literature. First, we provide a theoretical result to characterize the condition for the existence of an asset bubble in an infinite-horizon small open economy. This result extends the existing results of Tirole (1985), Santos and Woodford (1997), Miao and Wang (2015b), and Miao, Wang, and Zhou (2015) for closed economies. Second, we study how foreign interest rate shocks affect the bubble existence condition and generate a sudden stop in the domestic economy. We emphasize how the rise and fall of asset bubbles can amplify the shocks.

Our paper is related to three strands of the literature. First, our paper is related to the literature on the role of foreign interest rate shocks in dynamic general equilibrium models of small open economies (e.g., Neumeyer and Perri (2005) and Uribe and Yue (2006)). This literature does not study asset bubbles. Unlike this literature, we use a deterministic model to derive analytical results. Second, our paper is related to the recent literature on sudden stops (e.g, Calvo (1998), Gopinath (2004), Martin and Rey (2006), Gertler, Gilchrist, and Natalucci (2007), and Mendoza (2010)).[3] This literature views credit frictions as the central feature of the transmission mechanism that drives sudden stops. Mendoza (2010) also emphasizes the amplification and asymmetry of macroeconomic fluctuations that result from the debt-deflation transmission mechanism. Asset prices in this literature typically refer to capital prices or equity prices in stock markets. By contrast, our paper focuses on land as an asset and land prices. In the model of Kiyotaki and Moore (1997), land is used as an input and also serves as collateral. A land bubble does not exist in their model. To focus on the role of asset bubbles, we simply assume it is intrinsically useless as in Kocherlakota (2009).

Third, our paper is related to the recent literature on asset bubbles in open economies (e.g., Caballero and Krishnamurthy (2006), Ventura (2012), Basco (2014), and Martin and Ventura (2015a,b)). This literature typically adopts the overlapping-generations (OLG) framework. Ventura (2012) develops a multi-country OLG model with asset bubbles and shows that bubbles tend to appear and expand in countries where productivity is low relative to the rest of the world. Martin and Ventura (2015a) provide a small open economy model with asset bubbles. They study the effects of asset bubbles during exogenously specified normal times and sudden stops and the role of the capital control policy. Martin and Ventura (2015b) analyze credit bubbles and the role of government policies in a multi-country OLG model. Caballero and Krishnamurthy (2006) show that real estate bubbles, which serve as domestic stores of value and reduce capital outflows, are beneficial. However, overinvestment in bubbles may expose the economy to bubble crashes and capital flow reversals. Basco (2014) develops a two-country OLG model to study the relationship between globalization and the emergence of rational bubbles.

Like our paper, this literature emphasizes the importance of credit constraints. Martin and Ventura (2012, 2015a,b) also discuss the crowd-in and crowd-out effects of asset bubbles, similar to those in our paper. Our paper differs from this literature in addressed questions and modeling details. More importantly, unlike the OLG models, our model is in the infinite-horizon growth framework, which can incorporate many standard ingredients in the dynamic general equilibrium literature, and is more amenable for a quantitative study (see Miao and Wang (2015a) and Miao, Wang, and Xu (2015)). In our infinite-horizon framework credit constraints are essential for the existence of asset bubbles in the sense that a bubble could not exist without credit constraints.

[3]See Korinek and Mendoza (2014) for a survey of this literature.

By contrast, a bubble can still exist in OLG models without credit constraints and the presence of credit constraints allows bubbles to exist in dynamically efficient economies (Farhi and Tirole (2012) and Martin and Ventura (2012)). Our infinite-horizon model complements the existing OLG models.

2 The Model

Consider a discrete-time infinite-horizon model of a small open economy populated by a continuum of identical households with a unit measure and a continuum of ex ante identical firms with a unit measure. There is no government or monetary authority. Each household is a family with two types of members: workers and bankers. Workers supply labor to firms. Financial transactions between domestic and foreign residents are intermediated by domestic financial institutions or simply bankers. Firms are owned by the households. Each firm is subject to idiosyncratic investment-efficiency shocks, so they are ex post heterogeneous. Suppose that there is no aggregate uncertainty and a law of large numbers holds so that aggregate variables are deterministic over time.

2.1 Firms

There are two types of goods: domestic goods and foreign goods. Firms in the small open economy use foreign goods as an input factor to produce domestic goods. As Mendoza (2010) emphasizes, imported inputs are important for the initial drop of output during a sudden stop. In each period $t = 0, 1, ...$, one unit of foreign goods can be exchanged for e_t units of domestic goods (e_t is called the real exchange rate). The total demand of the rest of the world for the domestic goods is exogenously given by

$$X_t = e_t^{\sigma} Y_t^*, \tag{1}$$

where $\sigma > 0$ and Y_t^* denotes an exogenous component of foreign demand. When e_t is larger, the domestic goods are cheaper and hence foreign demand is larger.

A domestic firm indexed by $j \in [0, 1]$ uses a constant-return-to-scale technology to produce output Y_{jt} according to

$$Y_{jt} = K_{jt}^{\alpha} (A_t N_{jt})^{1-\alpha-\gamma} M_{jt}^{\gamma}, \quad \alpha \in (0, 1), \quad \gamma \in (0, 1), \quad \alpha + \gamma \in (0, 1), \tag{2}$$

where A_t, K_{jt}, N_{jt} and M_{jt}, represent aggregate productivity, capital input, labor input, and imported material input, respectively. To ensure a positive steady-state interest rate, we suppose that A_t grows at rate $g > 0$. For balanced growth, suppose that Y_t^* also grows at rate g.

Firm j solves the following static labor and material input choice problem:

$$R_{kt} K_{jt} = \max_{N_{jt}, M_{jt}} \; K_{jt}^{\alpha} (A_t N_{jt})^{1-\alpha-\gamma} M_{jt}^{\gamma} - W_t N_{jt} - e_t M_{jt}, \tag{3}$$

where W_t is the wage rate and the capital return R_{kt} satisfies

$$R_{kt} = \alpha A_t^{\frac{1-\alpha-\gamma}{\alpha}} \left(\frac{1-\alpha-\gamma}{W_t} \right)^{\frac{1-\alpha-\gamma}{\alpha}} \left(\frac{\gamma}{e_t} \right)^{\frac{\gamma}{\alpha}}. \tag{4}$$

Firm j draws an investment-specific shock ε_{jt} at the beginning of each period t. It can make investment I_{jt} to increase its capital stock so that the law of the motion for its capital follows

$$K_{jt+1} = (1-\delta) K_{jt} + \varepsilon_{jt} I_{jt}, \tag{5}$$

where $\delta \in (0,1)$ represents the depreciation rate. Suppose that the cumulative distribution function of ε_{jt} is F and the density function is f on $[\varepsilon_{\min}, \varepsilon_{\max}] \subset [0, \infty)$ and ε_{jt} is independently and identically distributed across firms and over time. Assume that there is no insurance market against the idiosyncratic investment-specific shock and that investment is irreversible at the firm level so that $I_{jt} \geq 0$.

Firms can trade two types of assets: a one-period risk-free bond and a bubble asset. One unit of the bond delivers one unit of domestic goods in the next period. Let R_{ft} denote the domestic market interest rate between periods t and $t+1$. When firm j's bond holdings B_{jt+1} in period t satisfy $B_{jt+1} < (\geq) 0$, B_{jt+1} is interpreted as borrowing (saving). Firms can borrow or lend abroad, but this must be intermediated by the bankers only. Firms face borrowing constraints and can use their physical capital and the bubble asset as collateral. The bubble asset is intrinsically useless and we may think of it as land. The credit constraint is given by

$$\frac{B_{jt+1}}{R_{ft}} \geq -\mu K_{jt} - \theta P_t H_{jt}, \tag{6}$$

where $\mu \in (0,1)$ and $\theta \in (0,1)$ are pledgeability parameters and reflect frictions in the domestic financial market.[4]

Land is illiquid and cannot be shorted, so the following constraint must hold

$$H_{jt+1} \geq \omega H_{jt} \geq 0, \ \ \omega \geq \theta. \tag{7}$$

For the collateral to indeed be transferred to the creditors in case of default, we need to impose the constraint that $\omega \geq \theta$.

The flow-of-funds constraint for firm j is given by

$$D_{jt} = R_{kt} K_{jt} - I_{jt} - \frac{B_{jt+1}}{R_{ft}} + B_{jt} + P_t \left(H_{jt} - H_{jt+1} \right), \tag{8}$$

[4]Here we use the current value of capital and land as collateral to simplify algebra. This can be justified by a particular debt contract form. As Caballero and Krishnamurthy (2006), Miao and Wang (2015), and Miao, Wang, Zhou (2015) show, using future value as collateral as in Kiyotaki and Moore (1997) will complicate algebra without changing any key insights.

where D_{jt}, W_t, P_t, and H_{jt} denote dividends, wage rate, land price, and land holdings, respectively. Assume that equity financing is too costly for firms to raise new funds. Then firms face the equity constraint

$$D_{jt} \geq 0. \tag{9}$$

Now we describe firm j's decision problem by dynamic programming. Let $V_t(\varepsilon_{jt}, K_{jt}, H_{jt}, B_{jt})$ denote firm j's value function, where we suppress aggregate state variables as arguments. The dynamic programming problem is given by

$$V_t(\varepsilon_{jt}, K_{jt}, H_{jt}, B_{jt}) = \max_{I_{jt}\geq 0, H_{jt+1}, B_{jt+1}} D_{jt} + \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} V_{t+1}(\varepsilon_{jt+1}, K_{jt+1}, H_{jt+1}, B_{jt+1}),$$

subject to (5), (6), (7), (8), and (9). Here E_t represents the conditional expectation operator with respect to the idiosyncratic investment shock and Λ_t is the representative household's marginal utility.

2.2 Households

Each household is an extended family consisting of a continuum of ex ante identical workers of unit mass and a continuum of identical bankers also of unit mass. Bankers intermediate financial transactions internationally. The international financial transactions are subject to quadratic adjustment costs.[5] The flow-of-funds constraint of a representative banker is given by

$$D_{t+1}^b = \frac{B_{t+1}}{R_{ft} e_t} R_{ft}^* e_{t+1} - B_{t+1} - \frac{\Omega}{2Y_{t+1}} \left(B_{t+1} - B_t\right)^2, \tag{10}$$

where D_{t+1}^b is the profit that the banker gives to his family in period $t+1$, $B_t \geq (<) 0$ is the bond supply (demand) from the banker, R_{ft}^* is the exogenous foreign interest rate between periods t and $t+1$, and $\Omega > 0$ captures the size of adjustment costs. We can also interpret B_{t+1}/R_{ft} as the net international investment position (NIIP) in period t. Adjustment costs are normalized by the total domestic output Y_{t+1} to make analysis tractable. When $\Omega = 0$, we have free capital mobility and the interest rate parity condition holds. Suppose that total adjustment costs are paid to the households in a lump-sum manner.

The interpretation of (10) is as follows. The banker sells B_{t+1} units of bonds to domestic residents in period t at the price $1/R_{ft}$ and converts the proceed into foreign currency at the rate $1/e_t$. It then saves in a foreign bank and gets interest at the gross rate R_{ft}^*. After converting into domestic currency at rate e_{t+1}, the banker obtains returns $\frac{B_{t+1}^b}{R_{ft}e_t} R_{ft}^* e_{t+1}$. After subtracting debt

[5] Our modeling of bankers is similar to that described in Chapter 4 of Uribe and Schmitt-Grohe (2016). See Aoki, Benigno, and Kiyotaki (2015) for related modeling.

repayment B_{t+1}^* and adjustment costs in period $t+1$, we obtain the profit D_{t+1}^b given in equation (10). The banker chooses $\{B_{t+1}\}$ to maximize the present value of profits:

$$\sum_{t=0}^{\infty} \beta^{t+1} \frac{\Lambda_{t+1}}{\Lambda_0} D_{t+1}^b.$$

Each worker in the family supplies one unit of labor inelastically and hands in the wage income to the family. The family pools the income and dividends from the workers, bankers, and firms, and distributes them equally among family members. A representative household chooses the family consumption $\{C_t\}$ to maximize its lifetime utility,

$$\sum_{t=0}^{\infty} \beta^t \ln(C_t),$$

subject to

$$C_t = W_t N_t + D_t + D_t^b + \frac{\Omega}{2Y_t} \left(B_t - B_{t-1}\right)^2, \tag{11}$$

where $N_t = 1$ and $D_t = \int D_{jt} dj$ denotes total dividends from all firms. Given log utility, the marginal utility is $\Lambda_t = 1/C_t$. Here we have assumed that households do not trade bonds or bubble assets. Allowing them to trade these assets will not affect our results if we assume that households cannot borrow and face a short-sales constraint on land. In this case households will choose not to hold any assets because their equilibrium returns are too low (see Kiyotaki and Moore (2008) and Miao, Wang, and Zhou (2015) for a similar result). We will show this point in (20) later.

2.3 Competitive Equilibrium

Denote $K_t = \int K_{jt} dj$, $I_t = \int I_{jt} dj$, and $Y_t = \int Y_{jt} dj$. A competitive equilibrium consists of sequences of aggregate quantities $\{C_t, K_{t+1}, I_t, Y_t, B_{t+1}, H_{t+1}, M_t\}$ and prices $\{W_t, P_t, R_{kt}, R_{ft}, e_t\}$ such that:

(i) Households, firms, workers, and bankers optimize.

(ii) The markets for labor, land, bonds, domestic goods, and foreign goods all clear so that

$$N_t = \int_0^1 N_{jt} dj = 1, \; H_t = \int_0^1 H_{jt} dj = 1, \tag{12}$$

$$B_t = \int_0^1 B_{jt} dj, \; M_t = \int_0^1 M_{jt} dj, \tag{13}$$

$$Y_t = C_t + I_t + X_t. \tag{14}$$

(iii) The law of motion of aggregate capital follows

$$K_{t+1} = (1-\delta) K_t + \int_0^1 \varepsilon_{jt} I_{jt} dj. \tag{15}$$

3 Model Solution

We first derive the solution to the firm's decision problem and then characterize the equilibrium system.

3.1 Firms' Decision Problem

Define Tobin's (marginal) Q as

$$Q_t = \frac{\beta \Lambda_{t+1}}{\Lambda_t} \frac{\partial E_t \left[V_{t+1}(\varepsilon_{jt+1}, K_{jt+1}, H_{jt+1}, B_{jt+1}) \right]}{\partial K_{jt+1}}.$$

The following proposition characterizes firm j's optimal policy.

Proposition 1 *Denote* $\bar{\varepsilon}_t = 1/Q_t \in (\varepsilon_{\min}, \varepsilon_{\max})$.

(i) When $\varepsilon_{jt} \geq \bar{\varepsilon}_t$, *firm* j *makes real investment,*

$$I_{jt} = R_{kt} K_{jt} + \mu K_{jt} + B_{jt} + (1 - \omega + \theta) P_t H_{jt}, \tag{16}$$

sells its land as much as possible, i.e., $H_{jt+1} = \omega H_{jt}$, *and exhausts its borrowing limit.*

(ii) When $\varepsilon_{jt} < \bar{\varepsilon}$, *firm* j *makes no real investment and is willing to hold any amount of land and bonds as long as condition (6) holds.*

(iii) The Tobin's Q Q_t, *the land price* P_t *and the interest rate* R_{ft} *satisfy*

$$Q_t = \frac{\beta \Lambda_{t+1}}{\Lambda_t} \left[R_{kt+1} + (1-\delta) Q_{t+1} + (R_{kt+1} + \mu) \int_{\bar{\varepsilon}_{t+1}}^{\varepsilon_{\max}} (\varepsilon Q_{t+1} - 1) f(\varepsilon) d\varepsilon \right], \tag{17}$$

$$P_t = \frac{\beta \Lambda_{t+1}}{\Lambda_t} P_{t+1} \left[1 + (1 - \omega + \theta) \int_{\bar{\varepsilon}_{t+1}}^{\varepsilon_{\max}} (\varepsilon Q_{t+1} - 1) f(\varepsilon) d\varepsilon \right], \tag{18}$$

$$\frac{1}{R_{ft}} = \frac{\beta \Lambda_{t+1}}{\Lambda_t} \left[1 + \int_{\bar{\varepsilon}_{t+1}}^{\varepsilon_{\max}} (\varepsilon Q_{t+1} - 1) f(\varepsilon) d\varepsilon \right], \tag{19}$$

and the usual transversality conditions.

Equation (16) shows that investment by efficient firms is financed by four sources: internal funds $R_{kt} K_{jt}$, debt collateralized by capital, μK_{jt}, payoffs from bond holdings B_{jt}, debt collateralized by land $\theta P_t H_{jt}$, and sales of land $(1 - \omega) P_t H_{jt}$. Equations (17), (18), and (19) are the asset-pricing equations for capital, land and bond, respectively. The integral terms in these three equations capture the liquidity premium because capital, land (if its price is positive), and bond can raise the firm's net worth. Moreover, a land bubble can help the firm relax its borrowing constraint by raising its debt capacity. We focus on the integral term in equation (18). At time $t+1$, when the investment-specific shock $\varepsilon_{jt+1} \geq \bar{\varepsilon}_{t+1}$, firm j can sell a fraction $(1 - \omega)$ of its land and use a fraction θ of it as collateral to borrow and finance the real investment. Each dollar in the payoff can generate $(\varepsilon_{jt+1}Q_{t+1} - 1)$ units of benefits. Thus ex ante average profits are given by the integral term multiplied by $(1 - \omega + \theta)$. The interpretation for the liquidity premium provided by capital and bonds are similar. Note that due to the illiquidity and collateral role of land, the liquidity premiums for the land and the bond are different. When $\omega = \theta = 0$, the land and bond are perfect substitutes. In this case the only role of the land bubble is to raise the firm's net worth and a land bubble can still exist. This insight is first developed by Kiyotaki and Moore (2008) when land bubble is viewed as fiat money and applied by Hirano and Yanagawa (2013) to study economic growth in infinite-horizon models. When $\theta > 0$, there is an additional collateral role of the land bubble. This role will be important for the amplification of exogenous shocks.

To see the importance of the liquidity premium for the existence of a bubble, we write the asset-pricing equation for the bubble without the liquidity premium as follows

$$P_t = \frac{\beta\Lambda_{t+1}}{\Lambda_t}P_{t+1}.$$

This equation cannot hold in the steady state with a positive land price for $\beta \in (0, 1)$. In otherwise words, transversality condition will rule out bubbles.

By equations (18) and (19), we have

$$P_t > \frac{\beta\Lambda_{t+1}}{\Lambda_t}P_{t+1} \text{ and } \frac{1}{R_{ft}} > \frac{\beta\Lambda_{t+1}}{\Lambda_t}. \tag{20}$$

These inequalities imply that households will not buy any land bubbles or bonds in equilibrium because their returns are too low.

3.2 The Current Account

The home country imports foreign goods and exports domestic goods. The current account balance consists of trade surplus and net interest income from foreign investments. Specifically, the current account is given by

$$CA_t = X_t - e_t M_t + B_t \frac{R^*_{ft-1}}{R_{ft-1}} \frac{e_t}{e_{t-1}} - \frac{B_t}{R_{ft-1}}, \tag{21}$$

where $X_t - e_t M_t$ is the trade balance, $B_t \frac{R^*_{ft-1}}{R_{ft-1}} \frac{e_t}{e_{t-1}}$ is the principal plus interest from foreign investments and $\frac{B_t}{R_{ft-1}}$ is the principal. The current account is equal to the change in foreign assets or in the NIIP

$$CA_t = \frac{B_{t+1}}{R_{ft}} - \frac{B_t}{R_{ft-1}}. \tag{22}$$

Combining equation (21) and (22), we obtain

$$X_t - eM_t = \frac{B_{t+1}}{R_{ft}} - B_t \frac{R^*_{ft-1}}{R_{ft-1}} \frac{e_t}{e_{t-1}}. \tag{23}$$

This equation says that the trade balance is equal to the change in the NIIP net of the interest income from foreign assets.

3.3 Equilibrium System

After aggregating individual decision rules and imposing market-clearing conditions, we obtain the equilibrium system shown by the following proposition:

Proposition 2 *The equilibrium system is given by the following equations: (17), (18), (19), (23),* $\bar{\varepsilon}_t Q_t = 1$,

$$0 = \frac{\Lambda_{t+1}}{\Lambda_t}\left[\frac{R^*_{ft}}{R_{ft}}\frac{e_{t+1}}{e_t} - 1 - \frac{\Omega}{Y_{t+1}}\left(B_{t+1} - B_t\right)\right] + \beta\frac{\Lambda_{t+2}}{\Lambda_t}\frac{\Omega}{Y_{t+2}}\left(B_{t+2} - B_{t+1}\right), \tag{24}$$

$$I_t = \left[R_{kt}K_t + \mu K_t + B_t + (1-\omega+\theta)P_t\right]\int_{\bar{\varepsilon}_t}^{\varepsilon_{\max}} f(\varepsilon)d\varepsilon, \tag{25}$$

$$K_{t+1} = (1-\delta)K_t + \left[R_{kt}K_t + \mu K_t + B_t + (1-\omega+\theta)P_t\right]\int_{\bar{\varepsilon}_t}^{\varepsilon_{\max}} \varepsilon f(\varepsilon)d\varepsilon,, \tag{26}$$

$$Y_t = A_t^{1-\alpha-\gamma}K_t^{\alpha}M_t^{\gamma}, \tag{27}$$

$$e_t M_t = \gamma Y_t, \tag{28}$$

$$Y_t = C_t + I_t + X_t, \tag{29}$$

$$1 = A_t^{\frac{1-\alpha-\gamma}{\alpha}}\left(\frac{1-\alpha-\gamma}{W_t}\right)^{\frac{1-\gamma}{\alpha}}\left(\frac{\gamma}{e_t}\right)^{\frac{\gamma}{\alpha}}K_t, \tag{30}$$

$$R_{kt} = \alpha A_t^{1-\alpha-\gamma}K_t^{\alpha-1}M_t^{\gamma}, \tag{31}$$

for the endogenous variables $\{Q_t, \bar{\varepsilon}, P_t, R_{ft}, B_{t+1}, I_t, K_{t+1}, Y_t, M_t, C_t, X_t, e_t, W_t, R_{kt}\}$. *The usual transversality conditions hold.*

Equation (24) is the no-arbitrage condition for the international financial transaction derived from the banker's optimization problem. When there is no portfolio adjustment cost ($\Omega = 0$), we obtain the interest rate parity condition. This condition does not hold whenever there are adjustment costs $\Omega > 0$. Equation (25) gives aggregate investment, which is financed by internal funds $R_{kt}K_t$, borrowing collateralized by physical capital and land $\mu K_t + \theta P_t$, sales of land $(1-\omega)\,P_t$, and the repayment from bonds B_t. The integral term in this equation reflects the fact that only firms with efficiency levels higher than $\bar{\varepsilon}_t$ make investment. If there is a land bubble $P_t > 0$, then each efficient firm with its efficiency level higher than $\bar{\varepsilon}_t$ will make more investment. This the crowd-in effect of the land bubble. On the other hand, inefficient firms with their efficient levels lower than $\bar{\varepsilon}_t$ have to buy land from efficient firms and do not make investment. This is the crowd-out effect. The net effect of a land bubble on aggregate investment depends on the relative strength of the crowd-in and crowd-out effects. Equation (26) gives the law of motion for aggregate capital.

Equation (28) shows that import-to-output ratio is equal to γ due to the Cobb-Douglas production function. Thus GDP in our model is given by

$$GDP_t = C_t + I_t + X_t - \epsilon_t M_t = (1-\gamma) Y_t.$$

Note that $P_t = 0$ for all t always satisfies (18). We call such an equilibrium bubbleless equilibrium. If there is an equilibrium such that $P_t > 0$ for all t, we call it bubbly equilibrium. We will focus on these two types of equilibrium in this paper.[6]

4 Steady-State Equilibria

The economy in equilibrium features balanced growth. In particular, the variables e_t, $\bar{\varepsilon}_t$, R_{kt}, and Q_t do not have a trend, while all other equilibrium variables grow at the rate of technical progress. We thus normalize any growing variable, say Z_t, by A_t and use a lower case variable $z_t = Z_t/A_t$ to denote the detrended value. In the appendix we present detrended equilibrium system. In this section we study steady states, in which all detrended variables are constant over time. We use a variable without the time subscript t to denote its steady-state value. We use a subscript f to denote any variable in a bubbleless (fundamental) steady state, while we use a subscript b to denote a variable in a bubbly steady state.

We first derive some common equations in both types of steady state. We use equation (17) and $Q = 1/\bar{\varepsilon}$ to derive the steady-state rental rate of capital

$$R_k = \frac{(1+g-\beta+\beta\delta) - \beta\mu\left(\int_{\bar{\varepsilon}}^{\varepsilon_{\max}} (\varepsilon - \bar{\varepsilon}) f(\varepsilon) d\varepsilon\right)}{\beta\left[\bar{\varepsilon} + \int_{\bar{\varepsilon}}^{\varepsilon_{\max}} (\varepsilon - \bar{\varepsilon}) f(\varepsilon) d\varepsilon\right]}. \tag{32}$$

We then use (19) to derive the steady-state domestic interest rate

$$R_f = \frac{1+g}{\beta\left[1 + \int_{\bar{\varepsilon}}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\bar{\varepsilon}} - 1\right) f(\varepsilon) d\varepsilon\right]}. \tag{33}$$

Both variables are functions of $\bar{\varepsilon}$, which are denoted by $R_k(\bar{\varepsilon})$ and $R_f(\bar{\varepsilon})$. The following lemma is useful for the analysis.

Lemma 1 *The asset-to-output ratio in a steady state is*

$$\frac{b}{y} = \frac{R_f^*/R_f - 1}{\Omega \frac{g}{1+g}\left(1 - \frac{\beta}{1+g}\right)}, \tag{34}$$

where R_f is given by (33). The current-account-to-output ratio in a steady state is

$$\frac{ca}{y} = \frac{g}{R_f}\frac{b}{y}. \tag{35}$$

[6] It is possible to have other types of equilibrium such as sunspot equilibria (Weil (1987)).

This lemma says that the steady-state current-account-to-output ratio is determined by the interest rate differential. When the foreign interest rate R_f^* is higher (lower) than the domestic interest rate R_f, there is a capital account surplus (deficit) or capital outflow (inflow). When the foreign interest rate rises, capital flow reverses, that is, capital inflow decreases and capital outflow increases.

The two types of steady state differ in the investment threshold $\bar{\varepsilon}$. We now derive this threshold and other equilibrium variables.

4.1 Bubbleless Steady State

We impose the following assumption to ensure the existence of a bubbleless steady state.

Assumption 1 *Assume that*

$$\delta + g < \left(R_k \left(\varepsilon_{\min} \right) + \mu + \frac{b}{y} \left(\varepsilon_{\min} \right) \frac{R_k \left(\varepsilon_{\min} \right)}{\alpha} \right) E \left[\varepsilon \right],$$

where R_k is given by (32) and b/y is given by (34).

The following proposition characterizes the bubbleless steady state.

Proposition 3 *Let assumption 1 hold. If μ is sufficiently small, then the equation*

$$\delta + g = \left(R_k \left(\bar{\varepsilon} \right) + \mu + \frac{b}{y} \left(\bar{\varepsilon} \right) \frac{R_k \left(\bar{\varepsilon} \right)}{\alpha} \right) \int_{\bar{\varepsilon}}^{\varepsilon_{max}} \varepsilon f(\varepsilon) d\varepsilon \tag{36}$$

has a unique solution for $\bar{\varepsilon} \in (\varepsilon_{\min}, \varepsilon_{\max})$, denoted by $\bar{\varepsilon}_f$. If further

$$1 - \left(\alpha + \frac{\alpha \mu}{R_k \left(\bar{\varepsilon}_f \right)} + \frac{b}{y} \left(\bar{\varepsilon}_f \right) \right) \int_{\bar{\varepsilon}_f}^{\varepsilon_{\max}} f(\varepsilon) d\varepsilon > \gamma + \frac{b}{y} \left(\bar{\varepsilon}_f \right) \frac{1 + g - R_f^*}{R_f \left(\bar{\varepsilon}_f \right)} > 0, \tag{37}$$

then there is a unique bubbleless steady state.

Assumption 1 allows us to use the intermediate value theorem to derive a solution to equation (36). A sufficiently small μ ensures that $R_k \left(\bar{\varepsilon} \right)$ is a decreasing function of $\bar{\varepsilon}$ (see Lemma 2 in the appendix), which ensures the uniqueness of the solution to (36). This condition is not needed for the existence.

After obtaining $\bar{\varepsilon}$, we then solve for the equilibrium real exchange rate e_f using equations (23) and (28). Once $\bar{\varepsilon}_f$ and e_f are derived, other equilibrium variables can be easily determined. Since export demand by foreigners is exogenously given by equation (1), we have to impose the two inequalities in (37) to ensure exports and consumption are positive in the bubbleless steady state.

4.2 Bubbly Steady State

The following proposition characterizes the existence of the bubbly steady state.

Proposition 4 *Suppose that the assumptions in Proposition 3 hold so that there exists a unique bubbleless steady state with the investment threshold given by $\bar{\varepsilon}_f$. If there is a bubbly steady state, then the following condition holds*

$$1 < \beta \left[1 + (1 - \omega + \theta) \int_{\bar{\varepsilon}_f}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\bar{\varepsilon}_f} - 1 \right) f(\varepsilon) d\varepsilon \right]. \tag{38}$$

Conversely, if this condition holds, then the equation

$$1 = \beta \left[1 + (1 - \omega + \theta) \int_{\bar{\varepsilon}}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\bar{\varepsilon}} - 1 \right) f(\varepsilon) d\varepsilon \right] \tag{39}$$

has a unique solution for $\bar{\varepsilon} \in (\bar{\varepsilon}_f, \varepsilon_{\max})$, denoted by $\bar{\varepsilon}_b$, and if further

$$1 - \left(\alpha + \frac{\alpha \mu}{R_k(\bar{\varepsilon}_b)} + \frac{b}{y}(\bar{\varepsilon}_b) + (1 - \omega + \theta) \frac{p}{y_b} \right) \int_{\bar{\varepsilon}_b}^{\varepsilon_{\max}} f(\varepsilon) d\varepsilon > \gamma + \frac{b}{y}(\bar{\varepsilon}_b) \frac{1 + g - R_f^*}{R_f(\bar{\varepsilon}_b)} > 0, \tag{40}$$

where

$$(1 - \omega + \theta) \frac{p}{y_b} = \frac{\alpha}{R_k(\bar{\varepsilon}_b)} \left(\frac{\delta + g}{\int_{\bar{\varepsilon}_b}^{\varepsilon_{\max}} \varepsilon f(\varepsilon) d\varepsilon} - \mu \right) - \alpha - \frac{b}{y}(\bar{\varepsilon}_b), \tag{41}$$

then there exists a unique bubbly steady state with the investment threshold given by $\bar{\varepsilon}_b$ and the bubble-to-output ratio p/y_b given above.

The interpretation for (40) is similar to that for (37) and is omitted here. Condition (38) is the key bubble existence condition, similar to that in Miao, Wang, and Zhou (2015). The main difference is that here the foreign interest rate R_f^* affects (38) because it affects the bubbleless steady-state investment threshold $\bar{\varepsilon}_f$. We interpret condition (38) as follows: Suppose that the economy is initially in the bubbleless steady state. If there is a land bubble, i.e., land has a positive price, then the right-hand side of (38) represents the steady-state benefit of purchasing one unit of land, while the left-hand side is the cost of purchasing one unit of land. The benefit comes from the liquidity and collateral role of land discussed earlier for (18). If condition (38) holds, then the benefit is larger than the associated cost. Thus the firm is willing to pay a positive price to buy land.

How is condition (38) related to the traditional bubble existence condition that the interest rate in the bubbleless economy must be lower than the rate of economic growth (Tirole (1985))? We have shown before that the steady-state interest rate satisfies (33). Thus the interest rate in the bubbleless steady state is $R_{ff} = R_f(\bar{\varepsilon}_f)$. Since $\omega \geq \theta$, it follows from (38) that $R_{ff} < 1 + g$, consistent with Tirole's (1985) condition. However, unlike his OLG model, our infinite-horizon economy is not dynamically inefficient since there is no overaccumulation of capital.

Proposition 5 *The smaller the foreigen interest rate R_f^*, the more likely a domestic land bubble can exist. When R_f^* is sufficiently high, a land bubble cannot exist.*

For a smaller foreign interest rate R_f^*, it is more likely to have capital inflows (or smaller capital outflows) by Lemma 1. This tends to decrease the domestic interest rate and fuel a land bubble. More formally, condition (38) is more likely to hold when R_f^* is smaller. But when the foreign interest rate is sufficiently high, this condition can be violated and hence a land bubble cannot exist.

We now compare the bubbly steady state with the bubbleless steady state.

Proposition 6 *Suppose that the assumptions in Propositions 3 and 4 hold so that the bubbly and bubbleless steady states coexist, then $R_{fb} > R_{ff}$. If further*

$$\left(\frac{1}{R_{fb}} + \frac{1}{R_{ff}}\right)^{-1} < R_f^* < 1 + g, \tag{42}$$

then $y_b > y_f$ and $k_b > k_f$.

The existence of a land bubble crowds out the demand for bonds and hence raises the interest rate so that $R_{fb} > R_{ff}$. In the appendix we show that, when condition (42) holds, the change in NIIP net of interest income relative to output $\frac{b}{yR_f}(1+g-R_f^*)$ in the bubbly steady state is smaller than in the bubbleless steady state. It follows from (23) that the trade balance relative to output is also lower in the bubbly steady state than in the bubbleless steady state, providing relatively lower pressure on the real exchange depreciation in the bubbly steady state. Thus output and capital in the bubbly steady state are relatively higher.

Note that condition (42) says that the foreign interest rate R_f^* should not be too high or too low. If this condition were violated, then the comparison of the trade balance across the bubbly and bubbleless steady states would reverse the direction. This is because the term $\frac{b}{yR_f}(1+g-R_f^*)$ depends on the product of the net foreign interest rate $R_f^* - 1 - g$ and the interest rate differential $(R_f^*/R_f - 1)$ through the NIIP b/R_f by (34).

Note that the comparison of welfare across the two steady states is ambiguous. This is because a land bubble has both crowd-out and crowd-in effects. On the one hand, a land bubble provides collateral to firms and allow efficient firms to borrow more to make more investment. This is the crowd-in effect that allows output to rise. On the other hand, land is an asset and holdings of land by inefficient firms crowd out resources for investment. These firms actually do not invest at all. The net effect on aggregate investment and consumption is ambiguous. In the numerical example studied in section 5, we find that the crowd-in effect is stronger.

4.3 Foreign Interest Rate Shock

Now we discuss how an increase in the foreign interest rate affects the bubbly and bubbleless steady states.

Proposition 7 *Consider the bubbly steady state.*

1. *The rental rate of capital R_{kb}, the domestic interest rate R_{fb}, and the investment threshold $\bar{\varepsilon}_b$ do not change with the foreign interest rate R_f^*.*

2. *The bubble-to-output ratio $\frac{p}{y_b}$ decreases in R_f^* and the current-account-to-output ratio $\frac{ca_b}{y_b}$ increases in R_f^*.*

3. *For $R_f^* < (R_{fb} + 1 + g)/2$, the domestic goods depreciate as R_f^* rises, the export x_b increases in R_f^*, and output y_b, the capital stock k_b, investment i_b, consumption c_b and the import m_b decrease in R_f^*. The opposite result holds true for $R_f^* > (R_{fb} + 1 + g)/2$.*

As Proposition 4 shows, the investment threshold in the bubbly steady state is determined by the steady-state asset-pricing equation for the bubble (39), which does not depend on the foreign interest rate R_f^*. Thus R_{kb} and R_{fb} do not depend on R_f^* by equations (32) and (33). A rise in R_f^* leads to an increased capital outflow, an increased current account, and a decreased size of the land bubble. The fall of the land bubble is due to the fact that land bubble and foreign investments are substitutes for providing liquidity to the firms. When R_f^* is sufficiently high, the net international investment position is sufficiently high so that firms have enough liquidity to finance investment. Thus there is no room for the existence of a bubble.

As discussed in the previous subsection, the trade balance in the steady state is equal to $\frac{b}{yR_f}(1+g-R_f^*)$, which depends on product of the net foreign interest rate $(R_f^* - 1 - g)$ and the interest rate differential. Thus the trade balance is not monotonic in R_f^*. For $R_f^* < (R_{fb} + 1 + g)/2$, the trade balance increases with R_f^*, leading to a real depreciation. Although exports increase, output, investment, consumption, imports, and capital stock all decrease. The opposite holds true for $R_f^* > (R_{fb} + 1 + g)/2$. In this case an increase in the interest rate can deteriorate the trade balance because the net interest income from foreign assets is too high, dominating the increase in the current account balance.

Now we consider the bubbleless steady state, in which firms rely on internal funds, domestic borrowing, and payoffs from foreign investments to finance investment. When the foreign interest rate rises, payoffs from foreign investments rise and thus firms can finance more real investment. To keep the aggregate investment rate constant in a steady state, there must be fewer firms to make investment so that the investment threshold rises. This causes the liquidity premium to decrease and hence the bond price to fall or the interest rate to rise.[7]

The impact of the foreign interest rate on other equilibrium variables is mixed. On the one hand, an increase in the foreign interest rate encourages real investment and hence raises capital and output. On the other hand, an increase in the foreign interest rate causes capital outflow and real depreciation. Real depreciation makes imported goods more expensive. Since imported material is an input, firms production is adversely affected. Therefore the net effect on output and capital is ambiguous.

Given the analysis above, we find that the bubbly and bubbleless economies are differently affected by an increase in R_f^*: the bubbly economy is surely adversely affected, whereas the bubbleless economy may be favorably or adversely affected, depending on parameter values.

5 Transition Dynamics

We have analyzed properties of the steady states in the previous section. We now study how the economy responds to exogenous shocks along transition paths. We focus on the effects of a permanent rise in the foreign interest rate. We will use numerical examples to illustrate our model mechanism. We choose parameter values such that the model economy roughly corresponds to an emerging market economy. We do not intend to perform a serious calibration.

The model period corresponds to a quarter. We set the growth rate $g = 0.0104$ so that the annual economic growth rate is consistent with the average growth rate for emerging and developing economies between 2012 and 2015 according to the World Economic Outlook Database from the IMF. Set the subjective discount factor $\beta = 0.974$, the capital share $\alpha = 0.312$, and the capital depreciation rate $\delta = 0.028$. Set the share of imported goods $\gamma = 0.285$, which is consistent with the world average share of imported inputs in GDP between 2012 and 2015 as reported by the World Bank. Set the price elasticity of exported domestic goods $\sigma = 1.56$, which lies in the range of empirical estimates obtained by (Feenstra et al. (2014)). Let $F(\varepsilon) = 1 - \left(\frac{\varepsilon}{\varepsilon_{\min}}\right)^{\eta}$. Set $\eta = 13$ and $\varepsilon_{\min} = 12/13$ so that the mean of ε is 1 and the investment-to-output ratio is 0.17 in the bubbly steady state. Set the pledgeability parameters $\mu = 0.025$ and $\theta = 0.647$, and the parameter for the illiquidity of land trading $\omega = 0.902$. These three parameter values indicate that there are substantial frictions in the financial and land markets. Set the parameter for the portfolio adjustment costs $\Omega = 20.6$ so that the net-foreign-asset-to-GDP-ratio in the bubbly steady state is 29%, which is in line with the data from the World Economic Outlook Database. Set the initial

[7] This result can be formally proved. In part 4 of the proof of Proposition 6 in the appendix, we show that $\bar{\varepsilon}_f$ increases with R_f^*. It follows from (33) that R_{ff} increases with R_f^*.

foreign interest rate $R_f^* = 1.0005$ so that the annual foreign interest rate is 0.20%.

Given the preceding parameter values, the economy features a unique bubbly steady state and a unique bubbleless steady state. The bubbly and bubbleless steady-state annual domestic interest rates are 0.6% and 0.28%, respectively. There are capital inflows in both steady states. We can check numerically that both steady states are saddle points. We will focus on local dynamics around these two steady states.

5.1 A Small Unanticipated Rise in the Foreign Interest Rate

We first study the impact of an unanticipated permanent rise in the foreign interest rate. We consider two economies with bubbles and without bubbles respectively. The bubbly (bubbleless) economy initially stays at the bubbly (bubbleless) steady state until period 10. Suppose that the annual foreign interest rate is unexpectedly announced to be raised by 25 basis points in period 11. This rise of the foreign interest rate is small so that the bubble can be sustained in the long run (see Proposition 5). Figure 1 displays the transition dynamics in response to this policy change. The solid lines describe the transition dynamics of the bubbly economy, and the dashed lines describe the transition dynamics of the bubbleless economy. Both economies will converge to their new steady states.

[**Insert Figure 1 Here**]

We start by the bubbly economy. In response to a small rise in the foreign interest rate, bankers borrow from domestic firms by selling bonds to invest in the international financial market so that capital begins to flow out of the domestic economy. The initial capital inflow immediately turns into capital outflows. Capital outflows reduce resources for domestic firms to invest in the land bubble. As a result, the land bubble drops substantially. After a sudden drop, the land bubble gradually converges to a lower level in the new bubbly steady state. Despite the increase in the foreign interest rate, the domestic interest rate declines initially and then rises gradually. This is because the drop of the land bubble lowers the collateral value. As a result, firms can raise less debt backed by the lower collateral value to finance investment. That is, the bond supply by firms declines immediately so that the domestic interest rate falls immediately. Later on, the domestic interest rate rises as the total bond supply from domestic bankers rises.

The capital outflow increases the supply of domestic goods to the rest of the world, inducing real depreciation. The real depreciation increases exports and decreases imports. The increase in the trade surplus, together with the increase in the interest income from foreign investments, implies an increase in the current account.

The significant loss of collateral value from the asset bubbles depresses investment so that the capital stock declines. This is the crowd-in effect of bubbles. On the other hand, the fall of land prices cause fewer inefficient firms to hold bubbles and hence the investment threshold falls on impact. These firms do not make investment. This is the crowd-out effect. Our numerical solutions show that the crowd-in effect dominates so that aggregate investment falls when land prices fall. Output also decreases on impact due to the declines of imported material input. Even though capital is predetermined and labor is fixed in our model, output still drops initially due to the real exchange rate channel in that the real depreciation causes the imported material input drops immediately.[8] This results in declines of consumption.

Now we turn to the bubbleless economy. In this case the land bubble collateral channel disappears. When the foreign interest rate rises, domestic bankers borrow money from domestic agents by selling bonds to invest in the foreign financial market, causing the domestic interest rate to rise. In the meantime, capital flight starts, which causes real depreciation. The qualitative responses of other variables in the bubbleless economy are similar to those in the bubbly economy. The intuition is also similar. The key difference is that the responses of the bubbly economy are much larger than those of the bubbleless economy. The main reason is due to the land bubble collateral channel. Since there is a large drop of the value of the land collateral, the drop of investment is much larger in the bubbly economy than in the bubbleless economy. As a result, the responses of output, consumption, imports, exports, real exchange rate, and capital flows are much larger. This means that asset bubbles provide an important amplification mechanism.

5.2 A Large Unanticipated Rise in the Foreign Interest Rate

In this subsection we consider the effects of a large unanticipated permanent rise in the foreign interest rate. Again suppose that the annual foreign interest rate is initially 0.20% until period 10. In period 11, it is unexpectedly announced to be raised by 75 basis points which is high enough to make the bubble no longer sustainable in the new steady state (see Proposition 5). Figure 2 shows the transition dynamics of the initial bubbly economy and the bubbleless economy.

[**Insert Figure 2 Here**]

Comparing Figures 1 and 2, we find that the transition dynamics are qualitatively similar with two key differences. First, for the initial bubbly economy, the land bubble immediately bursts as soon as the foreign interest rate rises. This is because the bubble cannot exist in the new steady state after the foreign interest rate rises to the new high level in the long run. Rational agents

[8] Other ways to enhance initial responses of output include capacity utilization and labor hoarding.

anticipate this so that the bubble must burst immediately. Second, the dynamic responses to a large rise in the foreign interest rate are much larger for the bubbly economy. This is because the burst of the land bubble reduces the collateral value significantly and hence tightens credit constraints significantly. The large tightening of credit has a large adverse impact on the macroeconomy. For example, output initially drops by about 0.5% in response to a large rise in the foreign interest rate, which is much larger than the initial 0.3% drop in response to a small rise studied in the previous subsection.

5.3 An Anticipated Rise in the Foreign Interest Rate

In this subsection we study the dynamic responses to an anticipated rise in the foreign interest rate. We focus on the case in which the economy is initially in the bubbly steady state. Suppose that there is a permanent increase in the foreign interest rate from 0.20% to 0.45% in period 11. This policy is anticipated by domestic agents in period 0. Figure 3 plots the transition dynamics. The solid lines describe the case of anticipated policy and the dashed lines describe the case of unanticipated policy studied earlier.

[**Insert Figure 3 Here**]

In anticipation of the future rise in the foreign interest rate, rational agents respond immediately. The transition dynamics are qualitatively similar to those in the case of unanticipated policy with the difference that the responses are shifted to the initial period. Since the rise in the foreign interest rate is small, the land bubble is sustained at a lower level in the long run. The land bubble drops immediately and gradually transits to the new steady state.

[**Insert Figure 4 Here**]

Figure 4 plots the transition dynamics for the case of a large increase in the foreign interest rate rises from 0.25% to 1.25% in period 10. In this case the land bubble cannot be sustained in the long run. If this rise in the foreign interest rate is anticipated in period 0, then the land bubble will burst immediately. But if this rise is not anticipated, then the land bubble stays at the initial steady state level until period 10.

6 Conclusion

We have provided a theoretical model to study the impact of the foreign interest rate shocks on capital flows, asset bubbles, and the real economy in a small open economy. Our main theoretical result is the characterization of the condition for the existence of an asset bubble. We show that low foreign interest rates are conducive to bubble formation. A rise in the foreign interest rate can generate the collapse of asset bubbles and a sudden stop. Its impact on the macroeconomy is larger compared to the case without an asset bubble. Asset bubbles provide an important amplification channel.

One limitation of our model is that it is stylized and cannot be confronted with data. To match the data, one has to introduce aggregate shocks (e.g., Miao, Wang, and Xu (2015)). Moreover, we have not studied policy questions because of space limitation. It would be interesting to introduce monetary and fiscal policy to our model. We leave these topics for future research.

A Appendix

Proof of Proposition 1 : We first consider firm j's decision problem. Solving the static labor and imported material choice problem in (3) gives demand for labor and imported material

$$N_{jt} = A_t^{\frac{1-\alpha-\gamma}{\alpha}} \left(\frac{1-\alpha-\gamma}{W_t}\right)^{\frac{1-\gamma}{\alpha}} \left(\frac{\gamma}{e_t}\right)^{\frac{\gamma}{\alpha}} K_{jt}, \tag{A.1}$$

$$M_{jt} = A_t^{\frac{1-\alpha-\gamma}{\alpha}} \left(\frac{1-\alpha-\gamma}{W_t}\right)^{\frac{1-\alpha-\gamma}{\alpha}} \left(\frac{\gamma}{e_t}\right)^{\frac{\alpha+\gamma}{\alpha}} K_{jt}. \tag{A.2}$$

Substituting these equations into (3) yields (4).

Now we solve the firm's dynamic problem. Conjecture that the value function $V_t(\varepsilon_{jt}, K_{jt}, H_{jt}, B_{jt})$ takes the following form

$$V_t(\varepsilon_{jt}, K_{jt}, H_{jt}, B_{jt}) = \phi_t^K(\varepsilon_{jt})K_{jt} + \phi_t^H(\varepsilon_{jt})H_{jt} + \phi_t^B(\varepsilon_{jt})B_{jt}, \tag{A.3}$$

where $\phi_t^K(\varepsilon_{jt})$, $\phi_t^H(\varepsilon_{jt})$ and $\phi_t^B(\varepsilon_{jt})$ are to be determined. In this case Tobin's Q satisfies

$$Q_t = \beta\frac{\Lambda_{t+1}}{\Lambda_t}\int \phi_{t+1}^K(\varepsilon)dF(\varepsilon). \tag{A.4}$$

Conjecture that

$$P_t = \beta\frac{\Lambda_{t+1}}{\Lambda_t}\int \phi_{t+1}^H(\varepsilon)dF(\varepsilon), \tag{A.5}$$

$$\frac{1}{R_{ft}} = \beta\frac{\Lambda_{t+1}}{\Lambda_t}\int \phi_{t+1}^B(\varepsilon)dF(\varepsilon). \tag{A.6}$$

Substituting the flow-of-funds constraint and the conjectured value function into the right-hand side of the Bellman equation, we obtain

$$\begin{aligned}
& D_{jt} + \beta E_t\frac{\Lambda_{t+1}}{\Lambda_t}V_{t+1}\left(\varepsilon_{jt+1}, K_{jt+1}, H_{jt+1}, B_{jt+1}\right) \\
= \; & R_{kt}K_{jt} - I_{jt} - \frac{B_{jt+1}}{R_{ft}} + B_{jt} + P_t\left(H_{jt} - H_{jt+1}\right) \\
& +(1-\delta)Q_tK_{jt} + \varepsilon_{jt}Q_tI_{jt} + \frac{B_{jt+1}}{R_{ft}} + P_tH_{jt+1} \\
= \; & R_{kt}K_{jt} + (1-\delta)Q_tK_{jt} + B_{jt} + P_tH_{jt} + \left(\varepsilon_{jt}Q_t - 1\right)I_{jt}. \qquad \text{(A.7)}
\end{aligned}$$

If $\varepsilon_{jt} < 1/Q_t$, the firm would not invest, i.e. $I_{jt} = 0$. And the firm is indifferent between saving and borrowing, and is different between purchasing and selling land. If $\varepsilon_{jt} \geq 1/Q_t$, the firm makes real investment as much as possible. Thus it exhausts the borrowing limit and sells land to finance investment. By (6), (7), (8), and (9), we have

$$\begin{aligned}
I_{jt} \;&\leq\; R_{kt}K_{jt} - \frac{B_{jt+1}}{R_{ft}} + B_{jt} + P_t\left(H_{jt} - H_{jt+1}\right) \\
&\leq\; R_{kt}K_{jt} + \mu K_{jt} + \theta P_tH_{jt} + B_{jt} + (1-\omega)\,P_tH_{jt}.
\end{aligned}$$

We then obtain (16).

Plugging the decision rules in the Bellman equation, we obtain

$$V_t(\varepsilon_{jt}, K_{jt}, H_{jt}, B_{jt}) = \begin{cases} R_{kt}K_{jt} + (1-\delta)K_{jt} + B_{jt} + P_tH_{jt} \\ +(\varepsilon_{jt}Q_t - 1)\left[R_{kt}K_{jt} + \mu K_{jt} + B_{jt} + (1-\omega+\theta)P_tH_{jt}\right], & \text{if } \varepsilon_{jt} \geq \bar{\varepsilon}; \\ R_{kt}K_{jt} + (1-\delta)K_{jt} + B_{jt} + P_tH_{jt}, & \text{if } \varepsilon_{jt} < \bar{\varepsilon}. \end{cases}$$

Matching coefficients in the preceding equation and equation (A.3) and making use of equation (A.4), (A.5) and (A.6) yield the equations in Proposition 1. Q.E.D.

Proof of Proposition 2 : Equation (24) follows from the first-order condition for the banker's decision problem. We use the decision rule in Proposition 1 and the Law of Large Numbers to derive aggregate investment in equation (25)

$$\begin{aligned} I_t &= \int_{\varepsilon_{jt}<\bar{\varepsilon}} 0 \cdot dj + \int_{\varepsilon_{jt}\geq\bar{\varepsilon}} \left[R_{kt}K_{jt} + \mu K_{jt} + B_{jt} + (1-\omega+\theta)P_tH_{jt}\right] dj \\ &= \left[R_{kt}K_t + \mu K_t + B_t + (1-\omega+\theta)P_t\right]\left(\int_{\bar{\varepsilon}}^{\varepsilon_{max}} f(\varepsilon)d\varepsilon\right), \end{aligned}$$

where the last equality is due to the fact that ε is IID. Equation (26) follows from aggregating equation (5).

Substituting (A.1) into the expression of aggregate labor, we have

$$\begin{aligned} N_t &= \int_0^1 N_{jt}dj = \int_0^1 A_t^{\frac{1-\alpha-\gamma}{\alpha}} \left(\frac{1-\alpha-\gamma}{W_t}\right)^{\frac{1-\gamma}{\alpha}} \left(\frac{\gamma}{e_t}\right)^{\frac{\gamma}{\alpha}} K_{jt}dj \\ &= A_t^{\frac{1-\alpha-\gamma}{\alpha}} \left(\frac{1-\alpha-\gamma}{W_t}\right)^{\frac{1-\gamma}{\alpha}} \left(\frac{\gamma}{e_t}\right)^{\frac{\gamma}{\alpha}} K_t. \end{aligned}$$

Equation (30) follows from the preceding equation and the market-clearing condition $N_t = 1$.

Substituting (A.2) into the expression of aggregate imported material, we have

$$\begin{aligned} M_t &= \int_0^1 M_{jt}dj = \int_0^1 A_t^{\frac{1-\alpha-\gamma}{\alpha}} \left(\frac{1-\alpha-\gamma}{W_t}\right)^{\frac{1-\alpha-\gamma}{\alpha}} \left(\frac{\gamma}{e_t}\right)^{\frac{\alpha+\gamma}{\alpha}} K_{jt}dj \\ &= A_t^{\frac{1-\alpha-\gamma}{\alpha}} \left(\frac{1-\alpha-\gamma}{W_t}\right)^{\frac{1-\alpha-\gamma}{\alpha}} \left(\frac{\gamma}{e_t}\right)^{\frac{\alpha+\gamma}{\alpha}} K_t. \end{aligned} \tag{A.8}$$

Substituting (A.1) and (A.2) into the expression of the aggregate output Y_t, we can derive

$$\begin{aligned} Y_t &= \int_0^1 K_{jt}^{\alpha}(A_tN_{jt})^{1-\alpha-\gamma}M_{jt}^{\gamma}dj \\ &= \int_0^1 K_{jt}^{\alpha}A_t^{1-\alpha-\gamma}A_t^{\frac{1-\alpha-\gamma}{\alpha}(1-\alpha-\gamma)} \left(\frac{1-\alpha-\gamma}{W_t}\right)^{\frac{1-\gamma}{\alpha}(1-\alpha-\gamma)} \left(\frac{\gamma}{e_t}\right)^{\frac{\gamma}{\alpha}(1-\alpha-\gamma)} K_{jt}^{1-\alpha-\gamma} \\ &\quad A_t^{\frac{1-\alpha-\gamma}{\alpha}\gamma} \left(\frac{1-\alpha-\gamma}{W_t}\right)^{\frac{1-\alpha-\gamma}{\alpha}\gamma} \left(\frac{\gamma}{e_t}\right)^{\frac{\alpha+\gamma}{\alpha}\gamma} K_{jt}^{\gamma}dj \\ &= A_t^{\frac{1-\alpha-\gamma}{\alpha}} \left(\frac{1-\alpha-\gamma}{W_t}\right)^{\frac{1-\alpha-\gamma}{\alpha}} \left(\frac{\gamma}{e_t}\right)^{\frac{\gamma}{\alpha}} K_t. \end{aligned}$$

Using the last equation and the preceding expressions of N_t and M_t, we can derive equations (27) and (28).

We substitute the flow-of-funds constraints for the firms and bankers, (8) and (10) into the budget constraint of the households (11), and obtain

$$C_t = Y_t - e_t M_t - I_t - \frac{B_{t+1}}{R_{ft}} + B_t \frac{R^*_{ft-1}}{R_{ft-1}} \frac{e_t}{e_{t-1}}.$$

Then comparing it with the resource constraint will directly leads to (23). Finally, we use (4), (30), and (A.8) to derive (31). Q.E.D.

Detrended Equilibrium System: The variables Q_t, $\bar{\varepsilon}_t$, e_t and R_{ft} do not have trend. All other equilibrium variables in Proposition 2 grows at the rate of technical progress except for Λ_t. Let $\Lambda_t = \lambda_t / A_t$ and any other growing variable $Z_t = z_t A_t$. We then obtain the detrended equilibrium system:

$$(1+g)\,Q_t = \beta \frac{\lambda_{t+1}}{\lambda_t}\left[R_{kt+1} + (1-\delta)\,Q_{t+1} + (R_{kt+1}+\mu)\int_{\bar{\varepsilon}}^{\varepsilon_{\max}} (\varepsilon Q_{t+1} - 1)\, f(\varepsilon) d\varepsilon\right], \tag{A.9}$$

$$p_t = \beta \frac{\lambda_{t+1}}{\lambda_t} p_{t+1}\left[1 + (1-\omega+\theta)\int_{\bar{\varepsilon}}^{\varepsilon_{\max}} (\varepsilon Q_{t+1} - 1)\, f(\varepsilon) d\varepsilon\right], \tag{A.10}$$

$$\frac{1+g}{R_{ft}} = \beta \frac{\lambda_{t+1}}{\lambda_t}\left[1 + \int_{\bar{\varepsilon}_t}^{\varepsilon_{\max}} (\varepsilon Q_{t+1} - 1)\, f(\varepsilon) d\varepsilon\right], \tag{A.11}$$

$$\begin{aligned} 0 &= \frac{\lambda_{t+1}}{\lambda_t}\left[\frac{R^*_{ft}}{R_{ft}}\frac{e_{t+1}}{e_t} - 1 - \frac{\Omega}{y_{t+1}}\left(b_{t+1} - \frac{b_t}{1+g}\right)\right] \\ &\quad + \frac{\beta}{1+g}\frac{\lambda_{t+2}}{\lambda_t}\frac{\Omega}{y_{t+2}}\left(b_{t+2} - \frac{b_{t+1}}{1+g}\right), \end{aligned} \tag{A.12}$$

$$i_t = [R_{kt}k_t + \mu k_t + b_t + (1-\omega+\theta)p_t]\int_{\bar{\varepsilon}}^{\varepsilon_{\max}} f(\varepsilon) d\varepsilon, \tag{A.13}$$

$$(1+g)k_{t+1} = (1-\delta)k_t + [R_{kt}k_t + \mu k_t + b_t + (1-\omega+\theta)p_t]\left(\int_{\bar{\varepsilon}}^{\varepsilon_{max}} \varepsilon f(\varepsilon) d\varepsilon\right), \tag{A.14}$$

$$y_t = k_t^{\alpha} m_t^{\gamma}, \tag{A.15}$$

$$e_t m_t = \gamma y_t, \tag{A.16}$$

$$y_t = c_t + i_t + x_t, \tag{A.17}$$

$$x_t = e_t^{\sigma} y_t^*, \tag{A.18}$$

$$x_t - e m_t = (1+g)\frac{b_{t+1}}{R_{ft}} - b_t \frac{R^*_{ft-1}}{R_{ft-1}}\frac{e_t}{e_{t-1}}, \tag{A.19}$$

$$1 = \left(\frac{1-\alpha-\gamma}{w_t}\right)^{\frac{1-\gamma}{\alpha}}\left(\frac{\gamma}{e_t}\right)^{\frac{\gamma}{\alpha}} k_t, \tag{A.20}$$

$$R_{kt} = \alpha k_t^{\alpha-1} m_t^{\gamma}, \tag{A.21}$$

for the endogenous variables $\{Q_t, \bar{\varepsilon}_t, p_t, R_{ft}, b_{t+1}, i_t, k_{t+1}, y_t, m_t, c_t, x_t, e_t, w_t, R_{kt}\}$, where $\lambda_t = 1/c_t$. The usual transversality conditions hold.

Proof of Lemma 1 Equation (A.12) in the steady states is

$$0 = \frac{R_f^*}{R_f} - 1 - \Omega \frac{g}{1+g} \frac{b}{y} + \Omega \frac{\beta}{1+g} \frac{g}{1+g} \frac{b}{y}. \tag{A.22}$$

We then obtain (34). According to the definition in Section 3.2, the current account in the detrended equilibria is

$$ca_t = (1+g)\frac{b_{t+1}}{R_{ft}} - \frac{b_t}{R_{ft-1}}$$

which leads to (35) in the steady state directly. Q.E.D.

Lemma 2 *When $\mu > 0$ is sufficiently small,* $\frac{\partial R_k(\bar{\varepsilon})}{\partial \bar{\varepsilon}} < 0$.

Proof. In steady state, substituting $Q = 1/\bar{\varepsilon}$ into (A.9), we can derive

$$\begin{aligned} R_k &= \frac{(1+g-\beta+\beta\delta)/\bar{\varepsilon} - \beta\mu \int_{\bar{\varepsilon}}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\bar{\varepsilon}} - 1\right) f(\varepsilon) d\varepsilon}{\beta\left[1 + \int_{\bar{\varepsilon}}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\bar{\varepsilon}} - 1\right) f(\varepsilon) d\varepsilon\right]} \\ &= \frac{(1+g-\beta+\beta\delta) - \beta\mu \int_{\bar{\varepsilon}}^{\varepsilon_{\max}} (\varepsilon - \bar{\varepsilon}) f(\varepsilon) d\varepsilon}{\beta\left[\bar{\varepsilon} + \int_{\bar{\varepsilon}}^{\varepsilon_{\max}} (\varepsilon - \bar{\varepsilon}) f(\varepsilon) d\varepsilon\right]}, \end{aligned} \tag{A.23}$$

which is equation (32).

The partial derivative of R_k with respect to $\bar{\varepsilon}$ is

$$\frac{\partial R_k}{\partial \bar{\varepsilon}} = \frac{\mu\beta \int_{\bar{\varepsilon}}^{\varepsilon_{max}} \varepsilon f(\varepsilon) d\varepsilon - (1+g-\beta+\beta\delta) F(\bar{\varepsilon})}{\beta\left[\bar{\varepsilon} + \int_{\bar{\varepsilon}}^{\varepsilon_{max}} (\varepsilon - \bar{\varepsilon}) f(\varepsilon) d\varepsilon\right]^2}.$$

When $\mu = 0$, it is negative. So when μ is sufficiently small, it is also negative. ■

Proof of Proposition 3 In the bubbleless steady state with $p = 0$, we divide both sides of equation (A.14) by k to derive

$$\delta + g = \left(R_k + \mu + \frac{b}{k}\right) \int_{\bar{\varepsilon}}^{\varepsilon_{\max}} \varepsilon f(\varepsilon) d\varepsilon.$$

Since $R_k = \alpha y/k$, we then obtain (36). In that equation, R_k is a decreasing function of $\bar{\varepsilon}$. By (33), R_f is an increasing function of $\bar{\varepsilon}$. Thus, the expression on the right-hand side of equation (36) is a decreasing function of $\bar{\varepsilon}$. This expression takes value 0 when $\bar{\varepsilon} = \varepsilon_{\max}$ and is larger than $\delta + g$ when $\bar{\varepsilon} = \varepsilon_{\min}$ by assumption 1. Thus it follows from the intermediate value theorem that there is a unique solution for $\bar{\varepsilon} \in (\varepsilon_{\min}, \varepsilon_{\max})$, denoted by $\bar{\varepsilon}_f$, to the equation (36).

After determining $\bar{\varepsilon}_f$, we can derive $Q_f = 1/\bar{\varepsilon}_f$, $R_{kf} = R_k\left(\bar{\varepsilon}_f\right)$, and $R_{ff} = R_f\left(\bar{\varepsilon}_f\right)$ by definition and equations (32) and (33). By Lemma 1, $b_f/y_f = b/y\left(\bar{\varepsilon}_f\right)$. By (A.13), we can derive

$$\frac{i_f}{y_f} = \left[R_{kf}\frac{k_f}{y_f} + \mu\frac{k_f}{y_f} + \frac{b_f}{y_f}\right]\int_{\bar{\varepsilon}_f}^{\varepsilon_{max}} f(\varepsilon)d\varepsilon.$$

Since $R_{kf} = \alpha y_f/k_f$ by (A.21), we have

$$\frac{i_f}{y_f} = \left[\alpha + \frac{\alpha\mu}{R_{kf}} + \frac{b_f}{y_f}\right]\int_{\bar{\varepsilon}_f}^{\varepsilon_{max}} f(\varepsilon)d\varepsilon.$$

Substituting (A.16) into equation (A.19) and dividing both sides by y_t, we obtain

$$\frac{x_t}{y_t} = \gamma + (1+g)\frac{b_{t+1}}{y_t R_{ft}} - \frac{b_t}{y_t}\frac{R^*_{ft-1}}{R_{ft-1}}\frac{e_t}{e_{t-1}}. \tag{A.24}$$

In the bubbleless steady state, the equation above implies

$$\frac{x_f}{y_f} = \gamma + \frac{b_f}{y_f}\frac{1+g-R^*_f}{R_{ff}}. \tag{A.25}$$

Since exports are positive by equation (1), we need the expression above to be positive, which gives the second inequality in condition (37).

We use the resource constraint to derive

$$\begin{aligned}\frac{c_f}{y_f} &= 1 - \frac{i_f}{y_f} - \frac{x_f}{y_f} \\ &= 1 - \alpha\left(1 + \frac{\mu}{R_{kf}} + \frac{b_f}{y_f}\right)\left(\int_{\bar{\varepsilon}}^{\varepsilon_{\max}} f(\varepsilon)d\varepsilon\right) - \gamma - \frac{b_f}{y_f}\frac{1+g-R^*_f}{R_{ff}}.\end{aligned} \tag{A.26}$$

To ensure $c_f > 0$, we impose the first inequality in condition (37).

We now determine the real exchange rate. By (A.16) and $R_{kt} = \alpha y_t/k_t$, we have

$$\frac{m_t}{k_t} = \frac{R_{kt}}{\alpha}\frac{\gamma}{e_t}.$$

Thus

$$R_{kt} = \alpha k_t^{\alpha-1} m_t^{\gamma} = \alpha k_t^{\alpha-1}\left(\frac{R_{kt}}{\alpha}\frac{\gamma}{e_t}k_t\right)^{\gamma},$$

which leads to

$$k_t = \left(\frac{\alpha}{R_{kt}}\right)^{\frac{1-\gamma}{1-\alpha-\gamma}}\left(\frac{\gamma}{e_t}\right)^{\frac{\gamma}{1-\alpha-\gamma}}, \tag{A.27}$$

and

$$m_t = \frac{R_{kt}}{\alpha}\frac{\gamma}{e_t}k_t = \left(\frac{\alpha}{R_{kt}}\right)^{\frac{\alpha}{1-\alpha-\gamma}}\left(\frac{\gamma}{e_t}\right)^{\frac{1-\alpha}{1-\alpha-\gamma}}. \tag{A.28}$$

Using these two equations, we can derive

$$y_t = k_t^{\alpha} m_t^{\gamma} = \left(\frac{\alpha}{R_{kt}}\right)^{\frac{\alpha}{1-\alpha-\gamma}}\left(\frac{\gamma}{e_t}\right)^{\frac{\gamma}{1-\alpha-\gamma}}. \tag{A.29}$$

Using the equation above and (A.18), we obtain

$$\frac{x_t}{y_t} = e_t^{\sigma} y_t^* \left(\frac{R_{kt}}{\alpha}\right)^{\frac{\alpha}{1-\alpha-\gamma}} \left(\frac{e_t}{\gamma}\right)^{\frac{\gamma}{1-\alpha-\gamma}}. \tag{A.30}$$

In the bubbleless steady state, this equation becomes

$$\frac{x_f}{y_f} = e_f^{\sigma} y^* \left(\frac{R_{kf}}{\alpha}\right)^{\frac{\alpha}{1-\alpha-\gamma}} \left(\frac{e_f}{\gamma}\right)^{\frac{\gamma}{1-\alpha-\gamma}}.$$

Combining this equation with equation (A.25), we obtain

$$e_f^{\sigma} y^* \left(\frac{R_{kf}}{\alpha}\right)^{\frac{\alpha}{1-\alpha-\gamma}} \left(\frac{e_f}{\gamma}\right)^{\frac{\gamma}{1-\alpha-\gamma}} = \gamma + \frac{b_f}{y_f}\frac{1+g-R_f^*}{R_{ff}}.$$

This equation can be used to solve for a closed-form expression for e_f given that R_{kf}, R_{ff}, and b_f/y_f are determined by $\bar{\varepsilon}$.

After e_f is determined, we can use equation (A.18) to solve for x_f. Then k_f, m_f and y_f can be derived directly using equations (A.27), (A.28) and (A.29). Since we have already solved b_f/y_f, i_f/y_f, c_f/y_f, x_f/y_f, and y_f, we can derive b_f, i_f, c_f, and x_f. Finally, we use (A.20) to solve for w_f. Q.E.D.

Proof of Proposition 4 : Suppose that a bubbly steady state exists. We want to derive condition (40). In the bubbly steady state with $p > 0$, we use (A.10) to derive (39). We use (A.14) to derive

$$\begin{aligned}(1-\omega+\theta)\frac{p}{y_b} &= \frac{\alpha}{R_{kb}}\left(\frac{\delta+g}{\int_{\bar{\varepsilon}_b}^{\varepsilon_{\max}}\varepsilon f(\varepsilon)d\varepsilon} - \mu\right) - \alpha - \frac{b_b}{y_b} \\ &= \frac{\alpha}{R_{kb}}\left(\frac{\delta+g}{\int_{\bar{\varepsilon}_b}^{\varepsilon_{max}}\varepsilon f(\varepsilon)d\varepsilon} - \mu\right) - \alpha - \frac{\frac{R_f^*}{R_{fb}} - 1}{\Omega\frac{g}{1+g}\left(1-\frac{\beta}{1+g}\right)} > 0.\end{aligned} \tag{A.31}$$

In the bubbleless steady state, we use (A.14) to derive

$$0 = \frac{\alpha}{R_{kf}}\left(\frac{\delta+g}{\int_{\bar{\varepsilon}_f}^{\varepsilon_{\max}}\varepsilon f(\varepsilon)d\varepsilon} - \mu\right) - \alpha - \frac{\frac{R_f^*}{R_{ff}} - 1}{\Omega\frac{g}{1+g}\left(1-\frac{\beta}{1+g}\right)}. \tag{A.32}$$

Since R_k decreases in $\bar{\varepsilon}$ by Lamma 2 and R_f increases in $\bar{\varepsilon}$ by (33), we can deduce that

$$\frac{\alpha}{R_k}\left(\frac{\delta+g}{\int_{\bar{\varepsilon}}^{\varepsilon_{\max}}\varepsilon f(\varepsilon)d\varepsilon} - \mu\right) - \alpha - \frac{\frac{R_f^*}{R_f} - 1}{\Omega\frac{g}{1+g}\left(1-\frac{\beta}{1+g}\right)}$$

increases in $\bar{\varepsilon}$. It follows from (A.31) and (A.32) that $\bar{\varepsilon}_b > \bar{\varepsilon}_f$. Thus we use (39) to deduce condition (40).

Now suppose that (40) holds. We want to derive a bubbly steady state. The expression on the right-hand side of equation (39) is a decreasing function of $\bar{\varepsilon}$. It takes the value $\beta < 1$ when

$\bar{\varepsilon} = \varepsilon_{\max}$ and a value larger than 1 when $\bar{\varepsilon} = \bar{\varepsilon}_f$ by (40). By the intermediate value theorem, there is a unique solution, denoted by $\bar{\varepsilon}_b$, to equation (39).

After determining $\bar{\varepsilon}_b$, we can derive $Q_b = 1/\bar{\varepsilon}_b$, $R_{kb} = R_k(\bar{\varepsilon}_b)$, and $R_{fb} = R_f(\bar{\varepsilon}_b)$ by definition and equations (32) and (33). By Lemma 1, $b_b/y_b = b/y(\bar{\varepsilon}_b)$. We then use (A.14) to derive

$$\begin{aligned}
(1-\omega+\theta)\frac{p}{y_b} &= \frac{\alpha}{R_{kb}}\left(\frac{\delta+g}{\int_{\bar{\varepsilon}_b}^{\varepsilon_{\max}}\varepsilon f(\varepsilon)d\varepsilon}-\mu\right)-\alpha-\frac{b}{y}(\bar{\varepsilon}_b) \\
&> \frac{\alpha}{R_{kf}}\left(\frac{\delta+g}{\int_{\bar{\varepsilon}_f}^{\varepsilon_{\max}}\varepsilon f(\varepsilon)d\varepsilon}-\mu\right)-\alpha-\frac{b}{y}(\bar{\varepsilon}_f) \\
&= 0,
\end{aligned} \tag{A.33}$$

where the inequality is due to the fact that R_k and $\int_{\bar{\varepsilon}}^{\varepsilon_{max}}\varepsilon f(\varepsilon)d\varepsilon$ decrease in $\bar{\varepsilon}$ and R_f increases in $\bar{\varepsilon}$. The last equality follows from (36) and $R_{kf} = R_k(\bar{\varepsilon}_f)$. Thus $p > 0$.

Next we solve for the bubbly steady-state real exchange rate e_b. We use a similar procedure in the proof of Proposition 3. Equations (A.24) and (A.30) hold in the bubbly equilibrium. They give the steady state equation

$$e_b^{\sigma}y^*\left(\frac{R_{kb}}{\alpha}\right)^{\frac{\alpha}{1-\alpha-\gamma}}\left(\frac{e_b}{\gamma}\right)^{\frac{\gamma}{1-\alpha-\gamma}} = \gamma+\frac{b_b}{y_b}\frac{1+g-R_f^*}{R_{fb}}.$$

When the second inequality in (40) holds, the expression on the right-hand side is positive so that the equation above gives a unique solution for e_b.

We can then solve for x_b using equation (A.18), and solve for k_b, m_b and y_b using equations (A.27), (A.28), and (A.29). Since we have solved for b_b/y_b and y_b, we can derive b_b. We can solve for p using (A.33) and solve for i_b and w_b using equations (A.13) and (A.20). Finanlly, we solve for c_b using equation (A.17). We need the first inequality in (40) to ensure $c_b > 0$. Q.E.D.

Proof of Proposition 5: It follows from Lemma 1 that b/y increases with R_f^*. By (36), $\bar{\varepsilon}_f$ must increase with R_f^*. When R_f^* is small, $\bar{\varepsilon}_f$ is small and hence the expression on the right-hand side of (38) is large. Thus condition (38) is likely to hold so that a land bubble is likely to exist. By (33), the domestic bubbleless steady-state interest rate R_{ff} also decreases as R_f^* decreases. But when R_f^* is sufficiently high, $\bar{\varepsilon}_f$ approaches $\varepsilon_{\max}$. In this case condition (38) cannot hold and hence a bubble cannot exist. Q.E.D.

Proof of Proposition 6: By Propositions 3 and 4, we know that $\bar{\varepsilon}_b > \bar{\varepsilon}_f$. Thus $Q_b < Q_f$. By Lemma 2, $R_{kb} < R_{kf}$. By (33), $R_{fb} > R_{ff}$. By (A.24) and (A.30), we know that

$$\frac{x_t}{y_t} = e_t^{\sigma}y_t^*\left(\frac{R_{kt}}{\alpha}\right)^{\frac{\alpha}{1-\alpha-\gamma}}\left(\frac{e_t}{\gamma}\right)^{\frac{\gamma}{1-\alpha-\gamma}} = \gamma+(1+g)\frac{b_{t+1}}{R_{ft}}-b_t\frac{R_{ft-1}^*}{R_{ft-1}}\frac{e_t}{e_{t-1}}. \tag{A.34}$$

Now show that
$$\frac{b_b}{y_b}\frac{1+g-R_f^*}{R_{fb}} < \frac{b_f}{y_f}\frac{1+g-R_f^*}{R_{ff}}, \tag{A.35}$$
when condition (42) holds. We consider three cases.

Case (i): When $R_f^* < R_{ff}$, $b_b < 0$ and $b_f < 0$ by Lemma 1. Applying Lemma 1 again and (42), we deduce that

$$\begin{aligned}
& \frac{b_b}{y_b}\frac{1+g-R_f^*}{R_{fb}} - \frac{b_f}{y_f}\frac{1+g-R_f^*}{R_{ff}} \\
= & \frac{1+g-R_f^*}{\Omega\frac{g}{1+g}\left(1-\frac{\beta}{1+g}\right)}\left[\left(\frac{R_f^*}{R_{fb}}-1\right)\frac{1}{R_{fb}} - \left(\frac{R_f^*}{R_{ff}}-1\right)\frac{1}{R_{ff}}\right] \\
= & \frac{1+g-R_f^*}{\Omega\frac{g}{1+g}\left(1-\frac{\beta}{1+g}\right)}\left(\frac{1}{R_{fb}}-\frac{1}{R_{ff}}\right)\left(\frac{R_f^*}{R_{fb}}+\frac{R_f^*}{R_{ff}}-1\right) \\
< & \ 0.
\end{aligned} \tag{A.36}$$

Case (ii): When $R_{ff} < R_f^* < R_{fb}$, $b_b < 0$ and $b_f > 0$. We then immediately obtain (A.35).

Case (iii): When $R_f^* > R_{fb}$, $b_b > 0$ and $b_f > 0$. Since $R_{fb} > R_{ff}$, condition (42) implies (A.35).

Using (A.34) and (A.35), we deduce the following inequality for the bubbly and bubbleless steady states:

$$\begin{aligned}
\frac{x_f}{y_f} &= e_f^{\sigma+\frac{\gamma}{1-\alpha-\gamma}}\frac{y^*}{\gamma^{\frac{\gamma}{1-\alpha-\gamma}}}\left(\frac{R_{kf}}{\alpha}\right)^{\frac{\alpha}{1-\alpha-\gamma}} = \gamma + \frac{b_f}{y_f}\frac{1+g-R_f^*}{R_{ff}} \\
> \frac{x_b}{y_b} &= e_b^{\sigma+\frac{\gamma}{1-\alpha-\gamma}}\frac{y^*}{\gamma^{\frac{\gamma}{1-\alpha-\gamma}}}\left(\frac{R_{kb}}{\alpha}\right)^{\frac{\alpha}{1-\alpha-\gamma}} = \gamma + \frac{b_b}{y_b}\frac{1+g-R_f^*}{R_{fb}}.
\end{aligned} \tag{A.37}$$

Suppose $y_b < y_f$. Given the inequality above, we must have $x_b < x_f$. Since $x = e^\sigma y^*$ in a steady state, then $e_b < e_f$. Given $R_{kb} < R_{kf}$ and $e_b < e_f$, we have $y_b > y_f$ by equation (A.29). This leads to a contradiction. Therefore we must have $y_b > y_f$. Since $R_{kb} = \alpha\frac{y_b}{k_b} < \alpha\frac{y_f}{k_f} = R_{kf}$, we have $k_b > k_f$. Q.E.D.

Proof of Proposition 7:

1. When bubble exists, $\bar{\varepsilon}_b$ is determined by equation (39) which does not depend on R_f^*. Thus Q_b, R_{kb} and R_{fb} do not depend on R_f^* either. They all remain constant when R_f^* varies.

2. By Lemma 1, $\frac{b_b}{y_b}$ and $\frac{ca_b}{y_b}$ increase with R_f^*. By (41), $\frac{p}{y_b}$ decreases with R_f^*.

3. By Lemma 1,
$$\frac{b_b}{y_b}\frac{1+g-R_f^*}{R_{fb}} = \frac{1}{\Omega\frac{g}{1+g}\left(1-\frac{\beta}{1+g}\right)}\left(\frac{R_f^*-R_{fb}}{R_{fb}}\right)\frac{1+g-R_f^*}{R_{fb}}.$$

It follows that the expression above increases in R_f^* for $R_f^* < (R_{fb} + 1 + g)/2$. Since the real exchange rate e_b is determined by the last equation in (A.37). It follows that e_b increases with R_f^* for $R_f^* < (R_{fb} + 1 + g)/2$. Thus x_b increases with R_f^* by (A.18). Since R_{kb} does not depend on R_f^*, it follows from (A.29) that y_b decreases in R_f^*. Equation (A.16) implies that m_b also decreases with R_f^*. Given $R_{kb} = \alpha \frac{y_b}{k_b}$ is independent of R_f^*, k_b decreases with R_f^*.

Comparing equation (A.13) and (A.14) gives us

$$\frac{i_b}{(\delta + g)k_b} = \frac{\int_{\bar{\varepsilon}_b}^{\varepsilon_{\max}} f(\varepsilon)d\varepsilon}{\int_{\bar{\varepsilon}_b}^{\varepsilon_{\max}} \varepsilon f(\varepsilon)d\varepsilon}.$$

Since $\bar{\varepsilon}_b$ does not depend on R_f^* and k_b decreases with R_f^*, i_b also decreases with R_f^*. At the same time, equation (A.13) also implies that

$$\frac{i_b}{y_b} = \frac{\int_{\bar{\varepsilon}_b}^{\varepsilon_{\max}} f(\varepsilon)d\varepsilon}{\int_{\bar{\varepsilon}_b}^{\varepsilon_{\max}} \varepsilon f(\varepsilon)d\varepsilon} \frac{\alpha(\delta + g)}{R_{kb}}.$$

Thus $\frac{i_b}{y_b}$ does not vary with R_f^*. By equation (A.37), $\frac{x_b}{y_b}$ increases with R_f^*. Thus

$$\frac{c_b}{y_b} = 1 - \frac{i_b}{y_b} - \frac{x_b}{y_b}$$

decreases with R_f^*. As y_b decreases with R_f^*, c_b must too.

The opposite result holds when $R_f^* > (1 + g + R_{fb})/2$. Q.E.D.

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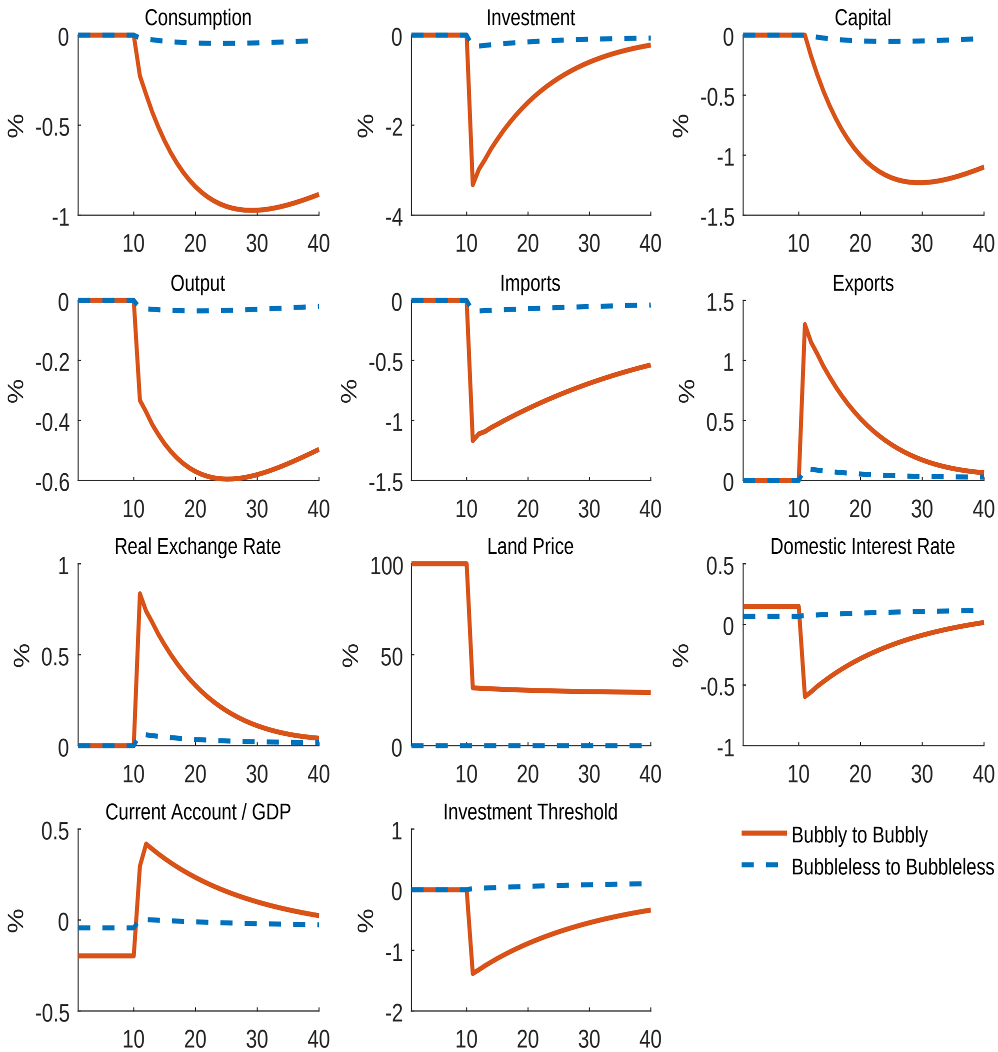

Figure 1: This figure plots the transition paths for the domestic economy when it is initially at the bubbly or bubbleless steady state until period 10 and the foreign interest rate is unexpectedly raised from 0.20% to a permanent level of 0.45% starting from period 11. The domestic interest rate is annualized and expressed as levels in percentage. The vertical axis in the panel for the current-account-to-GDP ratio is measured in percentage. Other vertical axes are expressed as percentage deviations from the initial steady state.

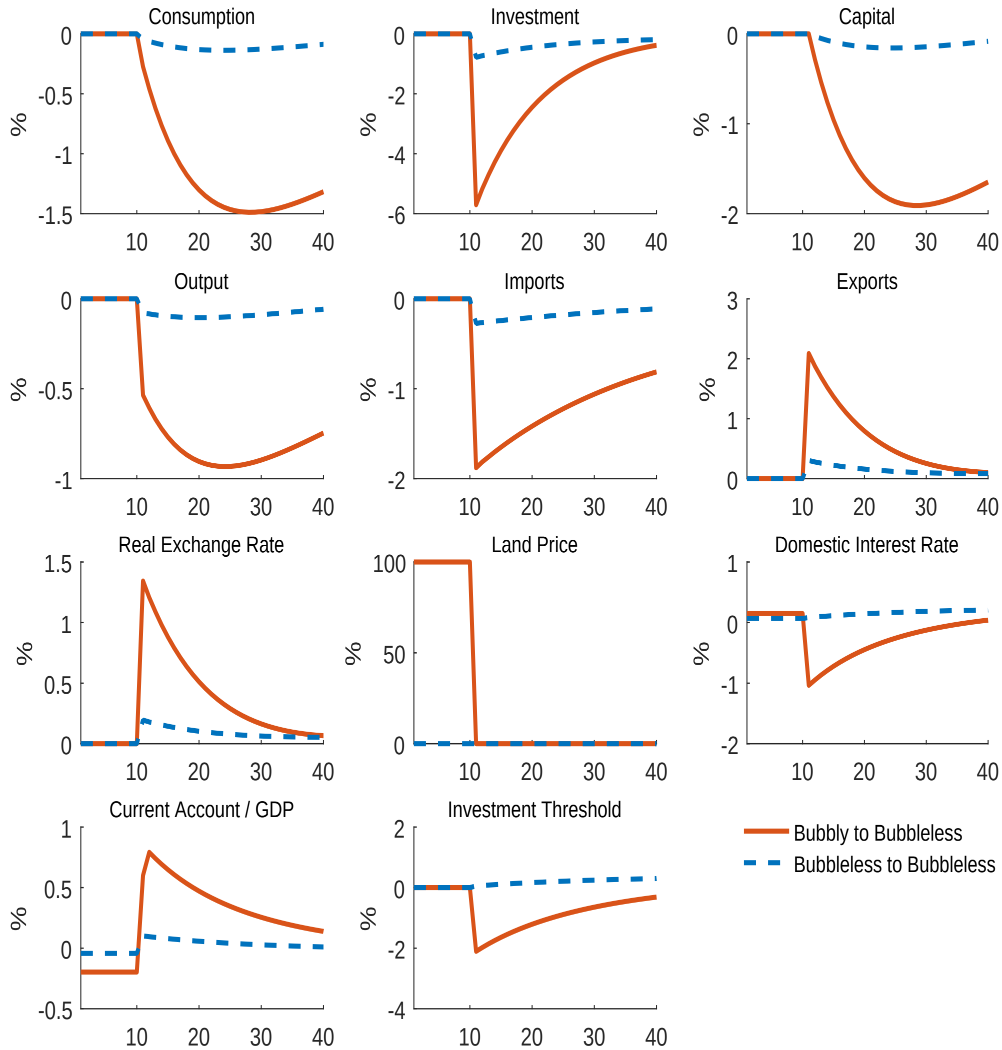

Figure 2: This figure plots the transition paths for the domestic economy when it is initially at the bubbly or bubbleless steady state until period 10 and the foreign interest rate is unexpectedly raised from 0.20% to a permanent level of 0.95% starting from period 11. The domestic interest rate is annualized and expressed as levels in percentage. The vertical axis in the panel for the current-account-to-GDP ratio is measured in percentage. Other vertical axes are expressed as percentage deviations from the initial steady state.

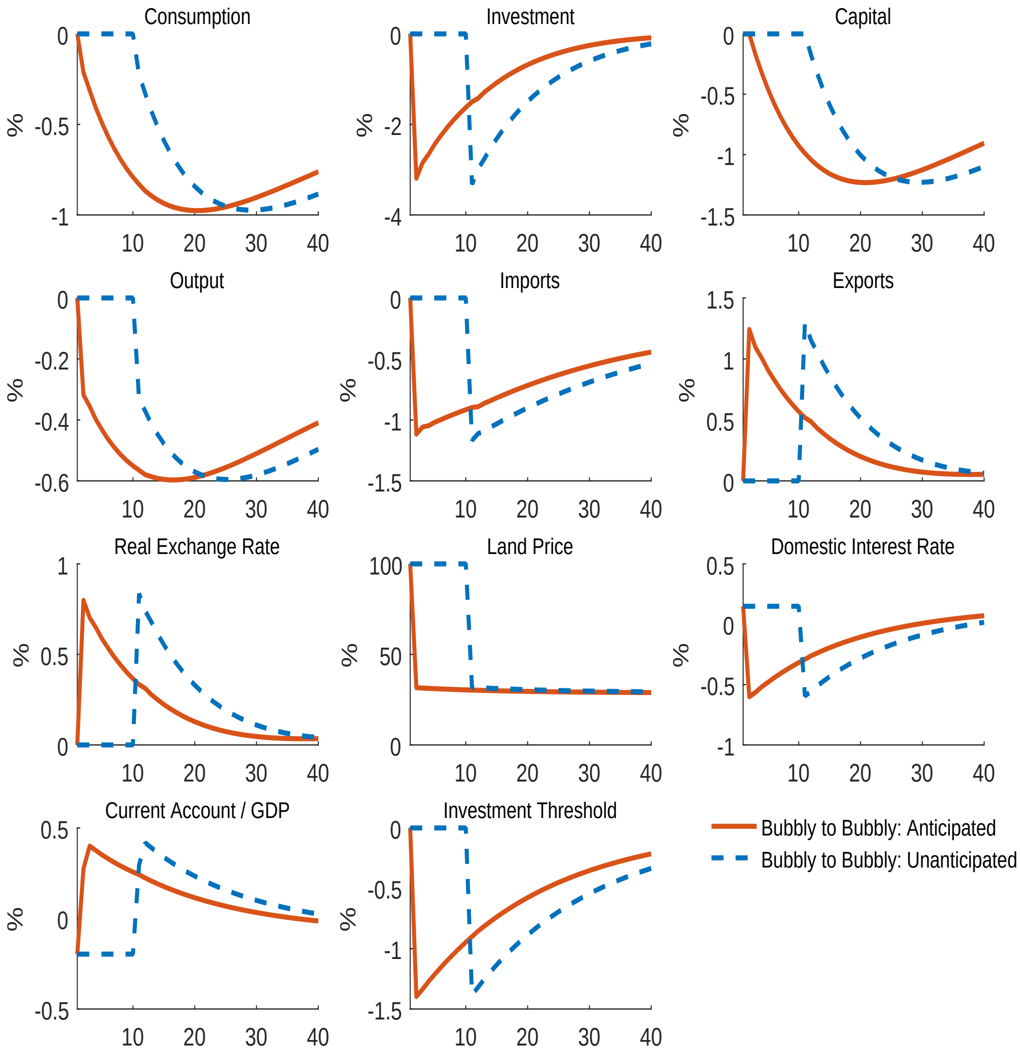

Figure 3: This figure plots the transition paths for the domestic economy when it is initially at the bubbly or bubbleless steady state until period 10 and the foreign interest rate is raised from 0.20% to a permanent level of 0.45% starting from period 11. The solid (dashed) lines describes the case where the rise of the foreign interest rate is anticipated (unanticipated) by domestic agents in period 0. The domestic interest rate is annualized and expressed as levels in percentage. The vertical axis in the panel for the current-account-to-GDP ratio is measured in percentage. Other vertical axes are expressed as percentage deviations from the initial steady state.

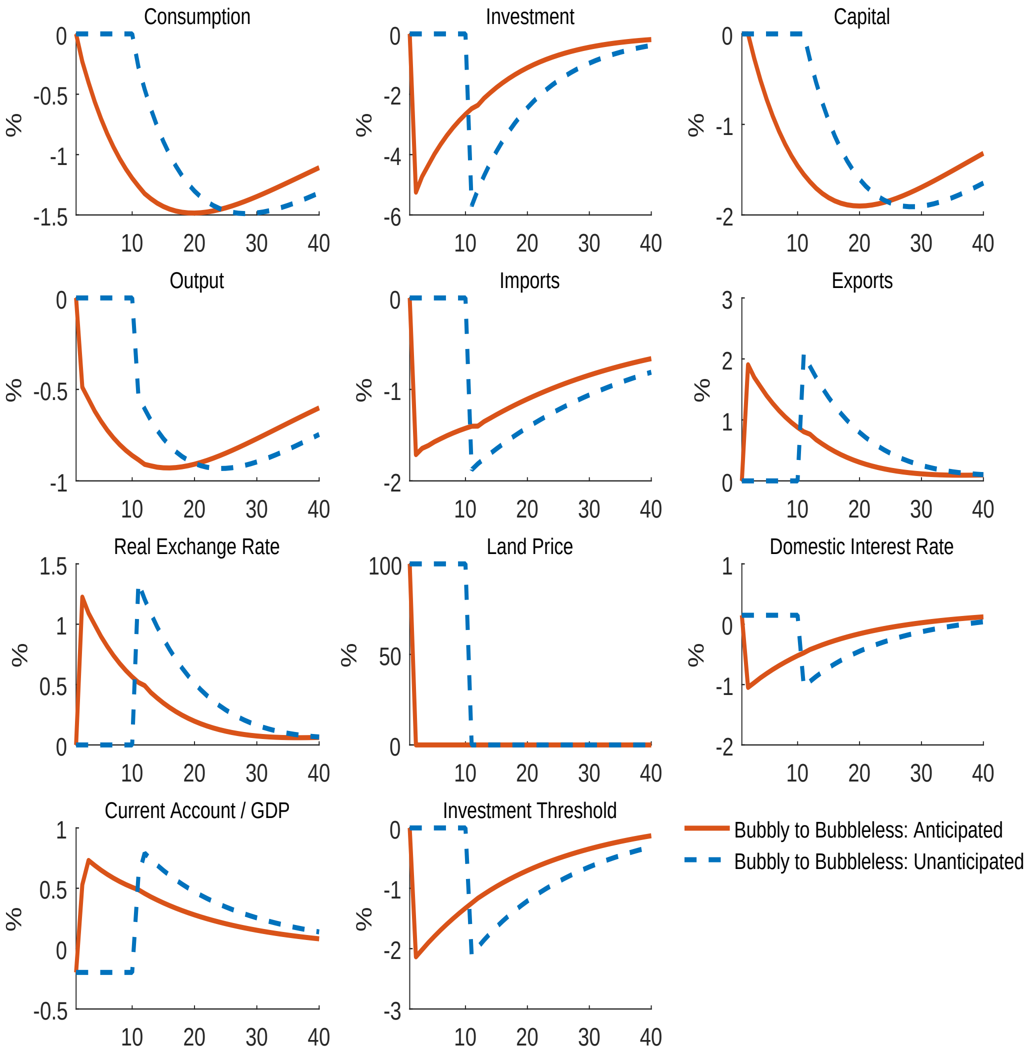

Figure 4: This figure plots the transition paths for the domestic economy when it is initially at the bubbly or bubbleless steady state until period 10 and the foreign interest rate is raised from 0.20% to a permanent level of 0.95% starting from period 11. The solid (dashed) lines describes the case where the rise of the foreign interest rate is anticipated (unanticipated) by domestic agents in period 0. The domestic interest rate is annualized and expressed as levels in percentage. The vertical axis in the panel for the current-account-to-GDP ratio is measured in percentage. Other vertical axes are expressed as percentage deviations from the initial steady state.