

# Network Risk and Key Players: A Structural Analysis of Interbank Liquidity\*

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## Abstract

We estimate the liquidity multiplier and study systemic liquidity risk using a network model of the interbank market. Banks' daily liquidity holding decisions are modelled as a game on a borrowing and lending network. At the Nash equilibrium, each bank's contributions to the network liquidity level and risk are distinct functions of its indegree and outdegree Katz–Bonacich centrality measures, and the network can damp or amplify the shocks to individual banks. Using a sterling interbank database we structurally estimate the model and find a substantial, and time varying, network generated risk: before the Lehman crisis, the network was cohesive, liquidity holding decisions were complementary, and there was a large network liquidity multiplier; during the 2007–08 crisis, the network became less clustered and liquidity holding less dependent on the network; during Quantitative Easing, the network liquidity multiplier became negative, implying a lower potential for the network to generate liquidity.

*Keywords:* financial networks; liquidity; interbank market; key players; systemic risk.

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# I Introduction

The collapse of Lehman Brothers and the subsequent great recession made it clear that financial intermediation plays an important role in the creation of money and liquidity. New theories of money propose that financial intermediaries generate “inside” liquid money which is used to fund long term illiquid investment. The ability of financial intermediaries to create inside money is crucial for economic growth. However, this ability is determined by the health of the banking system and the existence of profitable investment opportunities. During a recession, when the economy receives a negative productivity shock and banks’ balance sheet conditions are worsened, banks have to deleverage, the risk premium rises, and the money multiplier in the economy shrinks, which magnifies the negative real shocks in the economy. The opposite happens during a boom (Brunnermeier and Sannikov (2015); He and Krishnamurthy, (2013)). However, there is not much empirical evidence on how the liquidity multiplier changes in a banking network over the business cycle.

In this paper, we empirically estimate the liquidity multiplier in a banking network guided by a network model of banks’ liquidity holding decisions. In our model, a stand-alone bank might, to meet its liquidity shocks, need to maintain a different size of liquidity buffer than a bank that has access to an interbank borrowing and lending network. An interbank network, through its ability to intermediate liquidity shocks, affects banks’ choices of liquidity buffer stocks and this influence is heterogenous with respect to the location in the network of a bank. In our model, the interbank network multiplies or reduces the liquidity shocks of individual banks. The network multiplier acts as the liquidity multiplier, and we use these two terms interchangeably in the paper. Furthermore, in addition to estimating liquidity multipliers, we analyse the role that the interbank network plays in banks’ liquidity holding decisions and explore the implications for the endogenous formation of systemic liquidity risk (i.e. the volatility of aggregate liquidity).

Understanding the implications for systemic risk of an interbank network becomes also more relevant from a policy perspective. Through recent events, it is evident that banks are interconnected and decisions by individual banks in a banking network could have ripple effects leading to increased risk across the financial system. Instead of traditional regulatory tools that examine banks’ risk exposure in isolation and focus on bank-specific risk variables (e.g. capital ratios), it is now urgent to develop macro-prudential perspectives that assess the systemic implications of an individual bank’s behaviour in an interbank network.<sup>1</sup> In this paper, we contribute towards this endeavour.

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<sup>1</sup>Basel III is putting in place a framework for G-SIFI (Globally Systemically Important Financial Institutions). This will increase capital requirements for those banks which are deemed to pose a systemic risk. (See <http://www.bis.org/publ/bcbs207cn.pdf>).

The underlying economic mechanism in the paper is the externality in the interbank network. That is, neighbouring banks' liquidity holding decisions are not only dependent on their own balance sheet characteristics, but also on their neighbours' liquidity choices. Consequently, their location in the interbank network matters in their contribution to the systemic liquidity in the network. Using a linear-quadratic model, we outline an amplification mechanism for liquidity shocks originating from individual banks, and show its implications for aggregate liquidity level and risk. Based on this amplification mechanism, we estimate the network multiplier, construct the network impulse response function to decompose the aggregate network liquidity risk, and identify the the liquidity *level* key players (banks whose removal would result in the largest liquidity reduction in the overnight interbank system) and the liquidity *risk* key players (banks whose idiosyncratic shocks have the largest aggregate effect) in the network. Based on the estimation of the network multiplier effect, we characterise the social optimum and contrast it with the decentralised equilibrium level of systemic liquidity level and risk. This analysis allows us to identify ways for a planner's intervention to achieve the social optimum.

Specifically, in our model, all banks decide simultaneously how much liquid assets to hold at the beginning of the day as a buffer stock for liquidity shocks that need to be absorbed intraday. By holding liquidity reserves, banks are able to respond immediately to calls on their assets without relying on liquidating illiquid securities. Banks, being exposed to liquidity valuation shocks, derive utility from holding a liquidity buffer stock. A borrowing and lending network allows banks to access others' liquidity stocks to smooth daily shocks. The links between banks are both directional (i.e. lending and borrowing links are different in nature), and weighted in terms of the probabilities of a bank's being able to borrow from any other given bank. It is this complex network that gives rise to the externalities in the model.

There are two opposing network effects. On the one hand, neighbouring banks' liquidity holdings may signal the value of holding an extra unit of liquid asset, and give rise to strategic complementarity in liquidity holding decisions between directly connected banks. This effect is stronger if the valuation of liquidity is correlated between banks in the network. On the other hand, banks are averse to the volatility of the liquidity available to them (directly or via borrowing on the network). The aversion to risk leads banks to make liquidity holding decisions less correlated with their neighbours, resulting in substitution effects between neighbouring banks' choices of liquidity buffer stocks. The equilibrium outcome depends on the tradeoff between these two network effects. The lower (higher) the risk aversion, the higher (lower) the correlation of valuations of liquidity holdings between banks. The lower (higher) the availability of uncollateralised borrowing, the more the equilibrium will

be characterised by strategy complementarity (substitutability).

The existing theoretical literature has mostly modelled the liquidity holding decisions of banks as strategic substitutes (e.g. Bhattacharya and Gale (1987)), while more recent theoretical contributions (e.g. Moore (2012)) have shown that strategic complementarity might arise in equilibrium. Our structural model is flexible enough to incorporate both strategic substitution and strategic complementarity and, when taken to the data, is able to identify when one or the other effect dominates. The combination of these two opposing network externalities is summarised, in equilibrium, by a network decay factor  $\phi$ , which also characterises the network multiplier of liquidity shocks in our paper. When  $\phi$  is positive (negative), the strategic complementarity (substitution) effect dominates. In our model, idiosyncratic liquidity shocks are not diversified away and the banking network may either amplify or reduce these shocks, depending on which strategic effect dominates (i.e. depending on the sign of  $\phi$ ). When  $\phi$  is positive, the systemic risk is larger than the sum of the idiosyncratic risks, since the network magnifies these shocks when they are transmitted through connecting banks. When  $\phi$  is negative, the systemic risk is smaller than the sum of the idiosyncratic risks, since the banking network absorbs these risks while they bounce around.

At the (unique interior) Nash equilibrium, the liquidity holding of each individual bank embedded in the network is proportional to its *indegree* Katz–Bonacich centrality measure. That is, the liquidity holding decision of a bank is related to how it is affected by its own shocks, the shocks of its neighbours, of the neighbours of its neighbours, etc., weighted by the distance between these banks in the network and the network attenuation factor,  $\phi^k$ , where  $k$  is the length of the path.<sup>2</sup> When banks are less (more) risk averse, the liquidity collateral/signal value is larger (smaller), the network attenuation factor  $\phi$  is larger (smaller), and liquidity multiplies faster (slower), resulting in a larger (smaller) aggregate liquidity level and systemic liquidity risk. We also characterise the volatility of the aggregate liquidity and find that the contribution by each bank to the network risk is related to its (analogously defined) *outdegree* Katz–Bonacich centrality measure weighted by the standard deviation of its own shocks. That is, it depends on how the individual bank’s shock propagates to its (direct and indirect) neighbours. These two centrality measures identify the key players in the determination of aggregate liquidity levels and systemic liquidity risk in the network. We also solve the central planner problem and characterise the wedge with respect to the market solution.

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<sup>2</sup>This centrality measure takes into account the number of both immediate and distant connections in a network. For more on the Bonacich centrality measure, see Bonacich ((1987),) and Jackson ((2003)). For other economic applications, see Ballester, Calvo-Armengol, and Zenou (2006) and Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2005). For an excellent review of the literature, see Jackson and Zenou (2012).

We apply the model to study the decisions to hold central bank reserves by banks who are members of the sterling large value payment system, CHAPS. On average, in 2009, £700 billions of transactions were settled every day in the two UK systems, CREST and CHAPS, which is the UK nominal GDP every two days. Almost all banks in CHAPS regularly have intraday liquidity exposures in excess of £1 billion to individual counterparties. For larger banks these exposures are regularly greater than £3 billions. The settlements in CHAPS are done intraday and in gross terms and hence banks (as well as the central bank) are more concerned about managing their liquidity risks and their exposure to the network liquidity risks. We consider the network of all member banks in the CHAPS (which consists of 11 banks) and their liquidity holding decisions. These banks play a key role in the sterling payment system since they make payments both on their own behalf and on behalf of banks that are not direct members of CHAPS.<sup>3</sup> We consider the banks' liquidity holding decisions in terms of the amount of central bank reserves that they hold along with assets that are used to generate intraday liquidity from the Bank of England (BoE).<sup>4</sup> These reserve holdings are the ultimate settlement asset for interbank payments, fund intraday liquidity needs, and act as a buffer to protect the bank against unexpected liquidity shocks.<sup>5</sup> The network that we consider between these banks is the sterling unsecured overnight interbank money market. This is where banks lend central bank reserves to each other, unsecured, for repayment the following day. As an unsecured market, it is sensitive to changes in risk perception. The strength of the link between any two banks in our network is measured using the fraction of borrowing by one bank from the other. Hence, our network is weighted and directional. As well as relying on their own liquidity buffers, banks can also rely on their borrowing relationships within the network to meet unexpected liquidity shocks. Using daily data from January 2006 to September 2010, we cast the theoretical model in a spatial error framework and estimate the network effect. Our parametrization is flexible and allows the network to exhibit either substitutabilities or complementarities, and to change its role over time. The estimation of the network externality effects in the interbank market allows us to understand the shock transmission mechanism in the interbank network and sources of systemic risk. For instance, we decompose the volatility of the total liquidity into the individual banks' contributions to this aggregate quantity. We show that the contribution of each bank can

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<sup>3</sup>We choose not to ignore the network links between clients of the 11 member banks because these network links potentially could affect member banks' buffer stock holding decisions.

<sup>4</sup>In addition to central bank reserves, payment system participants may also repo government bonds to the BoE to provide extra intraday liquidity.

<sup>5</sup>Note that the UK monetary framework allows individual banks to choose their own level of reserve holdings. However, post Quantitative Easing (QE), the BoE has targeted the purchase of assets, and so has largely determined the aggregate supply of bank reserves. In Appendix A.1, we provide background information on the monetary framework (i.e. reserve regimes) including QE, the payment system, and the overnight interbank money markets.

be measured by the network impulse response function (NIRF) to that bank’s individual shocks. The NIRFs are determined in equilibrium by both the network decay factor  $\phi$  and the banks’ locations within the system. This novel measure allows us to pin down each bank’s contribution to systemic risk.

The empirical estimation sheds light on network effects in the liquidity holding decision of the banks over the sample period. This paper shows that this effect is time varying: a multiplier effect during the credit boom prior to 2007, close to zero in the aftermath of the Bear Stearns collapse and during the Lehman crisis, but negative during the Quantitative Easing (QE) period. That is, banks’ liquidity holding decisions are strategic complements during a credit boom but strategic substitutes during the QE period. We find these results to be robust to various specifications and controls.

As the first paper that structurally estimates the network multiplier, our finding of a time-varying network effect is an important empirical result. The long standing notion in the theoretical interbank literature has assumed that banks have incentives to free ride on other banks in holding liquidity and that liquidity is a strategic substitute (Bhattacharya and Gale (1987)). Our finding that liquidity holding decisions among banks sometimes exhibit strategic complementarity indicates that this notion does not fully capture the network effect in the interbank market. We interpret this finding as supportive of the “leverage stack” view of the interbank network in Moore (2012). Specifically, Moore (2012) shows that collateralised borrowing facilitates liquidity’s moving from lenders to borrowers in a system. In our setting, as we are looking at the unsecured market and central bank reserve holdings, we interpret this as meaning that banks that hold more liquid assets have greater access to borrowing from other banks. Moreover, the large positive network multiplier that we estimate during the boom period can be interpreted as a large velocity of inside money (i.e. the ratio between the total value of all transactions and the buffer stock holdings). During this period, banks held smaller but correlated liquidity buffer stocks sustaining a high volume of payment activities. This indicates that the network generated large aggregate liquidity using a smaller stock of cash. However, the multiplier effect also amplified the shocks from each individual bank, creating a potentially excessive aggregate liquidity risk. As crises unfold, banks, as rational agents, lower their exposure to network risk by reducing the correlation of their liquidity decision with their neighbouring banks, and this in turn generates a substantially reduced estimate of the network multiplier. This in turn has a dampens the propagation of shocks between banks but also results in lower aggregate liquidity generated through the network interaction. This new finding enriches our understanding of the interbank market and poses new questions for future theoretical research.

Moreover, using the estimated network effects, we construct the network impulse response

functions and identify the risk key players, i.e. the banks that contribute the most to the aggregate liquidity risk. We find that although the network risk is dominated by a small number of banks for most of the sample period, there are substantial time varying differences in their contributions to the network risk. In fact, during the QE period, the more centrally located banks tended to absorb, rather than contribute to, the network risk. We also find that the key risk players in the network are not necessarily the largest net borrowers. In fact, during the credit boom, large net lenders and borrowers are equally likely to be key players. This set of findings is of policy relevance, and gives guidance on how to effectively inject liquidity into the system. For instance, during the QE period, we find that banks hoarded large liquid reserves but the estimated network multiplier was small, indicating that banks did not generate much inside money with the reserves injected by the policy maker.<sup>6</sup>

The remainder of this paper is organised as follows. In Section II, we discuss the related literature. In Section III, we present and solve a liquidity holding decision game in a network, and define key players in terms of level and risk. Section IV casts the equilibrium of the liquidity network game in the spatial econometric framework, and outlines the estimation methodology. In Section V, we describe the data, the construction of the network, and the basic network characteristics throughout the sample period. In Section VI, we present and discuss the estimation results. Section VII concludes.

## II Related Literature

Broadly speaking our work is closely related to three streams of research. First, we contribute to the literature on the endogenous creation of liquidity and inside money in financial markets. The theoretical literature on liquidity formation in interbank markets has evolved since Bhattacharya and Gale (1987) and focuses on the microstructure of the interbank market. In particular, Freixas, Parigi, and Rochet (2000) show that counter-party risk could cause a gridlock equilibrium in the interbank payment system even when all banks are solvent. Afonso and Shin (2011) calibrate a payment system based on the US Fedwire system and find a multiplier effect. Ashcraft, McAndrews and Skeie (2010) find theoretically and empirically that in response to heightened payment uncertainty, banks hold excess reserves in the Fed fund market. More recently Brunnermeier and Sannikov (2015) have renewed the academic focus on the generation of inside money, and stressed the role played by financial intermediaries in this context. Our paper contributes to this literature by modeling banks' liquidity holding decisions as the outcome of a network game and estimating the impact of

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<sup>6</sup>Our finding is related to Maggio, Kermani, and Palmer (2015), that shows that the blanket purchases of treasury securities during the US QE2 was less effective, in stimulating the creation of inside money, than the targeted purchase of mortgage backed securities.

the externalities generated by the network topology.

Second, there is a sparse (due to the relative unavailability of data) empirical literature that studies the liquidity formation and risk assessment in interbank markets. In particular, by examining large sterling settlement banks during the subprime crisis of 2007–08, Acharya and Merrouche (2010) find evidence of precautionary liquidity demands on the part of the UK banks.<sup>7</sup> Fecht, Nyborg and Rocholl (2010) study the German banks’ behaviour in ECB’s repo auctions from June 2000 to December 2001 and find that the rate a bank paid for liquidity depended on other banks’ liquidity and not just its own. We follow this line of the literature by empirically relating a bank’s reserve holding decision to both its payment characteristics and the decisions of its neighbouring banks in the overnight money market. As far as we know, we are the first to estimate the spatial (network) effect of liquidity holding decisions.<sup>8</sup> Our empirical finding of time-varying strategic interactions among banks’ liquidity holding decisions in the interbank market is new, and calls for further theoretical development of this literature. In term of systemic risk implications, our paper is also related to the literature on financial networks that studies contagion and systemic risks. The theoretical papers in this area include, but are not limited to: Allen and Gale (2000), Freixas, Parigi, and Rochet (2000), Furfine (2000), Leitner (2005), Babus (2009), Zawadowski (2012).<sup>9</sup> However, confronted with the theoretical results on the importance of contagion risk, empirical papers that study shock simulations on realistic banking networks have made a puzzling finding: contagion through interbank linkages contributes relatively little to the systemic risk (Elsinger, Lehar, and Summer (2006)). Our empirical decomposition of the time varying amplification mechanism in the network may potentially resolve this divide between theory and empirics. We show that, for risk generation, the change in the type of equilibrium is the dominant force (rather than the change in the network topology itself).

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<sup>7</sup>There is also extensive policy related research in the BoE on the Sterling payment systems and the money market. For example, Wetherilt, Zimmerman, and Soramaki (2010) document the network characteristics during the recent crisis. Benos, Garratt, and Zimmerman (2010) find that banks made payments at a slower pace after the Lehman failure. Ball, Denbee, Manning and Wetherilt (2011) examine the risks that intraday liquidity pose and suggest ways to ensure that regulation doesn’t lead banks to a bad equilibrium of delayed payments.

<sup>8</sup>We want to point out that ‘liquidity’ in our paper refers to liquidity buffer stocks held in the form of reserves by banks, rather than to the links of the interbank network. There is also a large (but separate) literature that studies the formation of interbank borrowing–lending relationships. For example, Allen, Carletti and Gale (2008) model liquidity hoarding among banks, i.e. the reduction in interbank lending driven by an increase in aggregate uncertainty. Afonso and Lagos (2012) use a search theoretical framework to study the interbank market and banks’ trading behaviour. Afonso, Kovner, and Schoar (2010) show that counterparty risk plays a role in the Fed funds market conditions during the financial crisis in 2008. In our paper, we study the impact of network externality on banks’ choices of daily liquidity buffer stocks, using the interbank borrowing and lending relationships to measure the extent of network externalities. We complement this branch of the literature by considering an additional dimension to the liquidity formation in the interbank market.

<sup>9</sup>Babus and Allen (2009) gives a comprehensive survey of this literature.



Third, our paper is also related to the theoretical and empirical network literature that uses the concept of the Katz–Bonacich centrality measure (see Katz (1953), Bonacich (1987)). We depart from the theoretical literature, building upon the linear-quadratic approach of Ballester, Calvo-Armengol, and Zenou (2006), by analysing how bank-specific shocks translate into (larger or smaller) aggregate network risks. Therefore, we are more closely related to the recent works on aggregate fluctuation generated by networks (Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2005); Acemoglu, Ozdaglar, and Tahbaz-Salehi (2012), Kelly, Lustig, and Nieuwerburgh (2013), Atkeson, Eisfeldt, and Weill (2015), and Ozdagli and Weber (2015)). There is also an emergence of empirical work that links the concept of the Katz–Bonacich centrality measure with banks’ profitability (Cohen-Cole, Patacchini and Zenou (2010)), potential key roles played in risk transmission (see Aldasoro and Angeloni (2013) who motivate the use of the input–output measures), banks’ vulnerability (Greenwood, Landier and Thesmar (2012), Duarte and Eisenbach (2013)), the provision of intermediation by a dealer network (Li and Shürhoff (2012)), and tail risk exposure (Hautsch, Schaumburg, and Schienle (2012)). Moreover, our work is related to Diebold and Yilmaz (2009, 2014), that uses generalised impulse response functions to identify network spillovers via the covariance structure of a reduced form VAR representation. Our paper differs by providing a structural approach to estimate the systemic liquidity level as well as the risk contributions from the banks in the network. We show that, in equilibrium, the network structure, and network multiplier, jointly determine the variance of the liquid holdings and total liquidity in the system. Furthermore, we show that the time variation in the network structure generates a time varying volatility.

### III The Network Model

In this section, in order to study how aggregate liquidity risk is generated within the interbank system, we present a network model of interbank liquidity holding decisions, where the network reflects bilateral borrowing and lending relationships.

*The network:* there is a finite set of  $n$  banks. The network, denoted by  $g$ , is endowed with an  $n$ -square adjacency matrix  $\mathbf{G}$  where  $g_{ii} = 0$  and  $g_{ij \neq i}$  is the fraction of borrowing by bank  $i$  from bank  $j$ . The network  $g$  is therefore weighted and directed.<sup>10</sup> Banks  $i$  and  $j$  are directly connected (in other words, they have a direct lending or borrowing relationship)

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<sup>10</sup>We also explore other definitions of the adjacency matrix, where  $g_{ij}$  is either the sterling amount of borrowing by bank  $i$  from bank  $j$ , or 1 (0) if there is (no) borrowing or lending between Bank  $i$  and  $j$ . Note that, in this latter case, the adjacency matrix is unweighted and undirected. In the theoretical part of the paper, we provide results and intuitions for the case when  $\mathbf{G}$  is a right stochastic matrix. However, the results are easily extendable to other forms of adjacency matrices with some restrictions on the parameter values which we will highlight when needed.

if  $g_{ij}$  or  $g_{ji} \neq 0$ . The coefficient  $g_{ij}$  can be interpreted as the frequentist estimate of the probability of bank  $i$ 's receiving one pound from bank  $j$  via direct borrowing.

The matrix  $\mathbf{G}$  is a (right) stochastic (hollow) matrix by construction, is not symmetric, and keeps track of all direct connections – links of order one – between network players. That is, it summarises all the paths of length one between any pair of banks in the network. Similarly, the matrix  $\mathbf{G}^k$ , for any positive integer  $k$ , encodes all links of order  $k$  between banks, that is, the paths of length  $k$  between any pair of banks in the network. For example, the coefficient in the  $(i, j)$ th cell of  $\mathbf{G}^k$  – i.e.  $\{\mathbf{G}^k\}_{ij}$  – gives the amount of exposure of bank  $i$  to bank  $j$  in  $k$  steps. Since, in our baseline construction,  $\mathbf{G}$  is a right stochastic matrix,  $\mathbf{G}$  can also be interpreted as a Markov chain transition kernel, implying that  $\mathbf{G}^k$  can be thought of as the  $k$ -step transition probability matrix, i.e. the matrix with elements given by the probabilities of reaching bank  $j$  from bank  $i$  in  $k$  steps.

*Banks and their liquidity preferences in a network:* we study the amount of liquidity buffer stocks banks choose to hold at the beginning of the day when they have access to the interbank borrowing and lending network  $g$ . We define the total liquidity holding by bank  $i$ , denoted by  $l_i$ , as the sum of two components: bank  $i$ 's liquidity holdings absent of any bilateral effects (i.e. the level of liquidity that a bank would hold if it were not part of a network), and bank  $i$ 's level of liquidity holdings made available to the network, which depends (potentially) on its neighbouring banks' liquidity contributions to the network. We use  $q_i$  and  $z_i$  to denote these two components respectively, and  $l_i = q_i + z_i$ .

Before modelling the network effect on banks' liquidity choices, we specify a bank's liquidity holdings in the absence of any bilateral effects related to its bank-specific as well as macro variables as

$$q_i = \alpha_i + \sum_{m=1}^M \beta_m x_i^m + \sum_{p=1}^P \beta_p x^p \quad (1)$$

where  $\alpha_i$  is a bank fixed-effect,  $x_i^m$  is a set of  $M$  variables accounting for observable differences in the individual bank  $i$ , and  $x_p$  is a set of  $P$  variables controlling for time-series variation in systematic risks. That is,  $q_i$  captures the liquidity need specific to each individual bank due to its balance sheet and fundamental characteristics (e.g. leverage ratio, lending and borrowing rate), and its exposure to macroeconomic shocks (e.g. aggregate economic activity, monetary policy, etc.).

To study a bank's endogenous choice of  $z_i$ , that is, its liquidity holdings in a banking network, we need to model the various sources of bilateral effects. To do so, we assume that banks are situated in different locations in the borrowing–lending network  $g$ . Each bank decides simultaneously how much liquid capital  $z$  to hold given  $g$ . The network  $g$  is

predetermined.<sup>11</sup>

We assume that banks derive utility from having an accessible buffer stock of liquidity, but at the same time they dislike the variability of this quantity. The accessible network liquidity for bank  $i$  has two components: direct holdings,  $z_i$ , and what can be borrowed from other banks connected through the network. This second component is proportional to the neighbouring banks direct holdings,  $z_j$ , weighted by the borrowing intensities,  $g_{ij}$ , and a technological parameter  $\psi$ , that is,  $\psi \sum_j g_{ij} z_j$ . This component can be thought as unsecured borrowing. The direct utility of this buffer stock of accessible liquidity for bank  $i$  is  $\tilde{\mu}_i$  per unit. The term  $\tilde{\mu}_i$  captures the valuation (not necessarily positive) of a unit of bank  $i$ 's accessible buffer stock of liquidity, and is affected by random shocks. In summary, the valuation of liquidity for bank  $i$  in network  $g$  is modelled as

$$\underbrace{\tilde{\mu}_i}_{\text{Per Unit Value}} \underbrace{\left( z_i + \psi \sum_{j \neq i} g_{ij} z_j \right)}_{\text{Accessible Liquidity}}$$

We specify bank  $i$ 's per unit liquidity valuation,  $\tilde{\mu}_i$ , as being the sum of a bank specific, and stochastic, component ( $\hat{\mu}_i$ ), plus a network generated component, i.e.  $\tilde{\mu}_i := \hat{\mu}_i + \delta \sum_j g_{ij} z_j$ . To motivate this specification, consider the following thought experiment. Suppose bank  $i$  learns about its own per unit value of the liquidity buffer stock  $\tilde{\mu}_i$  from both its own information,  $\hat{\mu}_i$ , and its neighbouring banks' liquidity holdings. Even though each bank might value liquidity buffer stocks differently, neighbouring banks' liquidity holding decisions are informative about the market value of liquid reserves. Specifically, bank  $i$  uses a simple updating rule about  $\tilde{\mu}_i$  given by  $\hat{\mu}_i + \delta \sum_j g_{ij} z_j$ . This updating rule is in the spirit of the boundedly-rational model of opinion formation considered in DeMarzo, Vayanos and Zwiebel (2003) (see also DeGroot (1974)).<sup>12</sup> In this specification the coefficient  $\delta$  reflects the discount or "haircut" on the information aggregated over neighbouring banks' holdings. The network weights are used to aggregate information in neighbouring bank's liquidity holding decisions: the stronger the connecting link, the more influence the corresponding neighbouring bank's liquidity holding decision exerts.

However, by establishing bilateral relationships in the banking network  $g$ , a bank also exposes itself to the shocks from its neighbouring banks. We assume that banks dislike the

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<sup>11</sup>Since we consider the liquidity holding decision at the daily level, it is intuitive to take  $g$  as given, as it is unlikely the network would change significantly within a day. In the empirical part of the paper, we let  $g$  vary with time.

<sup>12</sup>Note that this updating rule is not Bayesian. We choose this updating rule for expositional clarity in capturing two opposing network bilateral effects, as shown later. There is a separate but growing literature that studies the role of information aggregation in network settings (DeMarzo, Vayanos, and Zwiebel (2003); Babus and Kondor (2014)).

volatility of their own liquidity and of the liquidity they can access given their links, which can be modelled as

$$\left( z_i + \psi \sum_{j \neq i} g_{ij} z_j \right)^2.$$

Denoting the risk aversion parameter as  $\gamma > 0$ , we now can fully characterise bank  $i$ 's utility from holding liquidity as

$$u_i(\mathbf{z}|g) = \left( \hat{\mu}_i + \delta \sum_{j \neq i} g_{ij} z_j \right) \left( z_i + \psi \sum_{j \neq i} g_{ij} z_j \right) - \frac{1}{2} \gamma \left( z_i + \psi \sum_{j \neq i} g_{ij} z_j \right)^2. \quad (2)$$

The above has the same spirit as a mean-variance utility representation. The bilateral network influences are captured by the following cross derivatives for  $i \neq j$ :

$$\frac{\partial^2 u_i(\mathbf{z}|g)}{\partial z_i \partial z_j} = (\delta - \gamma \psi) g_{ij}.$$

If  $\delta > \gamma \psi$ , the above expression is positive, reflecting strategic complementarity in liquidity holdings among neighbouring banks. The source of strategic complementarity in the model comes from the information embedded in neighbouring banks' liquidity holding decisions. Banks in the network rely on their own signal and neighbouring banks' liquidity holding decisions to estimate the value of liquidity buffer stock to themselves. When liquidity valuations are correlated among neighbours, larger liquidity buffer stock put aside by the neighbouring banks would indicate a higher correlated liquidity valuation. Inferring this, the connected bank would increase its liquidity buffer stock as a response, resulting in complementarity.

The strategic complementary effect in the interbank market also arises in the leverage stack model of Moore (2012), although coming from a different source. In Moore (2012), the interbank lending market is used by individual banks to generate collateral that can then be used to raise more funds from households. In an alternative formulation of our model, we specify this effect by adding a "collateralised" liquidity term  $z_i \delta \sum_j g_{ij} z_j$  where  $\delta$  can be thought of as haircut for collateral.<sup>13</sup> This alternative specification is as follows:

$$u_i(\mathbf{z}|g) = \underbrace{\hat{\mu}_i \left( z_i + \psi \sum_{j \neq i} g_{ij} z_j \right)}_{\text{Accessible Liquidity}} - \underbrace{\frac{1}{2} \gamma \left( z_i + \psi \sum_{j \neq i} g_{ij} z_j \right)^2}_{\text{Accessible Liquidity Volatility}} + \underbrace{z_i \delta \sum_{j \neq i} g_{ij} z_j}_{\text{"Collateralised" Liquidity}}, \quad (3)$$

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<sup>13</sup>Note that the counter-party risk of this "collateralised" liquidity is negligible in our banking network since the network consists of top players in the UK banking system, hence we do not introduce a corresponding second order term.

where the last term reflects a reduced form of the “collateral” effect. Since banks in our paper are engaged in unsecured borrowing and lending, the liquidity buffer stock of a bank can be thought as “information collateral”, signalling its liquidity strength and trustworthiness: banks which hold more liquid assets, have in turn greater access to borrowing from other banks. The essence of our model does not change with this alternative specification, although the planner’s problem differs slightly. Since the two specifications are isomorphic (in terms of decentralised equilibrium), we use the interpretation of  $\delta$  as haircut on both information value and information collateral.

Conversely, if  $\delta < \gamma\psi$ , the cross derivative is negative, reflecting strategic substitution in liquidity holdings among neighbouring banks. That is, an individual bank sets aside a smaller amount of liquid assets when its neighbouring lending banks hold a lot of liquidity which it can draw upon. In our model, strategic substitutability arises from the fact that banks dislike volatility in their accessible liquidity, and therefore prefer to hold buffer stocks of liquidity that are less correlated with the ones of the neighbouring banks. The strategic substitution effect has been modelled extensively in the interbank literature ever since the seminal paper by Bhattacharya and Gale (1987).<sup>14</sup>

The bilateral network effect in our model combines these two strategic effects. When  $\gamma$  is relatively large, that is, when banks are very averse to liquidity risks in the network, it is likely that  $\delta < \gamma\psi$  and the strategic substitution effect dominates. Conversely, when  $\delta$  is relatively large, the haircut is small and inside money velocity (i.e. the transactions value to holdings ratio) is large and the collateral chains are long, it is likely that  $\delta > \gamma\psi$  and the strategic complementary effect dominates. In our paper, we are agnostic about the sign of  $\delta - \gamma\psi$  and estimate it empirically.

*Equilibrium behaviour:* We now characterise the Nash equilibrium of the game where banks choose their liquidity level  $z$  simultaneously. Each bank  $i$  maximises (2) and we obtain the following best response function for each bank:<sup>15</sup>

$$z_i^* = \frac{\hat{\mu}_i}{\gamma} + \left( \frac{\delta}{\gamma} - \psi \right) \sum_{j \neq i} g_{ij} z_j = \mu_i + \phi \sum_j g_{ij} z_{j \neq i} \quad (4)$$

where  $\phi := \delta/\gamma - \psi$  and  $\mu_i := \hat{\mu}_i/\gamma =: \bar{\mu}_i + \nu_i$ . The parameter  $\bar{\mu}_i$  denotes the average valuation of liquidity by bank  $i$  (absent any valuation spillovers) scaled by  $\gamma$ , and  $\nu_i$  denotes the i.i.d. shock of this normalised valuation, and its variance is denoted by  $\sigma_i^2$ . Note that  $\bar{\mu}_i$

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<sup>14</sup>Bhattacharya and Gale (1987) show that banks’ liquidity holdings are strategic substitutes for a different reason. In their model, setting liquidity aside comes at a cost of forgoing higher interest revenue from long-term investments. Banks would like to free-ride their neighbouring banks for liquidity rather than conducting precautionary liquidity saving themselves.

<sup>15</sup>Note that this is also the best response implied by the formulation in equation (3).

will be positive for banks that, on average, contribute liquidity to the network, while a large negative  $\bar{\mu}_i$  will characterise banks that, on average absorb liquidity from the system.

**Proposition 1** *Suppose that  $|\phi| < 1$ . Then, there is a unique interior solution for the individual equilibrium outcome given by*

$$z_i^*(\phi, g) = \{\mathbf{M}(\phi, \mathbf{G})\}_i \mu, \quad (5)$$

where  $\{\}_i$  is the operator that returns the  $i$ -th row of its argument,  $\mu := [\mu_1, \dots, \mu_n]'$ ,  $z_i$  denotes the bilateral liquidity holding by bank  $i$ , and

$$\mathbf{M}(\phi, \mathbf{G}) := \mathbf{I} + \phi \mathbf{G} + \phi^2 \mathbf{G}^2 + \phi^3 \mathbf{G}^3 + \dots \equiv \sum_{k=0}^{\infty} \phi^k \mathbf{G}^k = (\mathbf{I} - \phi \mathbf{G})^{-1}. \quad (6)$$

where  $\mathbf{I}$  is the  $n \times n$  identity matrix.

**Proof.** Since  $\gamma > 0$ , the first order condition identifies the individual optimal response. Applying Theorem 1, part b, in Calvo-Armengol, Patacchini, and Zenou (2009) to our problem, the necessary equilibrium condition becomes  $|\phi \lambda^{\max}(\mathbf{G})| < 1$  where the function  $\lambda^{\max}(\cdot)$  returns the largest eigenvalue. Since  $\mathbf{G}$  is a stochastic matrix, its largest eigenvalue is 1. Hence, the equilibrium condition requires  $|\phi| < 1$ , and in this case the infinite sum in equation (6) is finite and equal to the stated result (Debreu and Herstein (1953)). ■

To gain intuition about the above result, note that a Nash equilibrium in pure strategies  $\mathbf{z}^* \in \mathbb{R}^n$ , where  $\mathbf{z} := [z_1, \dots, z_n]'$ , is such that equation (4) holds for all  $i = 1, 2, \dots, n$ . Hence, if such an equilibrium exists, it solves  $(\mathbf{I} - \phi \mathbf{G}) \mathbf{z} = \mu$ . If the matrix is invertible, we obtain  $\mathbf{z}^* = (\mathbf{I} - \phi \mathbf{G})^{-1} \mu \equiv \mathbf{M}(\phi, \mathbf{G}) \mu$ . The rest follows by simple algebra. The condition  $|\phi| < 1$  in the above proposition states that network externalities must be small enough in order to prevent the feedback triggered by such externalities to escalate without bounds.

The matrix  $\mathbf{M}(\phi, \mathbf{G})$  characterising the equilibrium has an important economic interpretation: it aggregates all direct and indirect links among banks using an attenuation factor,  $\phi$ , that penalises (as in Katz (1953)) the contribution of links between distant nodes at the rate  $\phi^k$ , where  $k$  is the length of the path between nodes. In the infinite sum in equation (6), the identity matrix captures the (implicit) link of each bank with itself, the second term in the sum captures all the direct links between banks, the third term in the sum captures all the indirect links corresponding to paths of length two, and so on. The elements of the matrix  $\mathbf{M}(\phi, \mathbf{G})$ , given by  $m_{ij}(\phi, \mathbf{G}) := \sum_{k=0}^{+\infty} \phi^k \{\mathbf{G}^k\}_{ij}$ , aggregates all the exposures in the network of  $i$  to  $j$ , where the contribution of the  $k$ th step is weighted by  $\phi^k$ .

In equilibrium, the matrix  $\mathbf{M}(\phi, \mathbf{G})$ , contains the relevant information needed to characterise the centrality of the players in the network. That is, it provides a metric from which

the relevant centrality of the network players can be recovered. In particular, multiplying the rows (columns) of  $\mathbf{M}(\phi, \mathbf{G})$  by a vector of appropriate dimensions, we recover the indegree (outdegree) Katz–Bonacich centrality measure.<sup>16</sup> The indegree centrality measure provides the weighted count of the number of ties directed to each node, while the outdegree centrality measure provides the weighted count of ties that each node directs to the other nodes. That is, the  $i$ -th row of  $\mathbf{M}(\phi, \mathbf{G})$  captures how bank  $i$  loads on the network as whole, while the  $i$ -th column of  $\mathbf{M}(\phi, \mathbf{G})$  captures how the network as a whole loads on bank  $i$ .

Moreover, as equation (5) shows, the matrix  $\mathbf{M}(\phi, \mathbf{G})$ , jointly with the vector  $\mu$  containing banks' valuation of network liquidity, fully determines the equilibrium bilateral liquidity holding of each bank in a very intuitive manner. First,  $z_i^*$  is increasing in bank  $i$ 's own valuation of network liquidity ( $\mu_i$ ). Second, when banks' valuations of bilateral liquidity are non-negative (i.e.  $\mu_i \geq 0 \forall i$ ), the larger (smaller) is  $\phi$ , the larger (smaller) is the bilateral liquidity of each bank. This is due to the fact that, when  $\phi$  is large, the benefits of using network liquidity are also large (as long as other agents provide liquidity in the network, and this always happens when  $\mu_i \geq 0 \forall i$ ). This also implies that  $z_i^*$  is increasing in  $\delta$  (the parameters measuring the benefit of information “collateralised” liquidity), decreasing in  $\psi$  (since the higher  $\psi$  is, the more each bank can free ride on other banks' buffer stocks of liquidity), and decreasing in  $\gamma$  (since the higher  $\gamma$  is, the more each bank dislikes the volatility of network liquidity). Third, when  $\phi$  is positive (i.e. when the liquidity holding decisions of the banks are strategic complements),  $z_i^*$  is also nondecreasing in other banks' valuations of network liquidity ( $\mu_{j \neq i}$ ). This is due to the fact that when other banks' valuations of liquidity increase, their supply of liquidity in the network increases too, and this, in turn, when  $\phi \equiv \delta/\gamma - \psi > 0$ , has a larger impact on the benefits of information-collateralised liquidity (controlled by  $\delta$ ) than on the incentives to free ride on other banks' liquidity (controlled by  $\psi$ ) and on the disutility coming from the increased volatility of the network liquidity (controlled by  $\gamma$ ).

*Equilibrium properties:* We can decompose the network contribution to the total bilateral liquidity into a level effect and a risk effect. To see this, note that the total bilateral liquidity,  $Z := \sum_i z_i$ , can be written at equilibrium as

$$Z^* = \underbrace{\mathbf{1}' \mathbf{M}(\phi, \mathbf{G}) \bar{\mu}}_{\text{level effect}} + \underbrace{\mathbf{1}' \mathbf{M}(\phi, \mathbf{G}) \nu}_{\text{risk effect}} \quad (7)$$

where  $\bar{\mu} := [\bar{\mu}_1, \dots, \bar{\mu}_n]'$ ,  $\nu := [\nu_1, \dots, \nu_n]'$ . The first component captures the network level

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<sup>16</sup>Newman (2004) shows that weighted networks can in many cases be analysed using a simple mapping from a weighted network to an unweighted multigraph. Therefore, the centrality measures developed for unweighted networks apply also to the weighted cases.

effect, and the second component (that aggregates bank specific shocks) captures the network risk effect. It is clear that if  $\bar{\mu}$  has only positive entries, both the network liquidity level and liquidity risk are increasing in  $\phi$ . That is, a higher network multiplier leads the interbank network to produce more liquidity and also generate more risk.

The equilibrium solution in equation (7) implies that bank  $i$ 's marginal contribution to the volatility of aggregate liquidity can be summarised as

$$\frac{\partial Z^*}{\partial \nu_i} \sigma_i = \mathbf{1}' \{ \mathbf{M}(\phi, \mathbf{G}) \}_{.i} \sigma_i =: b_i^{out}(\phi, \mathbf{G}). \quad (8)$$

The above expression is the *outdegree* centrality for bank  $i$  weighted by the standard deviation of its own shocks. Moreover, the volatility of the aggregate liquidity level in our model is

$$Var(Z^*(\phi, \mathbf{G})) = vec(\{b_i^{out}(\phi, \mathbf{G})\}_{i=1}^n) vec(\{b_i^{out}(\phi, \mathbf{G})\}_{i=1}^n)' \quad (9)$$

$$= \mathbf{1}' \mathbf{M}(\phi, \mathbf{G}) diag(\{\sigma_i^2\}_{i=1}^n) \mathbf{M}(\phi, \mathbf{G})' \mathbf{1}. \quad (10)$$

Therefore, equation (8) provides a clear ranking of the riskiness of each bank from a systemic perspective. This allows defining the concept of “systemic risk key player” as follows.

**Definition 1** [*Risk key player*] The risk key player  $i^*$ , given by the solution of

$$i^* = \arg \max_{i=1, \dots, n} b_i^{out}(\phi, \mathbf{G}),$$

is the one that contributes the most to the volatility of the overall network liquidity.

Similarly, we can identify the bank that can cause the maximum expected level of reduction in the network liquidity when removed from the system.<sup>17</sup>

**Definition 2** [*Level key player*] The level key player is the player that, when removed, causes the maximum expected reduction in the overall level of bilateral liquidity. We use  $\mathbf{G}_{\setminus \tau}$  to denote the new adjacency matrix obtained by setting to zero all of  $\mathbf{G}$ 's  $\tau$ -th row and column coefficients. The resulting network is  $g_{\setminus \tau}$ . The level key player  $\tau^*$  is found by solving

$$\tau^* = \arg \max_{\tau=1, \dots, n} \mathbb{E} \left[ \sum_i z_i^*(\phi, g) - \sum_{i \neq \tau} z_i^*(\phi, g_{\setminus \tau}) \right] \quad (11)$$

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<sup>17</sup>This definition is in the same spirit as the concept of the key player in the crime network literature as defined in Ballester, Calvo-Armengol, and Zenou (2006). There, it is important to target the key player for maximum crime reduction. Here, it is useful to consider the ripple effect on the network liquidity when a bank fails. Bailouts for key level players might be necessary to avoid major disruptions to the banking network.



where  $\mathbb{E}$  defines unconditional expectations.

In this definition, the level key player is the one with the largest impact on the total expected bilateral liquidity, under the assumption that when the player  $\tau$  is removed, the remaining other banks do not form new links –i.e. we consider the short-run effect of removing a player from the network.

Using Proposition 1, we have the following corollary.

**Corollary 1** *A player  $\tau^*$  is the level key player that solves (11) if and only if*

$$\tau^* = \arg \max_{\tau=1,\dots,n} \{ \mathbf{M}(\phi, G) \}_{\tau, \bar{\mu}} + \sum_{i \neq \tau} m_{i\tau}(\phi, \mathbf{G}) \bar{\mu}_{\tau} .$$

This follows from the fact that when bank  $\tau$  is removed, the expected reduction in the total bilateral liquidity can be written as

$$\mathbb{E} \left[ \sum_i z_i^*(\phi, g) - \sum_{i \neq \tau} z_i^*(\phi, g_{\setminus \tau}) \right] = \underbrace{\{ \mathbf{M}(\phi, \mathbf{G}) \}_{\tau, \bar{\mu}}}_{\text{Indegree effect}} + \underbrace{\mathbf{1}' \{ \mathbf{M}(\phi, \mathbf{G}) \}_{\cdot, \tau} \bar{\mu}_{\tau}}_{\text{Outdegree effect}} - \underbrace{m_{\tau\tau}(\phi, \mathbf{G}) \bar{\mu}_{\tau}}_{\text{Double count correction}} \quad (12)$$

That is, the removal of the level key player results in a direct (indegree) effect on its own liquidity generation and an indirect (outdegree) bilateral effect on other banks' liquidity generation. Instead of being the bank with the largest amount of liquidity buffer stock (captured by the first term on the right-hand side of equation (12)), the level key bank is the one with the largest expected contribution to its own and as well as its neighbouring banks' liquidity. This discrepancy exists because, in the decentralised equilibrium, no bank internalises the effect of its own liquidity holding level on the utilities of the other banks in the network. That is, no bank internalises the spillover of its choice of liquidity on other banks' liquidity valuation. Therefore, a relevant metric for a planner to use when deciding whether to bail out a failing bank should not be merely based on the size of the bank's own liquidity, but should also include its indirect network impact on other banks' liquidity.

This discussion leads us to analyse formally a planner's problem in this networked economy. A planner that equally weights the utility of each bank (in equation (2)) chooses the network liquidity holdings by solving the following problem:

$$\max_{\{z_i\}_{i=1}^n} \sum_{i=1}^n \left[ \hat{\mu}_i \left( z_i + \psi \sum_{j \neq i} g_{ij} z_j \right) + z_i \delta \sum_{j \neq i} g_{ij} z_j - \frac{1}{2} \gamma \left( z_i + \psi \sum_{j \neq i} g_{ij} z_j \right)^2 + \delta \psi \left( \sum_{j \neq i} g_{ij} z_j \right)^2 \right] . \quad (13)$$

The first order condition for the liquidity holding of the  $i$ -th bank ( $z_i$ ) yields

$$\begin{aligned}
z_i = & \underbrace{\mu_i + \phi \sum_{j \neq i} g_{ij} z_j}_{\text{decentralised f.o.c.}} + \underbrace{\psi \sum_{j \neq i} g_{ji} \mu_j}_{\substack{\text{neighbors' valuations} \\ \text{of own liquidity}}} + \underbrace{\phi \sum_{j \neq i} g_{ji} z_j}_{\substack{\text{neighbors' indegree} \\ \text{i.e. own outdegree}}} - \underbrace{\psi \left( \psi - \frac{2\delta}{\gamma} \right) \sum_{j \neq i} \sum_{m \neq j} g_{ji} g_{jm} z_m}_{\substack{\text{volatility of neighbors'} \\ \text{accessible network liquidity}}}
\end{aligned} \tag{14}$$

In the above equation, the first two (indegree) terms are exactly the same as in the decentralised case, while the last three (outdegree) terms reflect that the fact that the planner internalises a bank's contribution to its neighbouring banks' utilities. In particular: the third term captures the neighbours' idiosyncratic valuation of the liquidity provided by agent  $i$ ; the fourth term reflects bank  $i$ 's contribution to its neighbouring banks' endogenous valuation of network liquidity; the fifth term measures bank  $i$ 's contribution to the volatility of the network liquidity accessible by neighbouring banks.

Rewriting equation (14) in matrix form, we obtain  $\mathbf{z} = (\mathbf{I} + \psi \mathbf{G}') \mu + \mathbf{P}(\phi, \psi, \delta, \mathbf{G}) \mathbf{z}$  where  $\mathbf{P}(\phi, \psi, \delta, \mathbf{G}) := \phi(\mathbf{G} + \mathbf{G}') - \psi(\psi - 2\delta/\gamma) \mathbf{G}'\mathbf{G}$ . This allows us to state the following result.

**Proposition 2** *Suppose  $|\lambda^{\max}(\mathbf{P}(\phi, \psi, \delta, \mathbf{G}))| < 1$ . Then, the planner's optimal solution is uniquely defined and given by*

$$z_i^p(\phi, \psi, \delta, g) = \{\mathbf{M}^p(\phi, \psi, \delta, \mathbf{G})\}_i \mu, \tag{15}$$

where  $\mathbf{M}^p(\phi, \psi, \delta, \mathbf{G}) := [\mathbf{I} - \mathbf{P}(\phi, \psi, \delta, \mathbf{G})]^{-1}(\mathbf{I} + \psi \mathbf{G}')$ .

**Proof.** The proof follows the same argument as in the proof of Proposition 1. ■

To see what drives the difference between the network liquidity in the decentralised equilibrium ( $z^*$ ) and in the planner's solution ( $z^p$ ), one can rewrite the planner's first order condition (14) as

$$\mathbf{z}^p = \mathbf{z}^* + \mathbf{M}(\phi, \mathbf{G}) \left[ \psi \mathbf{G}' \mu + \left( \phi \mathbf{G}' - \psi \left( \psi - \frac{2\delta}{\gamma} \right) \mathbf{G}'\mathbf{G} \right) \mathbf{z}^p \right] \tag{16}$$

and observe that there are extra terms (indicated by being included in square brackets) compared to the decentralised outcome. These terms arise from the bank's failure to internalise the externalities it generates. Intuitively, among these terms: the first one reflects the contribution to the neighbours' valuations of liquidity holdings; the second one measures the contribution to the neighbouring nodes' indegree centrality and hence their network liquidity production level (that is, the network nodes' own outdegree centrality); and the last

one is the contribution to their neighbouring nodes' indirect volatility (that is, the network nodes' own second-order degree centrality).<sup>18</sup> Therefore, the discrepancy between the planner's optimum and the decentralised equilibrium rests on the planner's tradeoff between the liquidity level and the liquidity risk in the network. When the planner cares more about the level of liquidity production than the liquidity risk in the network, the first two terms are more pronounced relative to the last term. In this case, banks that have higher outdegree centralities tend to hold less than the socially optimal amount of liquidity. The planner might subsidise or inject liquidity to these banks to increase the liquidity generated by the network. Conversely, when the planner cares more about the liquidity risk in the network (which happens when  $\psi \gg 2\delta/\gamma$ , e.g. very large  $\psi$  or  $\gamma$  and small  $\delta$ ), banks that have higher second-degree centralities tend to hold more than the socially optimal amount of liquidity. The planner might impose a tax on these banks to reduce the risk in the banking network.

As in the decentralised solution, one can solve for the aggregate network liquidity level and risk in the planner's problem:

$$Z^p = \mathbf{1}' \mathbf{M}^p(\phi, \psi, \delta, \mathbf{G}) \bar{\mu} + \mathbf{1}' \mathbf{M}^p(\phi, \psi, \delta, \mathbf{G}) \nu \quad (17)$$

$$Var(Z^p(\phi, \psi, \delta, \mathbf{G})) = \mathbf{1}' \mathbf{M}^p(\phi, \psi, \delta, \mathbf{G}) \text{diag}(\{\sigma_i^2\}_{i=1}^n) \mathbf{M}^p(\phi, \psi, \delta, \mathbf{G})' \mathbf{1}. \quad (18)$$

The following lemma characterises the wedge between the planner's solution and that of the decentralised equilibrium outcome.

**Lemma 1** *Let  $\mathbf{H} := \phi \mathbf{G}' - \psi(\psi - 2\delta/\gamma) \mathbf{G}' \mathbf{G}$ . Then, the aggregate network liquidity in the planner's solution can be expressed as*

$$Z^p = Z^* + \mathbf{1}' [\psi \mathbf{M} \mathbf{G}' + \mathbf{M} \mathbf{H} \mathbf{M} (\mathbf{I} - \mathbf{H} \mathbf{M})^{-1} (\mathbf{I} + \psi \mathbf{G}')] \mu \quad (19)$$

where  $Z^*$  denotes the aggregate bilateral liquidity in the decentralised equilibrium in equation (7) and  $\mathbf{M} := \mathbf{M}(\phi, \mathbf{G})$ . Moreover, if  $\mathbf{H}$  is invertible, we have

$$Z^p = Z^* + \mathbf{1}' [\psi \mathbf{M} \mathbf{G}' + \mathbf{M} (\mathbf{H}^{-1} - \mathbf{M})^{-1} \mathbf{M} (\mathbf{I} + \psi \mathbf{G}')] \mu. \quad (20)$$

**Proof.** If  $\mathbf{H}$  is invertible, observing that

$$\mathbf{M}^p(\phi, \psi, \delta, \mathbf{G}) \equiv [\mathbf{M}(\phi, \mathbf{G})^{-1} - \phi \mathbf{G}' + \psi(\psi - 2\delta/\gamma) \mathbf{G}' \mathbf{G}]^{-1} (\mathbf{I} + \psi \mathbf{G}')$$

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<sup>18</sup>Note that the term  $\phi \mathbf{G}' - \psi(\psi - 2\delta/\gamma) \mathbf{G}' \mathbf{G}$  vanishes only in the unlikely case of  $\frac{\phi}{\psi(\psi - 2\delta/\gamma)}$  being an eigenvalue of  $\mathbf{G}$ .

and using the Woodbury matrix identity (see, e.g. Henderson and Searle (1981)) gives

$$\mathbf{M}^p(\phi, \psi, \delta, \mathbf{G}) = \mathbf{M} + \mathbf{M}(\mathbf{H}^{-1} - \mathbf{M})^{-1}\mathbf{M}.$$

The result is immediate. If  $\mathbf{H}$  is not invertible, using equation (26) in Henderson and Searle (1981), we obtain

$$\mathbf{M}^p(\phi, \psi, \delta, \mathbf{G}) = \mathbf{M} + \mathbf{M}\mathbf{H}\mathbf{M}(\mathbf{I} - \mathbf{H}\mathbf{M})^{-1}$$

and the result follows. ■

The above implies that both  $\mathbb{E}[Z^p - Z^*]$  and  $\{\mathbb{E}[\mathbf{z}^p - \mathbf{z}^*]\}_i$  might be positive or negative depending on the parameters and the topology of the network. In particular, one can show that the sign of the discrepancy between the solution of the planner and the decentralised solution depends on the parameters and the eigenvalues of the canonical operator of  $\mathbf{G}$  (see, e.g. Gorodentsev (1994) for a definition of the canonical operator).<sup>19</sup>

## IV Empirical Methodology

In order to estimate the network model presented in Section III, we need to map the observed total liquidity holding of a bank at time  $t$ ,  $l_{i,t}$ , into its two components: the liquidity holding absent of any bilateral effects (defined in equation (1)) and the bank's liquidity holding level made available to the network (defined in equation (5)). This can be done by reformulating the theoretical model in the fashion of a spatial error model (SEM). That is, we decompose the total bank liquidity holdings into a function of the observables and a latent term that captures the spatial dependence generated by the network:

$$l_{i,t} = \alpha_t^{time} + \alpha_i^{bank} + \sum_{m=1}^M \beta_m^{bank} x_{i,t}^m + \sum_{p=1}^P \beta_p^{time} x_t^p + z_{i,t} \quad (21)$$

$$z_{i,t} = \bar{\mu}_i + \phi \sum_{j=1}^n g_{ij,t} z_{j,t} + \nu_{i,t} \sim iid(0, \sigma_i^2), \quad i = 1, \dots, n, \quad t = 1, \dots, T. \quad (22)$$

The only differences between the theoretical model and the econometric reformulation above are that: *i*) we have made explicit that one of the aggregate factors is a set of common time dummies,  $\alpha_t^{time}$ , meant to capture potential trends in the size of the overall interbank market; *ii*) we allow the network links,  $g_{ij}$ , to potentially vary over time (but we construct them, as explained in the data description section below, in a fashion that makes them pre-

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<sup>19</sup>The proof of this result is very involved, hence we present it in an appendix available upon request.

determined with respect to the information set for time  $t$ ).<sup>20</sup> The coefficients  $\beta_m^{bank}$  capture the effect of observable bank characteristics while the coefficients  $\beta_p^{time}$  capture the effects of systematic risk factors on the choice of liquidity.

Equation (22) describes the process of  $z_i$ , which is the residual of the individual bank  $i$ 's level of liquidity in the network that is not due to bank specific characteristics or systematic factors. Moreover, defining  $\epsilon_i$  as the demeaned version of  $z_i$ , we have that  $\sum_{j=1}^n g_{ij,t} \epsilon_{j,t}$  is a standard spatial lag term and  $\phi$  is the canonical spatial autoregressive parameter. That is, the model in equations (21)–(22) is a variation of the Anselin (1988) spatial error model (see also Elhorst (2010a, 2010b)). This specification makes clear the nature of the network as a shock propagation mechanism: the shock to the liquidity of any bank,  $\epsilon_{i,t}$ , is a function of all the shocks to the other banks' liquidity; the intensity of the shock spillover is a function of the intensity of the network links between banks captured by the network weights  $g_{ij}$ ; and whether the network amplifies or damps the effect of the individual liquidity shocks on aggregate liquidity depends, respectively, on whether the banks in the network act as strategic complements ( $\phi > 0$ ) or strategic substitutes ( $\phi < 0$ ). To illustrate this point, note that the vector of shocks to all banks at time  $t$  can be rewritten as

$$\epsilon_t = (\mathbf{I} - \phi \mathbf{G}_t)^{-1} \nu_t \equiv \mathbf{M}(\phi, \mathbf{G}_t) \nu_t \quad (23)$$

where  $\epsilon_t = [\epsilon_{1,t}, \dots, \epsilon_{n,t}]'$  and  $\nu_t = [\nu_{1,t}, \dots, \nu_{n,t}]$ . This implies that if  $\mathbf{G}_t$  is a right stochastic matrix<sup>21</sup> (and this is the case when we model the network weights  $g_{i,j}$  as the fraction of borrowing by bank  $i$  from bank  $j$ ), then a unit shock to the system equally spread across banks (i.e.  $\nu_t = (1/n) \mathbf{1}$ ) would imply a total change in aggregate liquidity equal to  $(1 - \phi)^{-1}$  – that is,  $\phi$  captures the ‘average’ network multiplier effect of liquidity shocks.

Moreover, equation (23) implies that any time variation in the network structure,  $\mathbf{G}$ , or in the network multiplier,  $1/(1 - \phi)$ , would result in a time variation in the volatility of total liquidity since the variance of the shocks to the total network liquidity ( $\mathbf{1}'\epsilon_t$ ) is

$$Var_t(\mathbf{1}'\epsilon_t) = \mathbf{1}'\mathbf{M}(\phi, \mathbf{G}_t) \Sigma_\nu \mathbf{M}(\phi, \mathbf{G}_t)' \mathbf{1}.$$

Here we have used the fact that  $\mathbf{G}_t$  is pre-determined with respect to the time  $t$  information,  $\Sigma_\nu := \mathbb{E}[\nu_t \nu_t']$  is a diagonal matrix with the variances of the idiosyncratic shocks  $\{\sigma_i^2\}_{i=1}^n$  on

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<sup>20</sup>To allow for potential time variation in  $\phi$  instead we also perform estimations in subsamples and over a rolling window.

<sup>21</sup>If  $\mathbf{G}_t$  is a right stochastic matrix, then  $\mathbf{G}_t \mathbf{1} = \mathbf{1}$ , and therefore

$$\mathbf{1} = (\mathbf{I} - \phi \mathbf{G}_t)^{-1} (\mathbf{I} - \phi \mathbf{G}_t) \mathbf{1} = (\mathbf{I} - \phi \mathbf{G}_t)^{-1} \mathbf{1} (1 - \phi) \implies \mathbf{M}(\phi, \mathbf{G}_t) \mathbf{1} = (1 - \phi)^{-1} \mathbf{1}.$$

the main diagonal.

As outlined in Section A.2.1 of the Appendix, we can estimate the parameters of the spatial error model jointly using a quasi-maximum likelihood approach. In order to elicit the time variation in the network coefficient  $\phi$ , we perform subsample and rolling window estimates. The estimation frequency is daily, with a lagged monthly update of the network matrix  $\mathbf{G}_t$ .

An estimation issue for network models is the well-known reflection problem (Manski (1993)): the neighbouring banks' decisions about their liquidity holdings affect each other, so that we cannot distinguish between whether a given bank's action is the cause or the effect of its neighbouring banks' actions. To address this problem, Bramoullé, Djebbari and Fortin (2009) have shown that the network effect  $\phi$  can be identified if there are two nodes in the network with different average connectivities of their direct connected nodes. This condition is satisfied in our data.<sup>22</sup>

As a test of the model specification of our theory-driven formulation, we also consider a more general specification that allows for a richer set of network interactions. That is, we model liquidity holding as a spatial Durbin model (SDM – see, e.g. LeSage and Pace (2009)) where bank specific liquidity is allowed to depend directly on other banks' liquidity and characteristics, and pairwise control variables

$$l_{i,t} = \alpha_t^{time} + \alpha_i^{bank} + \sum_{m=1}^M \beta_m^{bank} x_{i,t}^m + \sum_{p=1}^P \gamma_p^{time} x_t^p + \rho \sum_{j=1}^n g_{i,j,t} l_{j,t} + \sum_{j=1}^n g_{i,j,t} x_{i,j,t} \theta + \nu_{i,t} \sim iid(0, \sigma_i^2), \quad (24)$$

where  $x_{i,j,t}$  denotes match specific control variables and the characteristics of other banks. The above formulation allows a specification test of our structural model, since restricting  $\theta = \mathbf{0}$ , and setting  $x_{i,j,t} := vec(x_{j \neq i,t}^m)'$ ,  $\theta = -\phi vec(\beta_m^{bank})$ ,  $\gamma_p^{time} = (1 - \phi) \beta_p^{time} \forall p$ , and most importantly  $\psi = \rho$ , we are back to the SEM specification implied by our structural model. These restrictions are tested formally in Section VI.

With the SEM estimated parameter at hand, we can also identify the risk key players of the interbank liquidity market. To do so, we define the network impulse response function as follows.

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<sup>22</sup>The separate identification of the fixed effects  $\bar{\mu}_i$  and  $\alpha_i^{bank}$  is more complex, and is discussed in detail in the Appendix. In particular, when  $\mathbf{G}_t$  is a right stochastic matrix, the identification of  $\bar{\mu}_i$  and  $\alpha_i^{bank}$  requires at least one bank to not borrow from any other bank at some point in the sample (in our data, this happens 13.5% of the time). Alternatively, one can normalise one of the  $\mu_i$  to zero and identify the remaining ones in deviation from it. But note that the separate identification of the fixed effects does not affect the identification of  $\phi$ .

**Definition 3** *Network Impulse-Response Function.* Let  $L_t \equiv \mathbf{1}'l_t = [l_{1t}, \dots, l_{Nt}]$  denote the total liquidity in the interbank network. The network impulse response function of total liquidity,  $L_t$ , to a one standard deviation shock to a given bank  $i$ , is given by

$$NIRF_i(\phi, \sigma_i, \mathbf{G}_t) \equiv \frac{\partial L_t}{\partial \nu_{i,t}} \sigma_i = \mathbf{1}' \{\mathbf{M}(\phi, \mathbf{G}_t)\}_{.i} \sigma_i \quad (25)$$

where the operator  $\{\}_{.i}$  returns the  $i$ -th column of its argument.

The network impulse response is identical to the *outdegree* centrality of bank  $i$  defined in equation (8). Note that  $NIRF_i(\phi, 1, \mathbf{G}_t)$  is less than or greater than 1 depending on whether  $\phi$  is positive or negative – that is, if  $\phi > 1$  ( $< 1$ ) individual bank shocks are amplified (reduced) through the system.

The network impulse response provides a metric to identify which bank's shocks have the largest impact on the overall liquidity. Moreover, it does so taking into account both the size of the bank (via  $\sigma_i$ ), the network multiplier,  $\phi$ , and all the direct and indirect links between banks, since  $\mathbf{1}' \{\mathbf{M}(\phi, \mathbf{G}_t)\}_{.i}$  is the solution, for  $|\phi| < 1$ , of

$$\mathbf{1}' \{\mathbf{M}(\phi, \mathbf{G}_t)\}_{.i} = \mathbf{1}' \{\mathbf{I} + \phi \mathbf{G}_t + \phi^2 \mathbf{G}_t^2 + \dots\}_{.i} = \mathbf{1}' \left\{ \sum_{k=0}^{\infty} \phi^k \mathbf{G}_t^k \right\}_{.i}$$

where the first element in the series captures the direct effect of a unit idiosyncratic shock to bank  $i$ , the second element captures the effects through the first order network links, the third element captures the effect through the second order links, and so on. This also implies that  $\{\mathbf{M}(\phi, \mathbf{G}_t)\}_{ji}$  measures the total (direct and indirect) effect of a shock to bank  $i$  on the liquidity of bank  $j$ . Also, the network impulse response functions provide a natural decomposition of the variance of the total liquidity in the network system, since

$$Var_t(\mathbf{1}'\epsilon_t) \equiv vec(\{\mathbf{NIRF}_i(\phi, \sigma_i, \mathbf{G}_t)\}_{i=1}^n)' vec(\{\mathbf{NIRF}_i(\phi, \sigma_i, \mathbf{G}_t)\}_{i=1}^n).$$

We can also isolate the purely network part of the impulse response function, that is, the liquidity effect in excess of the direct effect of a shock to a bank:

$$NIRF_i^e(\phi, \sigma_i, \mathbf{G}_t) \equiv NIRF_i(\phi, \sigma_i, \mathbf{G}_t) - \sigma_i = \mathbf{1}' \{(\mathbf{I} - \phi \mathbf{G}_t)^{-1} \phi \mathbf{G}_t\}_{.i} \sigma_i, \quad (26)$$

and the above, setting  $\sigma_i = 1$ , i.e. considering a unit shock, is exactly the centrality measure of Katz (1953). Note that  $NIRF_i^e(\phi, \sigma_i, \mathbf{G}_t)$  has by construction the same sign as  $\phi$ .

Note also that it is straightforward to compute confidence bands for the estimated network impulse response functions, since they are simply a function of the distribution of  $\hat{\phi}$ , and

$\hat{\phi} - \phi_0$  has a canonical Quasi-MLE asymptotic Gaussian distribution (see Section A.2.2 in the Appendix).

## V Description of the Network and Other Data

We study the sterling interbank network over the sample period January 2006 to September 2010. The estimation frequency is daily, but we also use higher frequency data to construct several of the control variables defined below. The network of banks we consider is constituted by the banks in the CHAPS system during the sample – a set of 11 banks. These banks play a key role in the sterling large value payment system since they make payments both on their own behalf and on behalf of banks that are not direct members of CHAPS. The banks in the network are: Halifax Bank of Scotland (owned by Lloyds Banking Group); Barclays; Citibank (the consumer banking arm of Citigroup); Clydesdale (owned by National Australia Bank); Co-operative Bank (owned by The Co-operative Group); Deutsche Bank; HSBC (that incorporated Midland Bank in 1999 – one of the historical “big four” sterling clearing banks<sup>23</sup>); Lloyds TSB; Royal Bank of Scotland (including Natwest); Santander (formerly Abbey, Alliance & Leicester and Bradford & Bingley, owned by Banco Santander of Spain); and Standard Chartered.

We split our sample into three periods: Pre-crisis period: 1 January 2006 to 9 August 2007; Post Northern Rock/ Hedge Fund Crisis: 10 August 2007 to 19 September 2009; Post Asset Purchase Programme: 20 September 2009 to 30 September 2010. This is explained in more detail below.

Our proxies for the intensity of network links are the interbank overnight borrowing relations. This data is extracted from payment system data by applying an algorithm developed by Furfine (2000). This is an approach which is common to most papers on the interbank money market. The algorithm identifies pairs of payments between two payment system counterparties where the outgoing payment (the loan) is a multiple of 100,000 and the incoming payment (the repayment) happens the following day and is equivalent to the outgoing payment plus a plausible interest rate. This algorithm has been tested thoroughly, and tracks accurately the LIBOR rate on the whole. Furfine (2000) showed that the algorithm

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<sup>23</sup>For most of the 20th Century, the phrase “the Big Four” referred to the four largest sterling banks, which acted as clearing houses for bankers’ cheques. These were: Barclays Bank; Midland Bank (now part of HSBC); Lloyds Bank (now Lloyds TSB Bank and part of Lloyds Banking Group); and National Westminster Bank (“NatWest”, now part of The Royal Bank of Scotland Group). Currently, the largest four UK banks are Barclays, HSBC, Lloyds Banking Group and The Royal Bank of Scotland Group (with a combined market capitalization of more than £254bn) closely followed by Standard Chartered (with a market cap of over £37bn) – and all of these banks are part of our network.



accurately identifies the Fed Funds rate when applied to Fedwire data.<sup>24</sup>

The loan data are compiled to form an interbank lending and borrowing network. In particular, the elements  $g_{ij,t}$  of the adjacency matrix  $\mathbf{G}_t$  are given by the fraction of bank's  $i$  overnight loans that come from bank  $j$ . In the baseline specification, the weights at time  $t$  are computed as monthly averages for the previous month.

By construction,  $\mathbf{G}_t$  is a square right stochastic matrix. Its largest eigenvalue is therefore equal to one. This implies that the potential propagation of shocks within the system will be dominated by the second largest eigenvalue of the adjacency matrix.<sup>25</sup> The time series of the second largest eigenvalue of  $\mathbf{G}_t$  is presented in Figure 1. As can be seen in the figure, there was a substantial increase of the eigenvalue in the aftermath of the Northern Rock/Hedge Fund Crisis period (September 2007), but what is striking is the substantial increase in the volatility of the network links in the post Quantitative Easing period.

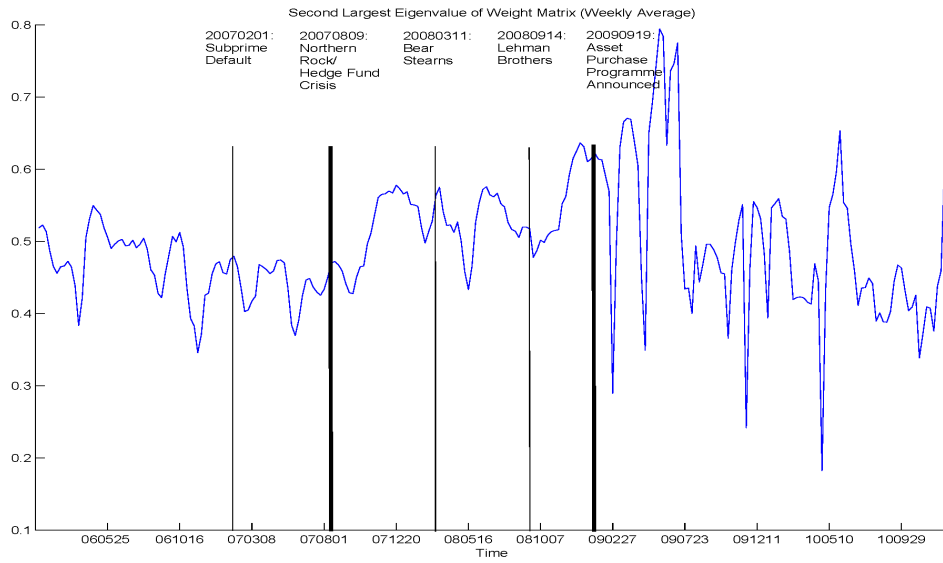


Figure 1: Second largest eigenvalue of  $\mathbf{G}_t$ .

One way to characterise time variation in the cohesiveness of the network is to examine the behaviour of the Average Clustering Coefficient (ACC – see Watts and Strogatz (1998))

<sup>24</sup>The data are not perfect. They are inferred data, so it is possible that there are some erroneous matches or that some loans are missing. We have no reason to expect this to introduce mechanical bias into the data. It is also necessarily incomplete. The data are only for banks which are participants in the payment systems. This creates two problems. First, some loans may be attributed to the settlement bank involved when in fact the payments are made on behalf of one of their customers. Second, where a loan is made between one customer of a settlement bank and another, this transaction will not be settled through the payment system but rather across the books of the settlement bank. This is a process known as internalisation. Internalised payments are invisible to the central bank, so they are a part of the overnight money market that will not be captured.

<sup>25</sup>This is because  $\mathbf{G}^k$  can be rewritten in Jordan normal form as  $\mathbf{P}\mathbf{J}^k\mathbf{P}^{-1}$  where  $\mathbf{J}$  is the (almost) diagonal matrix with eigenvalues (or Jordan blocks in case of repeated eigenvalues) on the main diagonal.

defined as

$$ACC_t = \frac{1}{n} \sum_{i=1}^n CL_i(\mathbf{G}_t), \quad CL_{i,t} = \frac{\#\{jk \in \mathbf{G}_t \mid k \neq j, j \in n_i(\mathbf{G}_t), k \in n_i(\mathbf{G}_t)\}}{\#\{jk \mid k \neq j, j \in n_i(\mathbf{G}_t), k \in n_i(\mathbf{G}_t)\}}$$

where  $n_i(\mathbf{G}_t)$  is the set of players that have a direct link with player  $i$  and  $\#\{.\}$  is the count operator. The numerator of  $CL_{i,t}$  is the number of pairs of banks linked to  $i$  that are also linked to each other, while its denominator is simply the number of pairs of banks linked to  $i$ . Therefore, the average clustering coefficient measures the average proportion of banks that are connected to bank  $i$  who are also connected to each other. By construction this value ranges from 0 to 1. The time series of the ACC is reported in Figure 2. The figure shows that at the beginning of the sample period, the network is highly cohesive since, on average, around 80% of the pairs of banks connected to any given bank are also connected to each other.

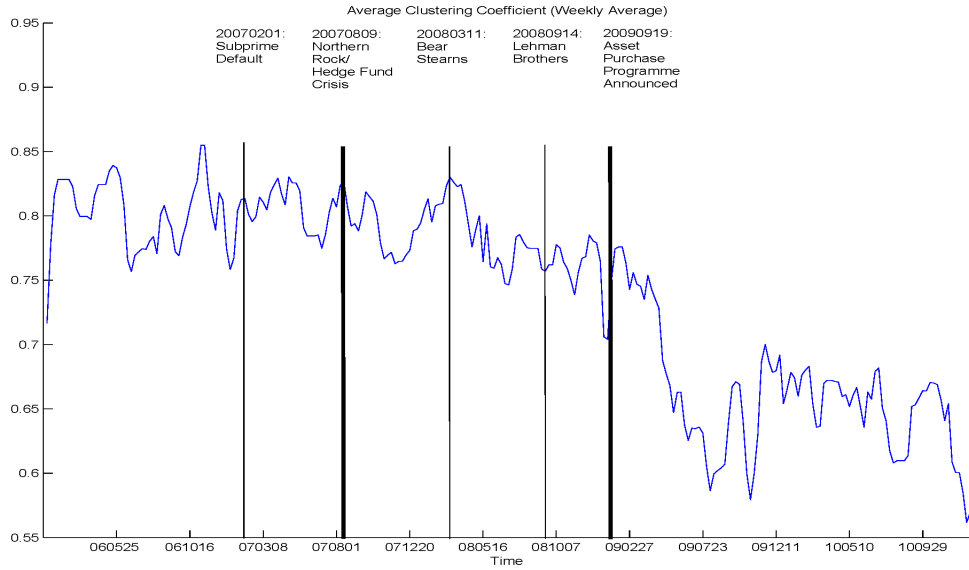


Figure 2: Average clustering coefficient of the interbank network.

The degree of connectedness seems to have a decreasing trend during 2007–2008, and a substantial and sudden decrease following the Asset Purchase Programme, when the average clustering coefficient decreased by about one-quarter of its pre-crisis average. This might be the outcome of reduced interbank borrowing needs during the Quantitative Easing thanks to the availability of additional reserves from the Bank of England (combined with a move towards increased collateralisation of borrowing and an overall deleveraging of banks balance sheets, see, e.g. Westwood (2011)). This interpretation is consistent with Figure 3, which depicts the (rolling monthly average of) daily gross overnight borrowing in the interbank network. The data record a substantial increase in overnight borrowing as the initial response

to the turmoil in the financial market, possibly caused by a shift to very short borrowing due to increased difficulties in obtaining long term financing (Wetherilt, Zimmerman, and Soramaki (2010)), and a substantial decrease in overnight borrowing after the beginning of Quantitative Easing.

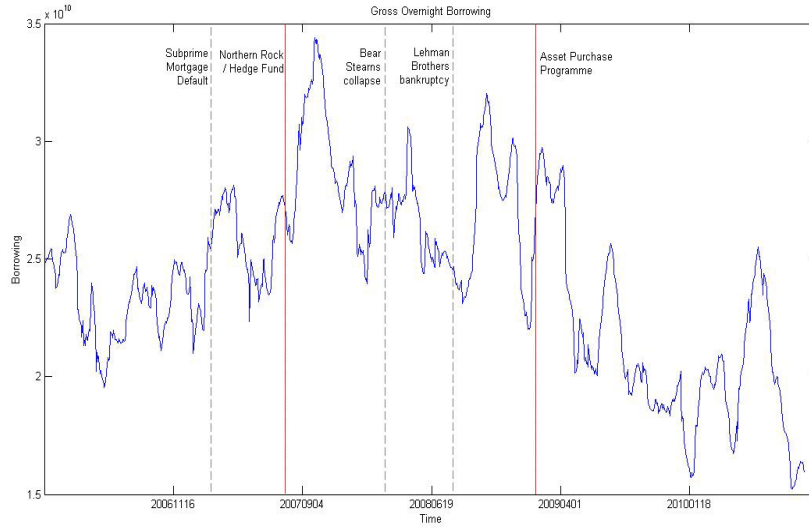


Figure 3: Daily gross overnight borrowing in the interbank network (rolling monthly average).

To measure the dependent variable  $l_{i,t}$ , that is, the liquidity holdings of each bank, we use central bank reserve holdings. We supplement this with the collateral that is repo'd with the Bank of England in return for intraday liquidity (these repos are unwound at the end of each working day). For robustness, we also analyse separately the behaviour of each of these two liquidity components. The weekly average of the total liquidity in the system is reported in Figure 4. The figure depicts a substantial upward trend in the available liquidity in the post subprime default subsample and during the various financial shocks registered in 2008–2009, consistently with the evidence of banks' hoarding liquidity in response to the financial crisis (Acharya and Merrouche (2010)), but this upward trend is dwarfed by the steep run-up registered in response to the Asset Purchase Programme (aka Quantitative Easing) that almost tripled the average liquidity in the system.

As covariates, in addition to common monthly time dummies meant to capture time effects, and bank fixed effects, meant to capture unobserved heterogeneity, we use a large set of aggregate ( $x_t^p$ ) and bank specific ( $x_{i,t}^m$ ) control variables. Note that since in the econometric specification in equations (21) and (22) the network effects are elicited through their contribution to the residuals, a potential overfitting of banks' liquidity choice through observable variables is a conservative approach.

*Aggregate Control Variables ( $x_t^p$ ):* All the common control variables, meant to capture

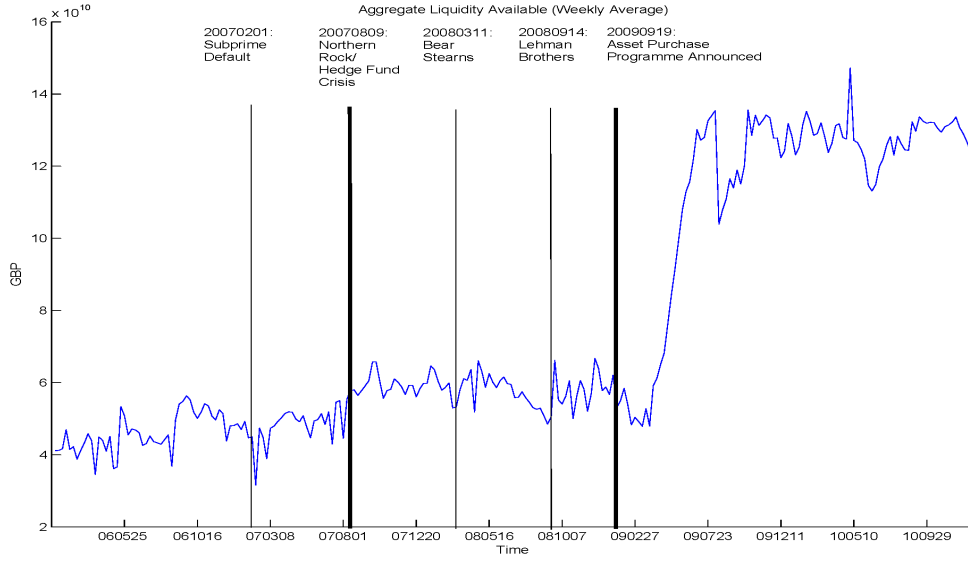


Figure 4: Weekly average of aggregate liquidity available at the beginning of the day.

aggregate market conditions, are lagged by one day so that they are predetermined with respect to time  $t$  innovations. To control for aggregate market liquidity condition we use the total liquidity in the previous day. To proxy for the overall cost of funding liquidity we use the lagged LIBOR rate and the interbank rate premium (computed as the difference between the overnight interest rate and the LIBOR rate).

Since banks' decisions to hold liquidity are likely to be influenced by the volatility of their daily payment outflows, we construct a measure of the intraday payments volatility, defined as

$$VolPay_t = \sqrt{\frac{1}{88} \sum_{\tau=1}^{88} (P_{t,\tau}^{out})^2} \quad (27)$$

where  $P^{out}$  denotes payment outflows and 88 is the number of time intervals within a day. The time series of this variable is reported in Figure 5. It is characterised by a strong upward trend before the subprime default crisis, and a distinctively negative trend during the period of financial turmoil preceding the beginning of QE. During the QE period, this variable has no clear trend but is characterised instead by a substantial increase in volatility.

We also control for the turnover rate in the payment system (see Benos, Garratt, and Zimmerman (2010)). This variable is

$$TOR_t = \frac{\sum_{i=1}^N \sum_{\tau=1}^{88} P_{i,t,\tau}^{out}}{\sum_{i=1}^N \max \{ \max_{\tau \in [1,88]} [CNP(\tau; i, t)], 0 \}}$$

where the cumulative net debit position (CNP) is defined as the difference between payment

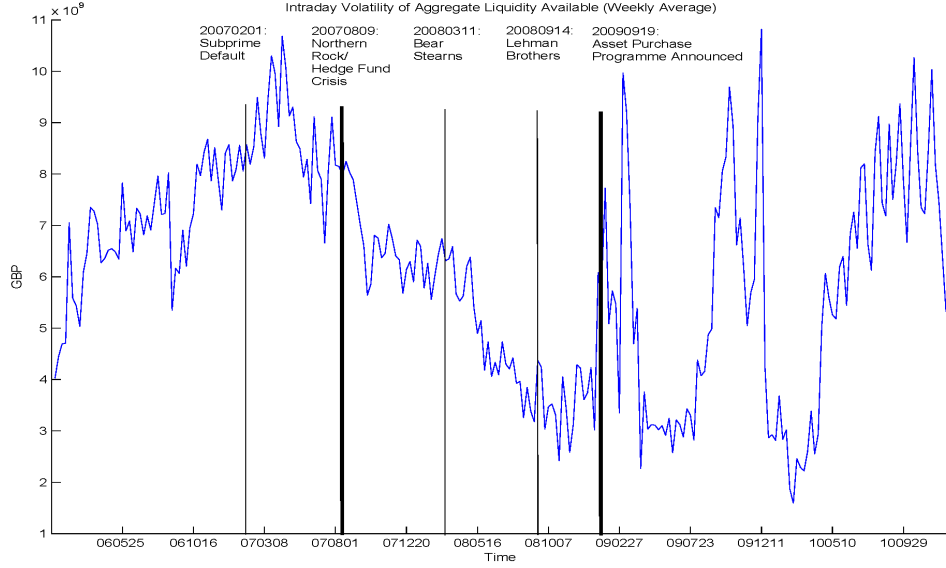


Figure 5: Intraday volatility of aggregate outflows.

outflows and inflows. The numerator captures the total payments in the system in a day, while the denominator is the sum of the maximum net debt positions of all banks in a given day. This variable is meant to capture the velocity of transactions within the interbank system and its time series is reported in Figure 11 of Appendix A.3, and indicates an increased turnover during the financial turmoil, followed by a reduction to levels below the historical average during the QE period.

Since banks have some degree of freedom in deciding on the timing of their intraday outflows, they could use this strategically. Therefore, we control for the right kurtosis<sup>26</sup> ( $rK_t$ ) of intraday payment times. The time series of this variable is reported in Figure 12 of Appendix A.3 and shows a substantial increase during the QE period.

*Bank Characteristics* ( $x_{i,t}^m$ ): As for the aggregate control variables, all bank characteristic variables are lagged so that they are predetermined with respect to innovations at time  $t$ . Despite the fact that we control for average interest rates (LIBOR and average overnight

<sup>26</sup>We define as right and left kurtosis (denoted, respectively, by  $rK_t$  and  $lK_t$ ) the fractions of kurtosis of payment times generated by payment times that are, respectively, above and below the average payment time of the day:

$$rK_t = \frac{\sum_{\tau > m_t} \left( \frac{\tau - m_t}{\sigma_t} \right)^4}{\sum_{\tau=1}^{88} \left( \frac{\tau - m_t}{\sigma_t} \right)^4}; \quad lK_t = \frac{\sum_{\tau < m_t} \left( \frac{\tau - m_t}{\sigma_t} \right)^4}{\sum_{\tau=1}^{88} \left( \frac{\tau - m_t}{\sigma_t} \right)^4};$$

where  $m_t$  and  $\sigma_t$  are defined as

$$m_t = \frac{1}{88} \sum_{\tau=1}^{88} \tau \left( \frac{P_{t,\tau}^{OUT}}{\sum_{t=1}^T P_{t,\tau}^{OUT}} \right), \quad \sigma_t^2 = \frac{1}{88-1} \sum_{\tau=1}^{88} \left[ \tau \left( \frac{P_{t,\tau}^{OUT}}{\sum_{t=1}^{88} P_{t,\tau}^{OUT}} \right) - m_t \right]^2.$$

borrowing rate), we also control for the bank specific overnight borrowing rate (computed as the average weighted by the number of transactions). We include these variables (reported in Figure 13 of Appendix A.3) because in response to each of the collapses, of Northern Rock and of Lehman Brothers, there was a substantial increase in the cross-sectional dispersion of the overnight borrowing rates, and this increase in dispersion persisted during the QE period (see Figure 14 of Appendix A.3). We also control for: bank specific right kurtosis of the time of intraday payments in ( $rK_{i,t}^{in}$ , to capture a potential incentive to increase bank liquidity) and out ( $rK_{i,t}^{out}$ , since banks in need of liquidity might have an incentive to delay their outflows); the intraday volatility of the used liquidity ( $VolPay_{i,t}$ , defined as in equation (27) but using bank specific flows); the total amount of intraday payments ( $LevPay_{i,t} \equiv \sum_{\tau=1}^{88} P_{i,t,\tau}^{out}$ ); the liquidity used ( $LU_{i,t}$ ) as defined in Benos, Garratt, and Zimmerman (2010);<sup>27</sup> the ratio of repo liabilities to total assets; the cumulative change in the ratio of retail deposits to total assets; the total lending and borrowing in the interbank market; the cumulative change in the 5-year senior unsecured credit default swap (CDS) premia; and a dummy variable for the top four banks in terms of payment activity.

## VI Estimation Results

For the first exercise, we estimate our empirical network model specified in equations (21) and (22) using three sub-periods of roughly equal size. These are the sub-period before the Northern Rock/Hedge Fund Crisis (Period 1), the sub-period immediately after the Northern Rock/Hedge Fund Crisis but before the announcement of the Assets Purchase Programme (Period 2), and the sub-period running from the announcement of the Assets Purchase Programme to the end of the period of study (Period 3). We split our period of study into these three parts since *a*) they correspond to very different overall market conditions, and *b*), as documented in Section V, the network structure and behaviour of these sub-periods seem to differ substantially. Period 1 is a relatively tranquil period for the banking sector. Period 2 is characterised by several significant events in world financial markets, such as: the run on Northern Rock (the first UK bank run in 150 years), the subprime mortgage hedge fund crisis, the Federal Reserve intervention in Bear Stearns and its subsequent sale to JP Morgan Chase, and the bankruptcy of Lehman Brothers. Period 3 is characterised by a real regime shift – the beginning of Quantitative Easing – in UK monetary policy.<sup>28</sup>

The estimation results for these three subsamples are reported in Table 1, where we

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<sup>27</sup>Liquidity used on day  $t$  is defined as  $LU_{i,t} = \max\{\max_{\tau \in [1,88]}[CNP(\tau; i, t)], 0\}$ .

<sup>28</sup>See Bank of England, ‘Financial Stability Report,’ No. 24, October 2008 and ‘Financial Stability Report,’ No. 25, June 2009.

**Table 1: Spatial Error Model Estimation**

|                      | <i>Period 1</i>   | <i>Period 2</i>  | <i>Period 3</i>    |
|----------------------|-------------------|------------------|--------------------|
| $\hat{\phi}$         | 0.8137<br>(21.47) | 0.3031<br>(1.90) | -0.1794<br>(-4.96) |
| $R^2$                | 66.01%            | 92.09%           | 91.53%             |
| $1/(1 - \hat{\phi})$ | 5.3677<br>(4.92)  | 1.4349<br>(4.37) | 0.8479<br>(32.61)  |

Estimation results for equations (21) and (22). Periods 1, 2 and 3, correspond, respectively, to before the Northern Rock/Hedge Fund Crisis, after the Hedge Fund Crisis but before the Asset Purchase Programme, and after the Asset Purchase Programme announcement. The  $t$ -statistics are reported in parentheses under the estimated coefficients. Standard errors are computed via block bootstrap. For the average network multiplier,  $1/(1 - \hat{\phi})$ , the delta method is employed.

report only the estimates of the spatial dependency parameter  $\phi$ , the  $R^2$  of the regression, as well as the implied average network multiplier  $1/(1 - \phi)$ .<sup>29</sup> Omitted from the table are the coefficient estimates associated with the control variables, which are reported in Table A1 of the Appendix. The first row of the panel reports the estimates of the network coefficient  $\phi$ . Recall that  $\phi > 0$  ( $< 0$ ) implies that banks' liquidity holding decisions are strategic complements (substitutes) and that this tends to amplify (reduce) the effect of bank specific liquidity shocks. In the first period, the point estimate of this coefficient is about 0.8137 (and highly significant) indicating the presence of a substantial network multiplier effect for liquidity shocks: a £1 idiosyncratic shock equally spread across banks would result in a  $1/(1 - \hat{\phi}) = £5.3677$  shock to aggregate liquidity. In the second period, the coefficient  $\phi$  is still statistically significant but it is substantially reduced in magnitude, to 0.3031, implying weak complementarity, with an (average) shock multiplier of about 1.4349. This finding suggests that in response to the turbulence in the financial market that have characterised Period 2, the banking system significantly reduced its own network risk exposure. In Period 3, the coefficient  $\phi$  becomes negative, -0.1794, but is still highly significant, implying an average network shock multiplier of about 0.8479. This is particularly interesting since a negative  $\phi$  implies strategic substitution in liquidity holdings, as in Bhattacharya and Gale (1987), that is, a situation in which individual banks decide to choose less liquidity when neighbouring banks have more liquidity, having a damping effect on the aggregate level of liquidity. That these negative point estimates are obtained in the Quantitative Easing sub-period suggests that the liquidity multiplier effect was not working at the time of the

<sup>29</sup>Note that from equation (23) we can compute the *average* network multiplier as the total impact on aggregated liquidity resulting from a unit shock equally spread across the  $n$  banks. This is given by

$$\mathbf{1}'\mathbf{M}(\phi, \mathbf{G}_t)\mathbf{1}\frac{1}{n} = \frac{1}{1 - \phi}$$

large inflow of liquidity provided by the Asset Purchase Programme of the Bank of England. The total liquidity injection from the program was, up to October 2011, of about £275 billion.<sup>30</sup> Overall, the fit of the model is quite good in all sub-periods, with an  $R^2$  in the range 66% – 92%.

With the subperiod estimates at hand, we can compute the network impulse response functions to identify the risk key players in the interbank market. The results for Period 1 are reported in the upper panel of Figure 6. In particular, in the upper panel we report the excess network impulse response functions to a unit shock  $NIRF^e(\hat{\phi}, 1, \bar{\mathbf{G}}_1)$  defined in equation (26) (where  $\bar{\mathbf{G}}_j$  denotes the average  $\mathbf{G}_t$  in the  $j$ -th subsample), as well as the two standard deviation error bands. Also, as a point of reference, we report in the same panel the average network multiplier in excess of the unit shock (i.e.  $(1 - \phi)^{-1} - 1$ ). As mentioned earlier, the point estimate in Period 1 implies a large average network multiplier of shocks to individual banks, and the picture shows that in response to a £1 idiosyncratic shock equally spread across banks, the final compounded shock to the overall liquidity would be increased by another £4.3677. Nevertheless, what the upper panel of Figure 6 stresses is that this large network amplification of shocks is due to a small subset of banks. In particular: a £1 idiosyncratic shock to the liquidity of either Bank 5 or Bank 9 would generate an excess reaction of aggregate liquidity of about £13.9 and £13.8; the same shock to Bank 6 would result in an excess reaction of aggregate liquidity of about £8.9; instead, a shock to Bank 4 would have an effect that is roughly of the same size as the average network multiplier while a shock to any of the remaining seven banks would be amplified much less by the network system. That is, the network impulse response functions stress that there is a small subset of key players in the interbank liquidity market that generate most of the network risk.

The central panel of Figure 6 shows the average net borrowing during Period 1. Comparing the upper and central panels of the figure, it is interesting to notice that simply looking at the individual lending and borrowing behaviour one cannot identify the riskiest players for the network. In particular, the two riskiest players identified through our structural estimation are not the largest net borrowers in the network – the largest net borrower, Bank 4, is instead an average bank in network risk terms. Moreover, Bank 5, one of the two risk key players, is not a net borrower – it is instead the second largest net lender. The comparison between the two panels also makes clear that the risk key players are not necessarily the net borrowing banks – net borrowers and net lenders are roughly as likely to be the network risk key players. This result is intuitive: negative liquidity shocks to a bank that lends liquidity to a large share of the network can be, for the aggregate liquidity level, as bad as a negative

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<sup>30</sup>See [http://www.bankofengland.co.uk/monetarypolicy/Pages/qe/qe\\_faqs.aspx](http://www.bankofengland.co.uk/monetarypolicy/Pages/qe/qe_faqs.aspx).



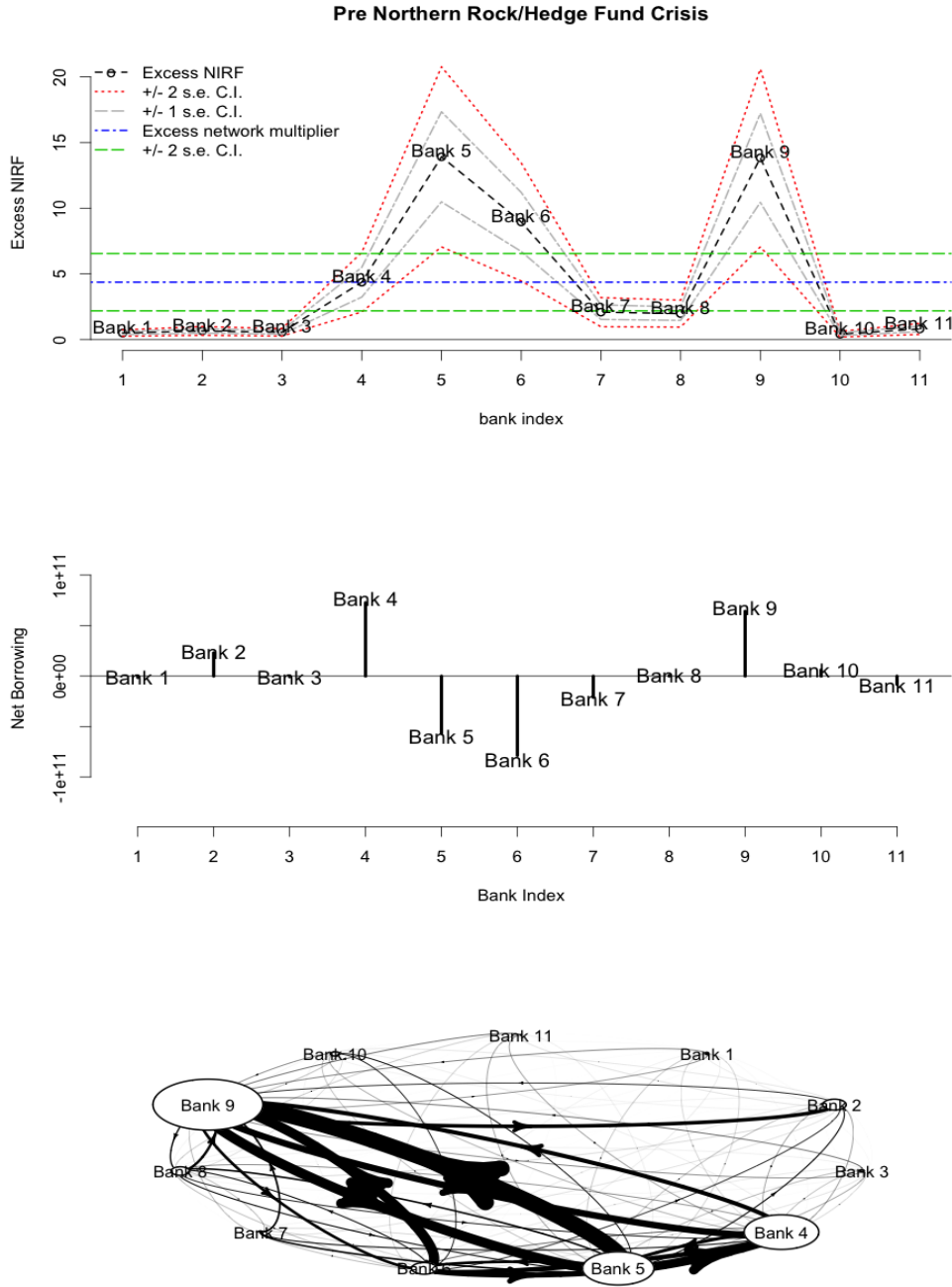


Figure 6: The period before the Northern Rock/Hedge Fund Crisis. Network excess impulse response functions to a unit shock (upper panel); net borrowing (central panel); borrowing and lending flows (lower panel) where the ellipses identifying individual banks are (log) proportional to their total gross borrowing in the system, incoming arrows to a node indicate borrowing flows to that node, while outgoing arrows indicate lending flows from that node, and the thickness of the arrows is (log) proportional to the sterling value of the flows.

shock to a bank that is borrowing liquidity from other banks. But the comparison between the two panels also makes it clear that simply looking at the largest players in terms of net borrowing or lending would not identify the key risk players for the system. The reasons behind this finding can be understood by looking at the lower panel of the figure, where we present the average network structure during Period 1. In particular, the size of the ellipses identifying individual banks are (log) proportional to their total gross borrowing in the system, incoming arrows to a node indicate borrowing flows to that node while outgoing arrows indicate lending flows from that node, and the thickness of the arrows is (log) proportional to the sterling value of the flows. The lower panel highlights that the key risk players are the players that tend to have the most borrowing and lending connections in the system, and are directly connected to banks that also have a lot of connections, but are not necessarily the players that borrow or lend more in either gross nor net terms, but are rather the players that borrow from, and lend to, more – well connected – players.

Figure 7 reports excess impulse response functions (upper panel), average net borrowing positions (central panel), and network flows (lower panel) for Period 2 – the period characterised by a high degree of stress in the financial market. The first thing to notice is that despite the overall increase in activity in the interbank borrowing and lending market, outlined by both the central and lower panels and by Figure 3, there is a drastic reduction in the average network multiplier reported in the first panel: the average excess network reaction to a unit shock is only about .43. That is, in a period of financial stress, banks seem, on average, to have radically reduced their network risk exposure, and they have done so despite having increased the amount of overnight borrowing and lending needed to fund their liquidity needs. Nevertheless, as stressed by the first panel, the network risk profile, even though substantially reduced overall, is still quite high for a small subset of banks. In particular, a unit shock to Bank 5, Bank 9 and Bank 6, would result, respectively, in an excess network liquidity change of 1.77, 1.36 and 0.85, while the same shock to Bank 4 would have an effect very similar to the average one, and a shock to the remaining banks would receive minimal amplification from the network system.

The results for Period 3 – the one starting at the onset of Quantitative Easing – are reported in Figure 8, and are radically different from the ones of the previous two periods. First, banks tend to behave as strategic substitutes in their liquidity holdings in this period, therefore the network has a reducing effect on individual bank shocks, implying a negative average excess multiplier of  $-0.15$ , that is, a unit liquidity shock equally spread across banks would result in a  $1 - 0.15 = 0.85$  shock to aggregate liquidity. But, once again, there is substantial heterogeneity among the banks, in the sense that for most banks (Bank 1, 3, 7, 8, 10 and 11) the network has basically no effect on how their own shocks propagate to the

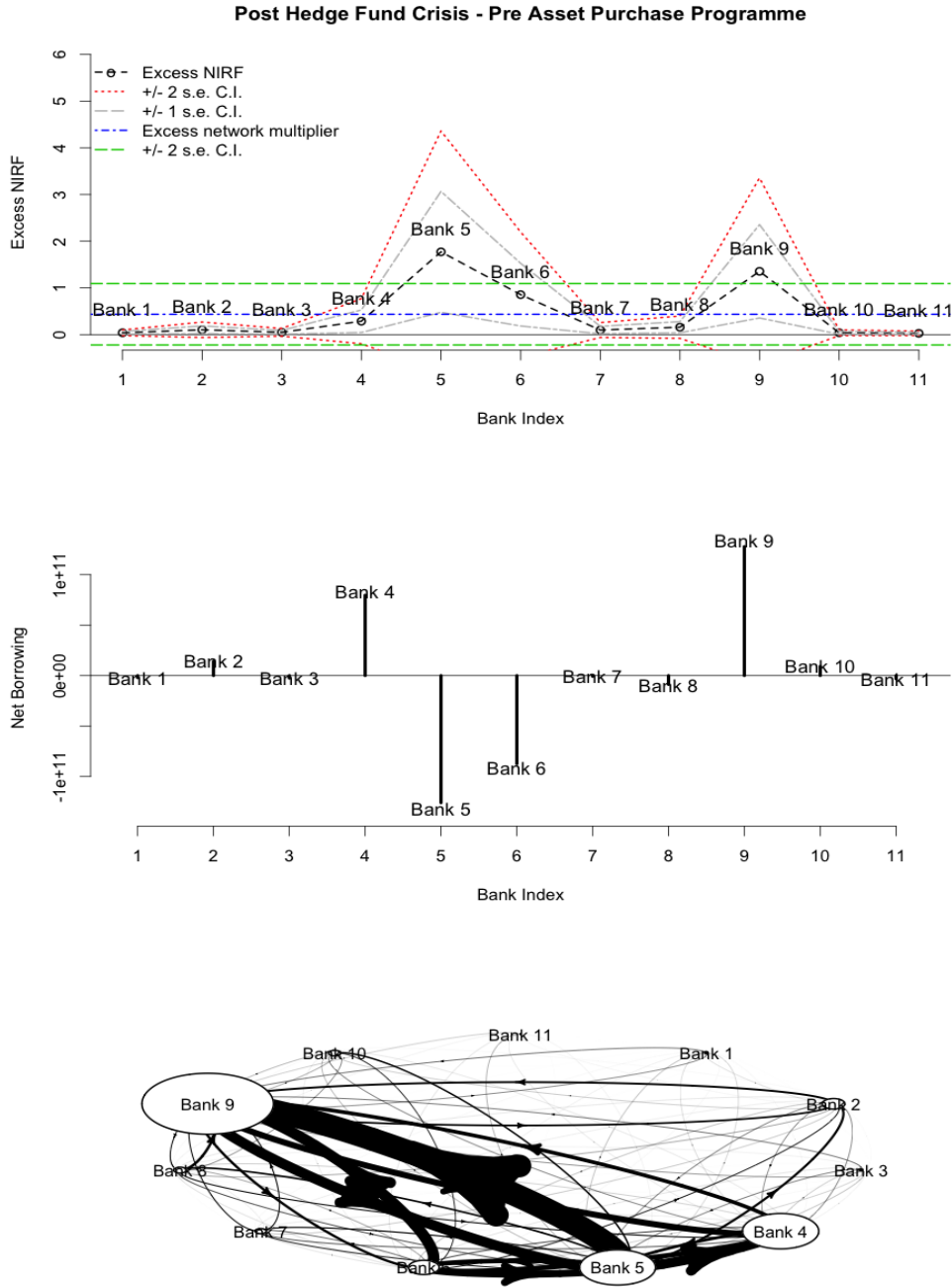


Figure 7: After the hedge fund crisis but before QE. Network excess impulse response functions to a unit shock (upper panel); net borrowing (central panel); borrowing and lending flows (lower panel), where the ellipses identifying individual banks are (log) proportional to their total gross borrowing in the system, incoming arrows to a node indicate borrowing flows to that node, while outgoing arrows indicate lending flows from that node, and the thickness of the arrows is (log) proportional to the sterling value of the flows.

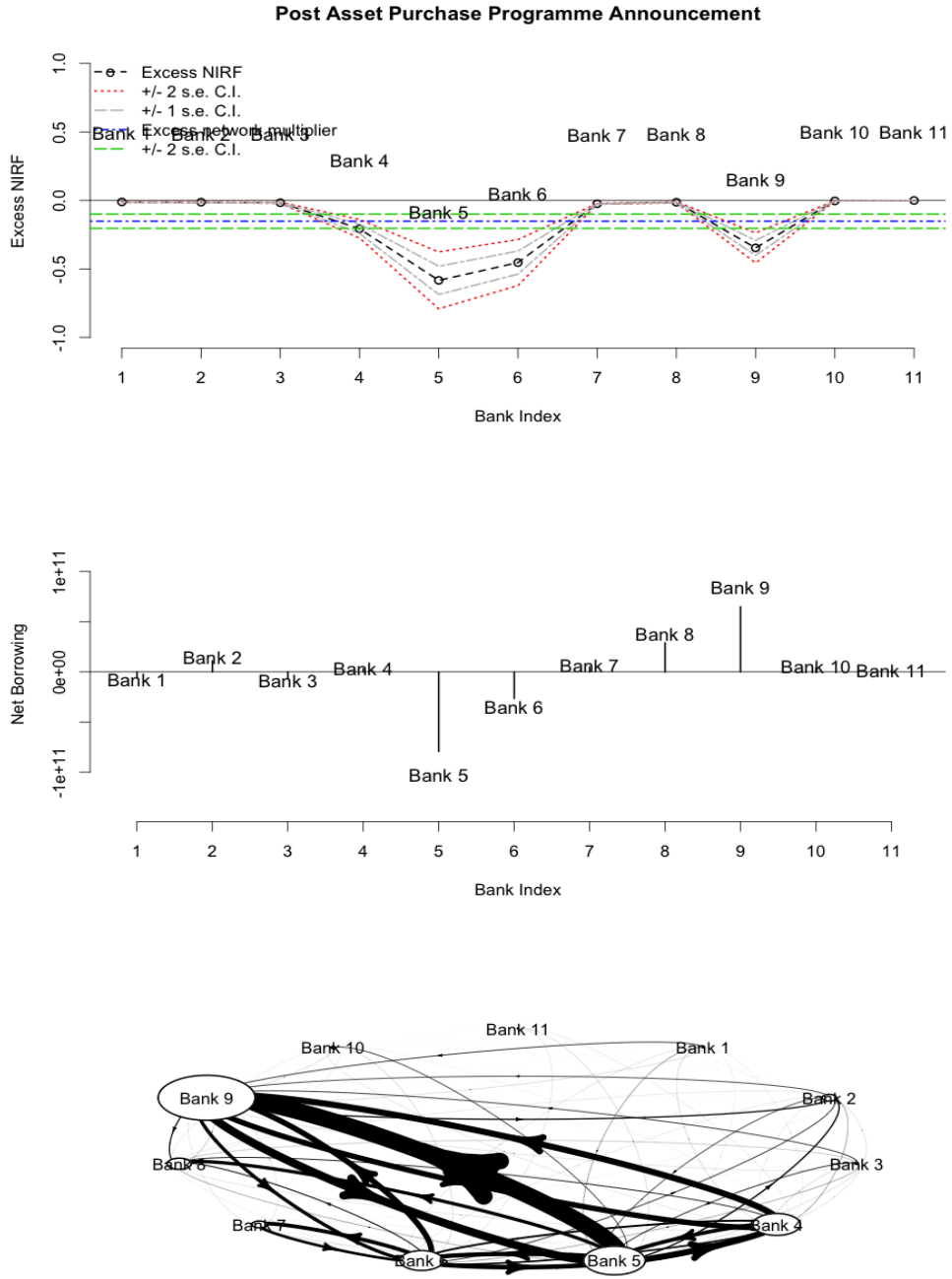


Figure 8: The QE period: Network excess impulse response functions to a unit shock (upper panel); net borrowing (central panel); borrowing and lending flows (lower panel) where the ellipses identifying individual banks are (log) proportional to their total gross borrowing in the system, incoming arrows to a node indicate borrowing flows to that node, while outgoing arrows indicate lending flows from that node, and the thickness of the arrows is (log) proportional to the sterling value of the flows.

system, while for a few other banks (4, 5, 6, and 9), the network structure helps reduce the impact of their own idiosyncratic shocks on aggregate liquidity.

This behaviour arises in a period in which the degree of connectedness of the network was substantially reduced (see Figure 2 and the lower panel of Figure 8), the gross borrowing in the system had been substantially reduced (see Figure 3), most banks held net borrowing positions close to zero (central panel of Figure 8), but at the same time the overall liquidity in the system had been substantially increased thanks to the Asset Purchase Programme (Figure 4). What is also interesting to notice is that the same banks that were the riskiest players in the previous two periods (Banks 5, 6 and 9) are now the less risky ones for the system since an idiosyncratic shock to these banks, thanks to both their centrality in the network and the overall strategic substitute behaviour of banks, would have a substantially damped effect on aggregate liquidity.

## VI.1 Central Planner vs Market Equilibrium

With the estimates of the structural parameters at hand, we can quantitatively assess the discrepancy, if any, between the banks' liquidity buffer holdings generated in response to their network exposures and borrowing ability, and the level of liquidity buffer that a benevolent central planner would have wanted the banks to hold in response to the liquidity risk in the interbank market. That is, from equations (7) and (17), we can compute the (average) difference between the central planner and the market liquidity buffer stock in the network as  $\mathbf{1}' [\mathbf{M}^p(\phi, \psi, \delta, \mathbf{G}) - \mathbf{M}(\phi, \mathbf{G})] \bar{\mu}$ .

Similarly, from equations (9) and (18) we can compute the difference between the level of volatility in the system desired by the central planner and that realised by the market:  $Var(Z^p(\phi, \mathbf{G})) - Var(Z^*(\phi, \psi, \delta, \mathbf{G}))$ .

The challenge in computing the above quantities is that we have consistent estimates of  $\phi$  and  $\bar{\mu}$ , but we cannot directly estimate  $\psi$  and  $\delta$ . Nevertheless, we can calibrate  $\psi$  to a natural benchmark:  $\psi = 1$ . This corresponds to the case in which each bank values in an identical manner the liquidity it holds directly in the network, and the liquidity available via its borrowing links to other banks. Moreover, with  $\psi = 1$  we have that  $\delta/\gamma = \phi + 1$ . Hence we do not need to choose values of  $\gamma$  and  $\delta$  if  $\psi = 1$  and  $\phi$  is set to the estimated value.

Table 2 reports the discrepancies between the central planner's solutions and the market equilibria, based on the point estimates of the structural parameters in Table 1, and the average value of the adjacency matrix  $\mathbf{G}$ , in the three sub-periods.

In Period 1 – when the (average) network multiplier was extremely large – the market equilibrium is characterised, from the perspective of a central planner, by excessive risk: the central planner would like the volatility of liquidity to be reduced by almost 91%. Moreover,

albeit marginally, the liquidity level in the system is also excessive. Given the high velocity of inside money in this period, the network has the capacity to greatly amplify the individual buffer stocks. Hence, a small reduction in the equilibrium buffer stock holdings, from central planner’s perspective, will come with a greater reduction in network volatility, therefore delivering a better level–risk tradeoff.

In Period 2, given the reduction in  $\phi$ , the market equilibrium produces *ceteris paribus* less volatility than in the Period 1. Nevertheless, the market volatility is still too large (by about 65%) from the central planner’s perspective. Moreover, the level of buffer stock network liquidity in this sub-period is much smaller than what is considered optimal by the central planner. That is, Period 2 is characterised by too much risk and too little buffer stock of liquidity. The latter phenomenon is partially due to the fact that the individual average valuations ( $\bar{\mu}$ ) are substantially reduced in this period, causing a significant reduction in buffer stock holdings in the decentralised equilibrium.

In the last sub-period, the (average) network multiplier in the market equilibrium is smaller than 1, hence overall the system dampens the volatility of shocks. From the central planner’s perspective, not enough volatility is generated (by about 31%) while at the same time the buffer stocks of network liquidity are too high. That is, in this period the velocity of inside money is too low and banks hold static reserves that are too large compared to the social optimum.

**Table 2: Central Planner vs. Market Equilibria**

|  | <i>Period 1</i> | <i>Period 2</i> | <i>Period 3</i> |
|--|-----------------|-----------------|-----------------|
| $\Delta\%$ Volatility of Total Liquidity | −90.8%          | −64.8%          | 30.7%           |
| $\Delta$ Network Liquidity               | −3.47           | 15.5            | −27.5           |

The three sub-periods are indicated by  $j = 1, 2, 3$ , and  $\bar{\mathbf{G}}_j$  is the average  $\mathbf{G}_t$  in sub-period  $j$ . The table reports: first row,  $100 \times \left[ \left( \frac{\text{Var}(Z^p(\hat{\phi}_j, \bar{\mathbf{G}}_j))}{\text{Var}(Z^*(\hat{\phi}_j, \psi=1, \bar{\mathbf{G}}_j))} \right)^{\frac{1}{2}} - 1 \right]$ ; second row,  $\mathbf{1}' \left[ \mathbf{M}^p(\hat{\phi}_j, \psi=1, \bar{\mathbf{G}}_j) - \mathbf{M}(\hat{\phi}_j, \bar{\mathbf{G}}_j) \right] \hat{\mu}_j$  (unit: £10bn).

## VI.2 Time Varying Network Effects

The results presented so far indicate a substantial change over time in the role played by the network interactions in determining aggregate liquidity level and risk. In this section, we analyse the drivers of this time variation.

### VI.2.1 The Drivers of the Time Variation in the Network Amplification

The network impulse response functions depicted in Figure (6)-(8) show substantial time variation in the amplification of shocks between sub-periods. This could be caused by either the time variation in the network topology  $\mathbf{G}$  or in the network multiplier  $\phi$ .

To examine these drivers, we compute the changes in the network impulse response functions between the three subperiods. In particular, Panel A of Figure 9 reports the total change in NIRF between Periods 1 and 2 ( $NIRF_i(\hat{\phi}_2, 1, \bar{\mathbf{G}}_2) - NIRF_i(\hat{\phi}_1, 1, \bar{\mathbf{G}}_1)$ , dashed line with circles), the change due to the variation of  $\mathbf{G}$  ( $NIRF_i(\hat{\phi}_1, 1, \bar{\mathbf{G}}_2) - NIRF_i(\hat{\phi}_1, 1, \bar{\mathbf{G}}_1)$ , dotted line with triangles), and the change due to the variation of  $\phi$  ( $NIRF_i(\hat{\phi}_2, 1, \bar{\mathbf{G}}_1) - NIRF_i(\hat{\phi}_1, 1, \bar{\mathbf{G}}_1)$ , dash-dotted line with +).

A striking feature of the graph is that most of the total change comes from the reduction in the network multiplier  $\phi$  for all banks. In fact, ceteris paribus, the outdegree centrality (hence the NIRF) of Bank 5 would have increased due to its increased borrowing and lending activity (captured by  $\bar{\mathbf{G}}_2$ ). However, this effect is dwarfed by the reduction of its NIRF caused by the change in  $\phi$ .

Panel B reports the same decomposition of the change in NIRFs between Periods 2 and 3. Once again the changes are mostly driven by the change in the network multiplier rather than the change in network topology.

Overall, Figure 9 shows that the time variation of the network multiplier has the first order effect on the network amplification mechanism.

### VI.2.2 Time Varying Network Multiplier

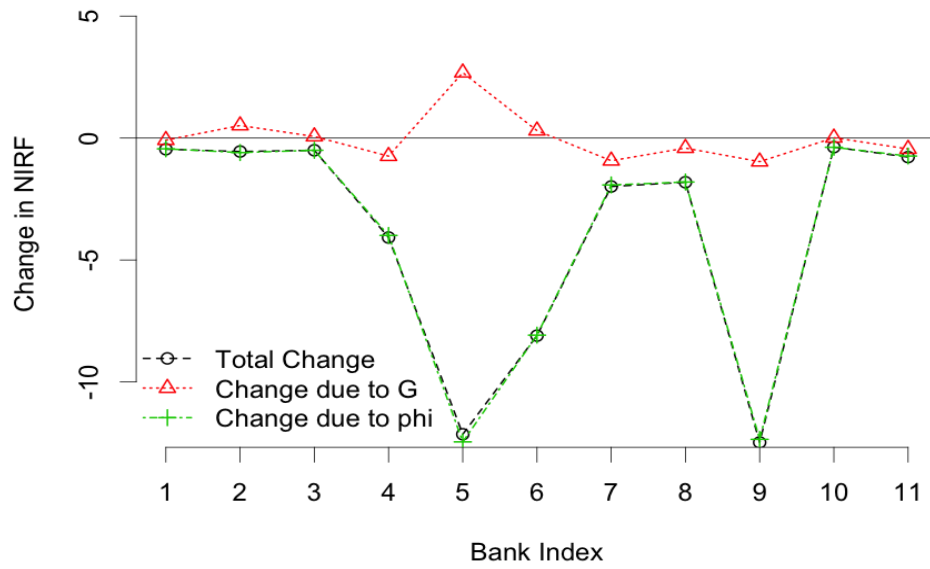
The results in the previous sections indicate the importance of the time variation of  $\phi$ . Therefore, to capture this time variation, we now estimate the structural model in equations (21) and (22) using a 6-month rolling window.<sup>31</sup> These rolling estimates of the network coefficient  $\phi$  are reported (blue line), together with 95% confidence bands (red lines), in Figure 10.

The figure also reports the rolling point estimates of the coefficient  $\phi$  implied by the spatial Durbin model (green line) in equation (24) since, if our theory-driven spatial error

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<sup>31</sup>Recall that when  $\mathbf{G}_t$  is a right stochastic matrix, separate identifications of the bank ( $\alpha^{bank}$ ) and network ( $\bar{\mu}$ ) fixed effects require that there is a subset of banks that does not borrow at least one point in time in each subsample. This condition is not satisfied in all the rolling sub-samples. But since the separate identification of these fixed effect does not affect the identification of  $\phi$ , we normalise the unidentified fixed effects. Moreover, given the very short length of the rolling window, we drop time fixed effects from the specification. Estimates with the full sets of fixed effects show a very similar behaviour, but with somewhat larger confidence intervals, hence making it easier not to reject the SEM specification. As a consequence, we focus on the more parsimonious specification.

**Panel A: NIRF change decomposition between Period 1 and 2**



**Panel B: NIRF change decomposition between Period 2 and 3**

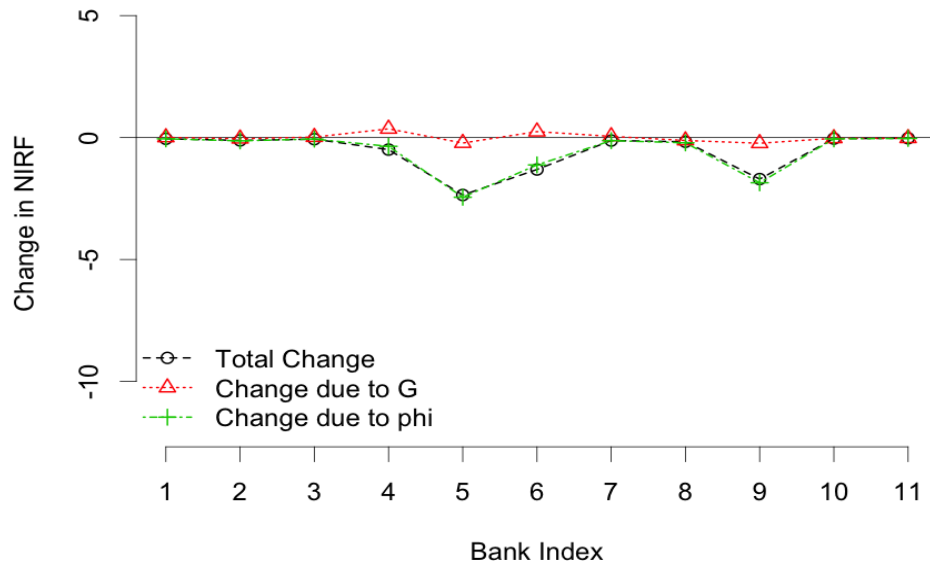


Figure 9: Decomposition of total change in the NIRFs between periods.



specification of the interbank network is correct, the two estimates should not be statistically different.

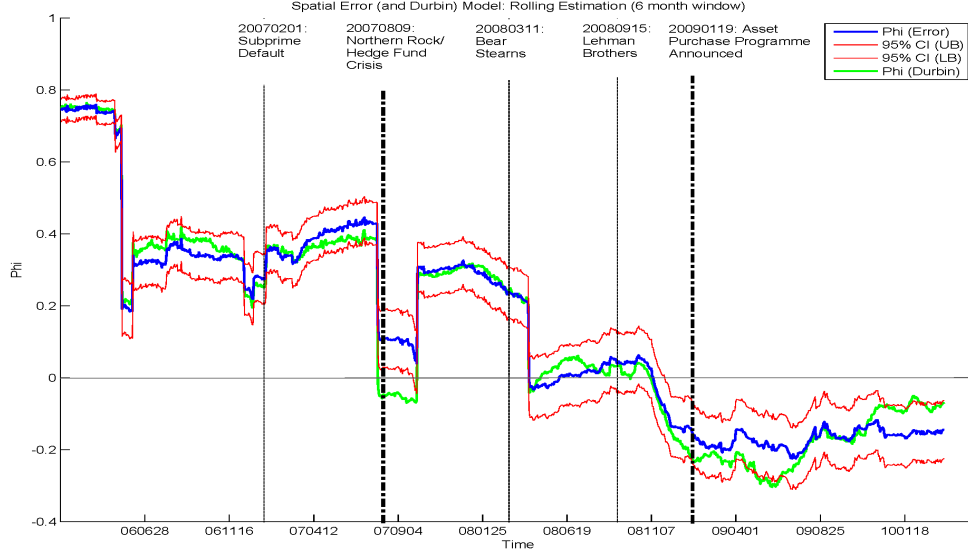


Figure 10: Spatial Error (blue line) and Durbin (green line) rolling estimates of  $\phi$ .

At the beginning of the sample, the figure shows an extremely large network coefficient,  $\phi$ , implying a substantial network amplification of shocks to banks in the system. The estimated coefficient has its first sharp reduction around the 18th May 2006 when the Bank of England introduced the reserve averaging system described in Section A.1. The network multiplier is relatively stable after May 2006, except for a temporary decrease during the 2007 subprime default, until the Northern Rock bank run when the network multiplier is drastically reduced for several months. After this reduction, the coefficient goes back to roughly the previous period average but shows a trend decline that culminates in a sharp drop following the Bear Stearns collapse. From this period onward, and until long after the Lehman Brothers bankruptcy, the coefficient is statistically indistinguishable from zero, implying a zero excess network multiplier of bank specific shocks. That is, the estimation suggests that in this period there was basically no added risk coming from the network structure of the interbank market, and that individual bank shocks would not be amplified by some sort of domino effect in the UK interbank market. This figure suggests that the banks' reaction to the financial market turmoil was to reduce the amplification of risk generated through the interbank network. This reduction could come from any of these three sources: *a*) a reduction in the availability of collateralization, i.e.  $\delta$ , *b*) an increase in risk aversion,  $\gamma$ , and *c*) an increased availability of accessible liquidity due to  $\psi$ .

Interestingly, the coefficient  $\hat{\phi}$  becomes negative, and statistically significant, right before the announcement of the Asset Purchase Programme, and remains stably so throughout

the Quantitative Easing period. This indicates that during the liquidity inflow (and also in expectation of it) coming from the Bank of England’s QE policy, banks started behaving as strategic substitutes in their liquidity holding decision (as implied by Bhattacharya and Gale (1987)). Note that this is a period in which the aggregate supply of central bank reserves was almost completely price inelastic since QE set a target level for asset purchases (and subsequent reserve creation) and let market forces determine their price. This overall change of the BoE supply of reserves is unlikely to be the driver of our estimates of the network multiplier coefficient during this period since: *a)* the change in  $\phi$  actually occurred before the announcement of QE; *b)* we estimate the identified optimal response of the banks to market conditions (effectively, the banks’ equilibrium demand function), and we control for variation in aggregate price and quantities of liquidity, as well as bank deposits held by the private sector.

Lastly, this figure outlines that the point estimates of  $\hat{\phi}$  coming from our theory-driven spatial error specification and the ones coming from the more general spatial Durbin model are always very similar, both numerically and in terms of their overall evolution during the sample. Moreover, testing formally for a discrepancy between the two types of estimates, we find that they are statistically different at the 5% confidence level less than 95% of the time, providing formal support for our formulation of the network model.

## VII Conclusions

In this paper, we develop and estimate a network model of interbank liquidity that is flexible enough to incorporate both strategic complementarity and substitution as potential network equilibria. Based on network topology and the estimated network effects, we construct measures of systemic risks and identify the network players that are most important in contributing to the aggregate liquidity and its risk in the banking system.

We find that the network effect varies significantly through the sample period, January 2006 to September 2010. Prior to the Northern Rock/Hedge Fund crisis, liquidity provision in the network was driven by the strategic complementarity of the holding decisions. That is, liquidity shocks were amplified by the network and each bank had a large exposure to network shocks. The high network amplification generated a higher velocity of inside money in the system. In contrast, during the crisis, the network itself also became less cohesive, and the network amplification was greatly reduced. Finally, during the Quantitative Easing period, in response to the large injection of liquidity in the system, the network became characterised by strategic substitution: that is, individual players were free riding on the aggregate increase in liquidity.

To the best of our knowledge, we are the first to estimate substantial time variation in the nature of the equilibrium in a financial network. Moreover, we show that, for risk generation, the change in the type of equilibrium is the dominant force (rather than the change in the network topology itself). This could rationalise the empirical puzzle of network changes having little impact on aggregate quantities in calibration/simulation exercises on interbank networks (Elsinger, Lehar, and Summer (2006)).

Moreover, we also solve for the benevolent central planner equilibrium. This allows us to estimate the gap between central planner and decentralised optima for both liquidity level and risk. In particular, we find that during both the pre-crises and crises periods the system was characterised by an excessive amount of risk and (during the crises) too little liquidity relative to the social optimum.

Last, but not least, we estimate the individual bank contributions to aggregate liquidity risk and document that most of the systemic risk is generated by a small subset of key players.

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# A Appendix

## A.1 Reserves Schemes, Payment Systems, and Interbank Overnight Borrowing

Banks in the UK choose the amount of central bank reserves that they wish to hold to support a range of short term liquidity needs. On a daily basis, reserve balances are used to fund intraday liquidity needs in the large value payment and settlement systems, and to protect against intraday liquidity shocks. Additionally, as central bank reserves are the most liquid asset, they are available to protect banks against a range of unexpected outflows of funds. They are also the ultimate settlement asset for interbank payments. Whenever payments are made between the accounts of customers at different commercial banks, they are ultimately settled by transferring central bank money (reserves) between the reserves accounts of those banks. Therefore, a bank's most liquid asset is its holdings of central bank reserves. Since 2006, the banks in our sample choose their own level of sterling reserve holdings and reserve holdings are not mandatory. However, their decisions to hold certain stocks of central bank reserves do not happen independently of the policy framework in which they operate.

### A.1.1 Monetary Policy Framework

Since the 1998 Banking Act, the Bank of England has had independent responsibility for setting interest rates to ensure that inflation, as measured by the Consumer Price Index (CPI), meets the inflation target of 2%. Each month the Monetary Policy Committee (MPC) meets to decide the appropriate level of the Bank rate (the policy interest rate) to meet the inflation target in the medium term. The Bank of England's main mechanism for influencing the inflation rate in the economy is the Sterling Monetary Framework. This framework uses the Bank of England's balance sheet to influence the level of short-term interest rates, and, through this, inflation. When banks decide upon the appropriate level of central bank reserves to hold, they do so within the constraints set by this framework. During our sample period (January 2006 to September 2010) the Bank of England had three distinct monetary frameworks: prior to 18 May 2006, the Bank of England operated an unremunerated reserve scheme; this was then replaced by a reserves average scheme; since March 2009 and the initiation of Quantitative Easing, the reserves average scheme has been suspended. *Pre-2006 Money Market Reform:* The Bank of England's Sterling Monetary Framework prior to the 2006 reforms was based upon a system of voluntary unremunerated reserves. In this system there were no reserve requirements and no reserve averaging over a maintenance period. The only binding requirement was that banks were obliged to maintain a minimum zero balance at the end of each day. In practice, due to uncertainties about managing end

of day cash positions, banks opted for small nonzero reserve balances. *Reserve Averaging:* In May 2006, the Bank of England undertook a major reform of the Sterling Monetary Framework. The new scheme was voluntary remunerated reserves with a period-average maintenance requirement. Each maintenance period – the period between each meeting of the Monetary Policy Committee – banks that participated in the reserve framework were required to decide upon a reserves target. This voluntary choice of reserve balances is a feature unique to the UK system. Over the course of each maintenance period, the banks would manage their balance sheets so that, on average, their reserve balances hit the target. Where banks were unable to meet the target, standing borrowing and deposit facilities were available. If a banks' balances were more than 1% above the target, the Bank of England would not remunerate the excess reserves; if they were more than 1% below the target, the Bank of England would impose a penalty, reducing the interest paid on the other reserves. At various points during the crisis, this  $\pm 1\%$  range was increased to give banks more flexibility to manage their liquidity. The amount paid on these reserves was the Bank rate, the interest rate determined by the MPC's monthly meeting. As each bank was free to choose its own reserves target, the level of reserves in the system was almost entirely demand driven.<sup>32</sup> In both schemes before Quantitative Easing (QE), the Bank of England would use short term Open Market Operations (OMOs), repo and reverse repo transactions backed by high quality liquid assets, to ensure that there were sufficient central bank reserves in the system so that each bank could achieve positive end of day balances or meet their reserves target. The Bank of England acts as the marginal supplier of funds to the banking system. Banks then use the private interbank money markets to ensure that the reserves are correctly distributed so that all banks meet their targets. *Post Quantitative Easing:* Quantitative Easing started in March 2009 when the MPC decided that in order to meet the inflation target in the medium term, it would need to supplement the use of interest rates to influence the price of money (which had hit the practical lower bound of 0.5%) with the purchase of assets using central bank reserves. This consisted of the BoE's boosting the money supply by creating central bank reserves and using them to purchase assets, predominantly UK gilts. As the quantity of reserves shifted from being demand driven to being influenced by QE, the BoE suspended the average reserve targeting regime, and now remunerates all reserves at the Bank rate.

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<sup>32</sup>There was a ceiling, expressed as the higher of a percentage of eligible liabilities and a fixed value, for the target level of reserves that any bank could choose. However, in practice banks typically chose lower targets, so this was not a binding constraint. The ceiling was raised in May 2008.

### A.1.2 Payment and Settlement Systems

Banks use central bank reserves to, inter alia, meet their demand for intraday liquidity in the payment and settlement systems. Reserves act as a buffer to cover regular timing mismatches between incoming and outgoing payments, and to cover unexpected intraday liquidity needs, for example, due to exceptionally large payments, operational difficulties, or stresses that impact upon a counterparty's ability, or willingness, to send payments. There are two major interbank payment systems in the UK: CHAPS and CREST. These two systems play a vital role in the UK financial system. On average, in 2011, £700 billion of transactions was settled every day within the two systems. This equates to the UK 2011 nominal GDP being settled every two days. CHAPS is the UK's large-value payment system. It is used for real time settlement of payments between its member banks.<sup>33</sup> These banks settle payments on behalf of hundreds of other banks through correspondent banking relationships. Typical payments are business-to-business payments, home purchases, and interbank transfers. Payments relating to unsecured interbank money markets are settled in CHAPS. CHAPS opens for settlement at 8 am and closes at 4:20 pm. Payments made on behalf of customers cannot be made after 4 pm. The system has throughput guidelines which require members to submit 50% of their payments by noon and 75% by 14:30. This helps ensure that payments are settled throughout the day and do not bunch towards the end of the day. In 2011 CHAPS settled an average of 135,550 payments each day valuing 254bn. CHAPS is a real-time gross settlement (RTGS) system. This means that payments are settled finally and irrevocably in real time. To fund these payments, banks have to access liquidity intraday. If a bank has, at any point during the day, cumulatively sent more payments than it has received, then it needs liquidity to cover this difference. This comes either from central bank reserves or intraday borrowing from the central bank. Furthermore, when a bank sends funds to another bank in the system, it exposes itself to liquidity risk. That is, the risk that the bank will not get those funds back during the day, and so will have to use other funds to fulfill its payment obligations. Therefore, it is important to choose an appropriate level of liquidity buffer to manage these intraday liquidity risks. Besides maintaining a liquidity buffer, banks can manage their intraday liquidity exposure to settlement banks by borrowing from and lending to each other in the unsecured overnight markets. The shortest term for these money markets is overnight. According to Bank of England estimates, payments relating to overnight unsecured money market activity (advances and repayments) account for about 20% of CHAPS values (Wetherilt, Zimmerman, and Soramaki (2010)). CREST, on the other hand, is a securities settlement system. Its Delivery-vs-Payment (DVP) mechanism

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<sup>33</sup>This includes the Bank of England and CLS. Some bank groups also have multiple memberships due to mergers.



ensures simultaneous transfer of funds and securities. Liquidity needs in CREST are largely met via Self Collateralising Repos (SCRs), in which the purchase of central bank eligible collateral automatically generates collateralised liquidity from the Bank of England, requiring few intraday resources from the purchasing bank.

### A.1.3 The Sterling Unsecured Overnight Money Market

Money markets are the markets where banks and other financial institutions borrow and lend assets, typically with maturities of less than one year. At the shortest maturity, overnight, banks borrow and lend interest-bearing central bank reserves. Monetary policy aims at influencing the rate at which these markets transact, so as to control inflation in the wider economy. There is very little information available about the size and the structure of the sterling money markets. The Bank of England estimates suggest that the overnight unsecured market is approximately £20–30 billion per day. Wetherilt, Zimmerman, and Soramaki (2010) describe the network of the sterling unsecured overnight money market. They find that the network has a small core of highly connected participants, surrounded by a wider periphery of banks loosely connected with each other, but with connections to the core. It is believed that prior to the recent financial crisis, the unsecured market was much larger than the secured one. But counterparty credit risk concerns, combined with new FSA liquidity regulations, which encourage banks to borrow secured and to increase the maturity of their funding, have increased the importance of the secured markets (Westwood 2011).

## A.2 Details of the Empirical Methodology

### A.2.1 Quasi-Maximum Likelihood Formulation and Identification Issues

Writing the variables and coefficients of the spatial error model in equations (21) and (22) in matrix form as<sup>34</sup>

$$B := [\alpha_1^{time}, \dots, \alpha_t^{time}, \dots, \alpha_T^{time}, \alpha_1^{bank}, \dots, \alpha_i^{bank}, \dots, \alpha_N^{bank}, \\ \beta_1^{bank}, \dots, \beta_m^{bank}, \dots, \beta_M^{bank}, \beta_1^{time}, \dots, \beta_p^{time}, \dots, \beta_P^{time}]',$$

$$L := [l_{1,1}, \dots, l_{N,1}, \dots, l_{i,t}, \dots, l_{1,T}, \dots, l_{N,T}], \quad \mathbf{z} := [z_{1,1}, \dots, z_{N,1}, \dots, z_{i,t}, \dots, z_{1,T}, \dots, z_{N,T}]'$$

$$\nu := [\nu_{1,1}, \dots, \nu_{N,1}, \dots, \nu_{i,t}, \dots, \nu_{1,T}, \dots, \nu_{N,T}]', \quad \underline{\mu} := \mathbf{1}_T \otimes [\bar{\mu}_1, \dots, \bar{\mu}_N]',$$

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<sup>34</sup>This is similar to the spatial formulation in Lee and Yu (2010).

$$\mathbf{G} := \text{diag}(\mathbf{G}_t)_{t=1}^T = \begin{bmatrix} \mathbf{G}_1 & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{G}_2 & \dots & \dots \\ \dots & \dots & \dots & \mathbf{0} \\ \mathbf{0} & \dots & \mathbf{0} & \mathbf{G}_T \end{bmatrix}, \quad \mathbf{X} := [\mathbf{D}, \mathbf{F}, \mathbf{X}^{bank}, \mathbf{X}^{time}],$$

where  $\mathbf{D} := I_T \otimes \mathbf{1}_N$ ,  $\mathbf{F} := \mathbf{1}_T \otimes I_N$ , and

$$\mathbf{X}^{time} = \begin{bmatrix} x_1^1 & \dots & x_1^p & \dots & x_1^P \\ \dots & \dots & \dots & \dots & \dots \\ x_t^1 & \dots & x_t^p & \dots & x_t^P \\ \dots & \dots & \dots & \dots & \dots \\ x_T^1 & \dots & x_T^p & \dots & x_T^P \end{bmatrix} \otimes \mathbf{1}_N, \quad \mathbf{X}^{bank} = \begin{bmatrix} x_{1,1}^1 & \dots & x_{1,1}^m & \dots & x_{1,1}^M \\ \dots & \dots & \dots & \dots & \dots \\ x_{N,1}^1 & \dots & x_{N,1}^m & \dots & x_{N,1}^M \\ \dots & \dots & \dots & \dots & \dots \\ x_{N,T}^1 & \dots & x_{N,T}^m & \dots & x_{N,T}^M \end{bmatrix},$$

we can then rewrite the empirical model as

$$L = \mathbf{X}B + \mathbf{z}, \quad \mathbf{z} = \underline{\mu} + \phi \mathbf{G}\mathbf{z} + \nu, \quad \nu_{i,t} \sim iid(0, \sigma_i^2).$$

This, in turn, implies that

$$\nu(B, \underline{\mu}, \phi) = (I_{N \times T} - \phi \mathbf{G})(L - \mathbf{X}B). \quad (28)$$

Finally, using the Gaussian distribution to model the exogenous error terms  $\nu$  yields the log likelihood

$$\ln \mathcal{L}(B, \phi, \underline{\mu}, \{\sigma_i^2\}_{i=1}^N) \equiv -\frac{TN}{2} \ln(2\pi) - \frac{T}{2} \sum_{i=1}^N \ln \sigma_i^2 - \sum_{i=1}^N \frac{1}{2\sigma_i^2} \sum_{t=1}^T \nu_{i,t}(B, \underline{\mu}, \phi)^2, \quad (29)$$

and the above can be estimated using standard optimization methods.

In the above formulation, the identification of  $\phi$  is ensured by the usual conditions on  $\mathbf{G}$  (see, e.g. Bramoullé, Djebbari, and Fortin (2009)). Instead, the separate identification of the bank fixed effects,  $\alpha^{bank} := [\alpha_1^{bank}, \dots, \alpha_N^{bank}]'$ , and the network-bank fixed effects,  $\bar{\mu} := [\bar{\mu}_1, \dots, \bar{\mu}_N]'$ , deserve some further remarks. Isolating the role of these fixed effects, equation (28) can be rewritten as

$$\begin{aligned} \nu(B, \underline{\mu}, \phi) &= (I_{N \times T} - \phi \mathbf{G}) \left( L - \tilde{\mathbf{X}}\tilde{B} - \mathbf{F}\alpha^{bank} \right) - \underline{\mu} \\ &= (I_{N \times T} - \phi \mathbf{G}) \left( L - \tilde{\mathbf{X}}\tilde{B} \right) - \mathbf{1}_T \otimes (\bar{\mu} + \alpha^{bank}) + \phi \mathbf{G}\mathbf{F}\alpha^{bank} \end{aligned}$$

where  $\tilde{\mathbf{X}} := [\mathbf{D}, \mathbf{X}^{bank}, \mathbf{X}^{time}]$  and  $\tilde{B}$  is simply the vector  $B$  without the  $\alpha^{bank}$  elements. Several observations are in order. First, the above implies that if  $\phi = 0$ , then  $\bar{\mu}$  and  $\alpha^{bank}$

cannot be separately identified (nevertheless the parameters  $\tilde{B}$  are still identified). Second, if there is no time variation in the network structure, i.e. if  $\mathbf{G}_t = \bar{\mathbf{G}} \forall t$ ,  $\bar{\mu}$  and  $\alpha^{bank}$  cannot be separately identified even if  $\phi \neq 0$ . Third, if a bank never lends to any other bank in the sample, its fixed effects  $\bar{\mu}_i$  and  $\alpha_i^{bank}$  cannot be separately identified. Fourth, if  $\mathbf{G}_t$  is a right stochastic matrix, separate identification of  $\bar{\mu}$  and  $\alpha^{bank}$  can be achieved only up to a parameter normalization, since for any scalar  $\kappa$  and vector  $\bar{\kappa} := \mathbf{1}_N \otimes \kappa$ , we have

$$\nu(B, \underline{\mu}, \phi) = (I_{N \times T} - \phi \mathbf{G}) \left( L - \tilde{\mathbf{X}} \tilde{B} \right) - \mathbf{1}_T \otimes (\bar{\mu} + \alpha^{bank} + \phi \bar{\kappa}) + \phi \mathbf{G} \mathbf{F} (\alpha^{bank} + \bar{\kappa})$$

The above also makes clear that a handy normalisation is to set one of the network-bank fixed effect (say the  $i$ -th one) to zero since it would imply the restriction  $\{\alpha^{bank} + \phi \bar{\kappa}\}_i = \{\alpha^{bank} + \bar{\kappa}\}_i$  that, for any  $\phi \neq 0$  and 1, can only be satisfied with  $\kappa = 0$ . Under this normalisation, the remaining estimated bank-network fixed effects are then in deviation from the normalised one. Fifth, note that the lack of separate identification for  $\bar{\mu}$  and  $\alpha^{bank}$  is due to the fact that when  $\mathbf{G}_t$  is a right stochastic matrix, and if all banks borrow from at least one bank at each point in time (i.e.  $\mathbf{G}_t$  has no rows of zeros), then  $\mathbf{G}_t \mathbf{1}_N = \mathbf{1}_N$  and  $\mathbf{G} \mathbf{1}_{N \times T} = \mathbf{1}_{N \times T}$ . Fortunately, in our dataset, the condition  $\mathbf{G}_t \mathbf{1}_N = \mathbf{1}_N$  does not hold every day in the sample because there are periods in which certain banks do not borrow (in this case, the corresponding rows of  $\mathbf{G}_t$  contain all zeros and sum to zero, instead of one). In our sample, except for bank 7 and bank 11, all the other banks borrow every period from at least one of their counterparties. There are fourteenn days when bank 7 does not borrow at all, and 145 days in which bank 11 does not borrow at all. Moreover, the no borrowing days of bank 7 and bank 11 do not overlap, so we have a total of 159 days in which either the sum of the 7th row of  $\mathbf{G}_t$  or the sum of the 11th row of  $\mathbf{G}_t$  is equal to zero, not one (13.5% of the days).

### A.2.2 Confidence Bands for the Network Impulse Response Functions

The  $\phi$  estimator outlined in the previous section has an asymptotic Gaussian distribution with variance  $s_\phi^2$  (that can be readily estimated as standard from the Hessian and gradient of the log likelihood in equation (29), or via bootstrap). That is,  $\sqrt{T}(\hat{\phi} - \phi_0) \xrightarrow{d} N(0, s_\phi^2)$ , where  $\phi_0$  denotes the true value of  $\phi$ . Writing

$$a_1(\phi) := \frac{\partial \mathbf{1}' \{(\mathbf{I} - \phi \mathbf{G})^{-1}\}_{.i}}{\partial \phi}, \quad a_2(\phi) = \frac{\partial \mathbf{1}' \{(\mathbf{I} - \phi \mathbf{G})^{-1} \phi \mathbf{G}\}_{.i}}{\partial \phi}$$

we have from Lemma 2.5 of Hayashi (2000) that

$$\begin{aligned}\sqrt{T} \left[ NIRF_i(\hat{\phi}, 1) - NIRF_i(\phi_0, 1) \right] &\xrightarrow{d} N(0, a_1(\phi_0)^2 s_\phi^2), \\ \sqrt{T} \left[ NIRF_i^e(\hat{\phi}, 1) - NIRF_i^e(\phi_0, 1) \right] &\xrightarrow{d} N(0, a_2(\phi_0)^2 s_\phi^2).\end{aligned}$$

Therefore, since  $a_j(\hat{\phi}) \xrightarrow{p} a_j(\phi_0)$ ,  $j = 1, 2$ , by the continuous mapping theorem, and by Slutsky's theorem,  $a_j(\hat{\phi}) \hat{s}_\phi^2 \xrightarrow{p} a_j(\phi_0)^2 s_\phi^2$ , where  $\hat{s}_\phi^2$  is a consistent variance estimator, we can construct confidence bands for the network impulse response functions using the sample estimates of  $\phi$  and  $s_\phi^2$ .

### A.2.3 Details of the Construction of the Variables

#### Macro control variables

- $rK_{t-1}$ : lagged right kurtosis of the intraday time of aggregate payment outflow:

$$rK_t = \frac{\sum_{\tau > m_t} \left( \frac{\tau - m_t}{\sigma_t} \right)^4}{\sum_{\tau=1}^{88} \left( \frac{\tau - m_t}{\sigma_t} \right)^4}$$

where

$$m_t = \frac{1}{88} \sum_{\tau=1}^{88} \tau \left( \frac{P_{t,\tau}^{OUT}}{\sum_{\tau=1}^{88} P_{t,\tau}^{OUT}} \right), \quad \sigma_t^2 = \frac{1}{88-1} \sum_{\tau=1}^{88} (\tau - m_t)^2 \left( \frac{P_{t,\tau}^{OUT}}{\sum_{\tau=1}^{88} P_{t,\tau}^{OUT}} \right)$$

and  $P_{t,\tau}^{OUT}$  is the aggregate payment outflow at time interval  $\tau$ . Note that transactions are recorded for 88 10-minute time intervals within each day (from 5:00 to 19:30). The variable  $m_t$  is the average of payment time weighted by the payment outflow.

- $\ln VolPay_{t-1}$ : intraday volatility of aggregate liquidity available (lagged and in logarithms). Liquidity available is defined in the following section on the bank specific control variables.
- $TOR_{t-1}$ : lagged turnover rate in the payment system. To define the turnover rate, we need first to define the Cumulative Net (Debit) Position (CNP):

$$CNP(T, i, s) = \sum_{t=1}^T (P_{i,s,t}^{OUT} - P_{i,s,t}^{IN}),$$

where  $P_{i,s,t}^{OUT}$  is bank  $i$ 's total payment outflow at time  $t$  in day  $s$ .  $P_{i,s,t}^{IN}$  is the payment

inflow. The turnover rate (in day  $s$ ) is defined as

$$TOR_s = \frac{\sum_{i=1}^N \sum_{t=1}^{88} P_{i,s,t}^{OUT}}{\sum_{i=1}^N \max\{\max_T[CNP(T; i, s)], 0\}}$$

The numerator is the total payment made in the system at day  $s$ . The denominator sums the maximum cumulative net debt position of each bank at day  $s$ . Note that in the denominator, if the cumulative net position of a certain bank is always below zero (that is, this bank's cumulative inflow always exceeds its cumulative outflow), this bank actually absorbs liquidity from the system. If there are banks absorbing liquidity from the system, there must be banks injecting liquidity into the system. When we calculate the turnover rate (the ratio between the total amount circulating and the base), we should only consider one of the two. That's why we take the first (outside) maximum operator. The reason for the inside operator goes as follows: Any increase in the cumulative net debit position (wherever positive) incurs an injection of liquidity into the system, so the maximum of the cumulative net position is the total injection from the outside to the payment system. And, the sum over the different banks gives the total injection through all the membership banks. A higher turnover rate means a more frequent reuse of the money injected from outside into the payment system.

- *LIBOR*: lagged LIBOR rate.
- *Interbank Rate Premium*: lagged average interbank rate in the market minus lagged LIBOR.

### Bank-specific variables

- Liquidity Available ( $LA$ ) is the amount of liquidity to meet payment requirements and is measured as the sum of reserves ( $SDAB$ , Start of Day Account Balance) plus the value of intraday repos with the BoE ( $PC$ , Posting of Collateral). As time passes, the liquidity available is calculated by subtracting the money moved to CREST from the liquidity available in the previous time interval. In this way, we can trace for bank  $i$  the liquidity available at any time  $t$  in day  $s$ :

$$LA(t, i, s) = SDAB_{i,s} + PC_{i,s} - \sum_{\tau=1}^t CREST_{i,s,\tau}$$

- Liquidity holding at the beginning at the day ( $l$ ): the logarithm of the cash balance plus posting of collateral (the value of intraday repo) at the start of the day.

- *Interbank Rate*: lagged interbank rate.
- $\ln LevPay_{i,t-1}$ : total intraday payment level (Yesterday, in logarithms).
- $rK_{i,t-1}^{in}$ : lagged right kurtosis of incoming payment time.
- $rK_{i,t-1}^{out}$ : lagged right kurtosis of outgoing payment time.
- $\ln VolPay_{i,t-1}$ : intraday volatility of liquidity available (lagged, in logarithms).
- $\ln LU_{i,t-1}$ : liquidity used (lagged, in logarithms). Liquidity Used:

$$LU(i, s) = \max\{\max_T[CNP(T; i, s)], 0\}.$$

A positive cumulative net debit position means that in this time interval the bank is consuming its own liquidity. If a positive cumulative net position never happens for a bank, this bank only absorbs liquidity from the system. That is the reason for the first (outside) maximum operator. The second (inside) maximum operator helps us to trace the highest amount of liquidity a bank uses.

- $\frac{repo\ Liability}{Assets}$ : Repo liability to total asset ratio (monthly).
- *Total Assets (log)*: total asset (monthly, in logarithms).
- $\frac{\Delta Deposit}{Assets}$ : cumulative change in ratio of retail deposits to total assets  $\times 100$  (monthly).
- *Total Lending and Borrowing (log)*: total lending and borrowing in the interbank market (in logarithms).
- *CDS (log)*: CDS relative price (lagged, in logarithms).
- *Stock Return (Inc. Dividend)*: stock return including dividends (lagged).

### A.3 Additional Figures and Tables

Figure 11: Turnover rate in the payment system.

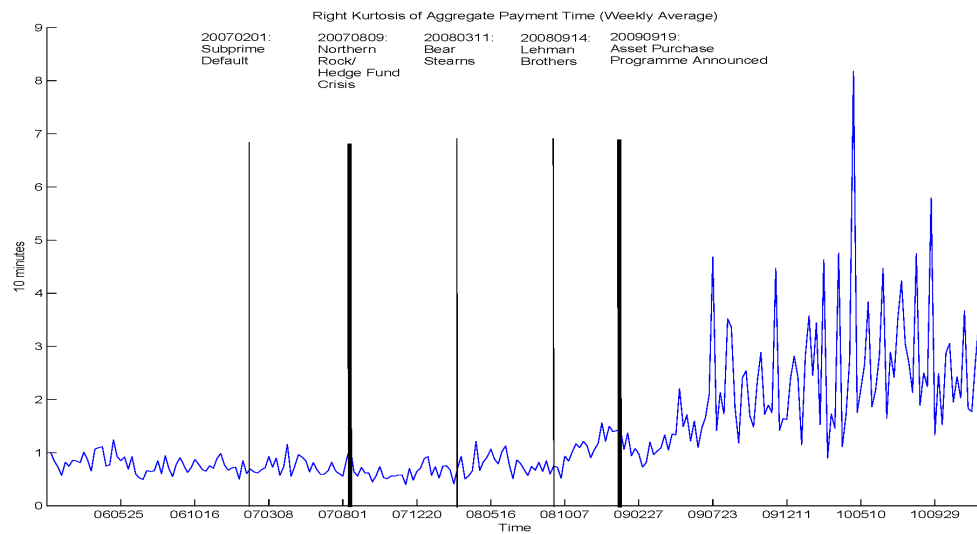
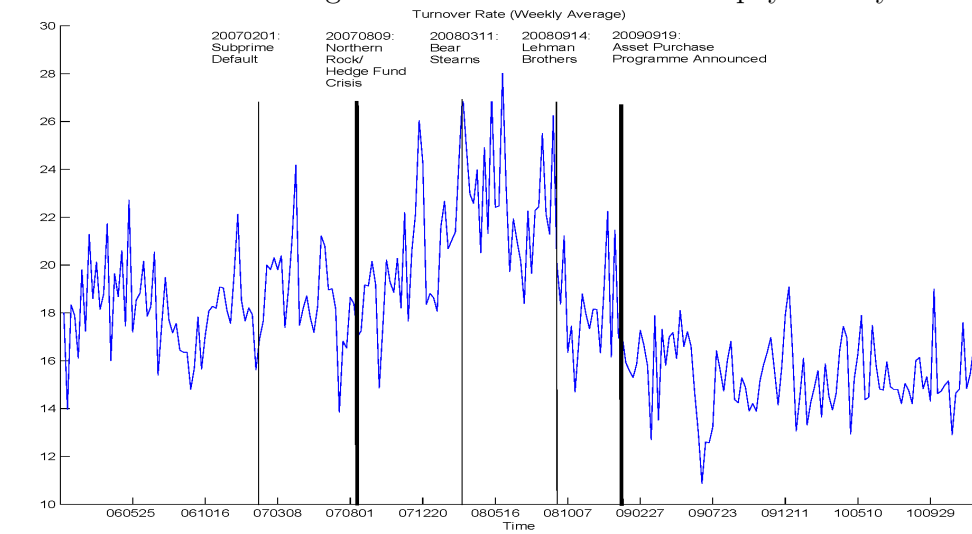


Figure 12: Weekly average of the right kurtosis of aggregate payment times.

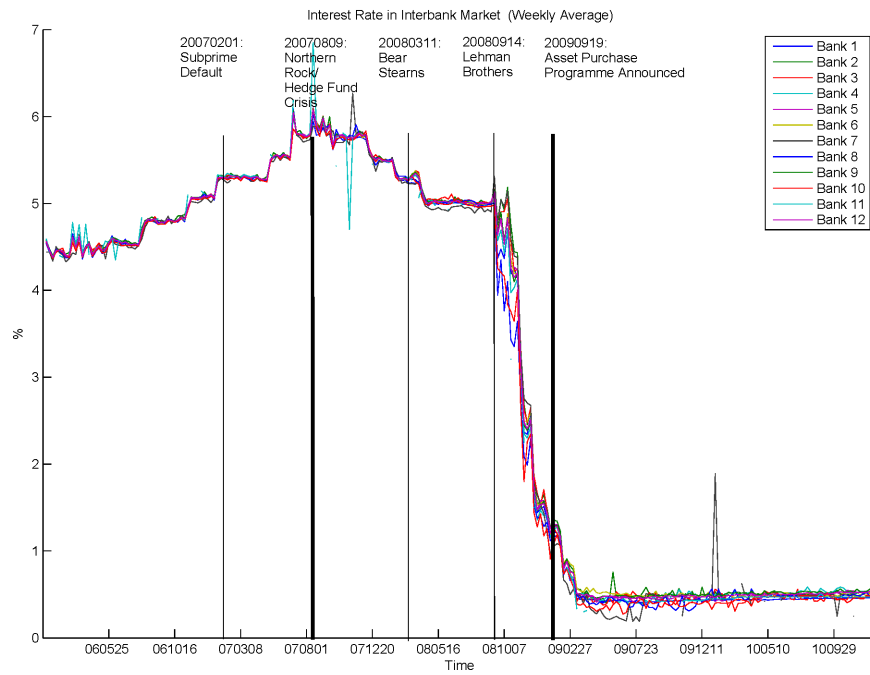


Figure 13: Interest rates in the interbank market.

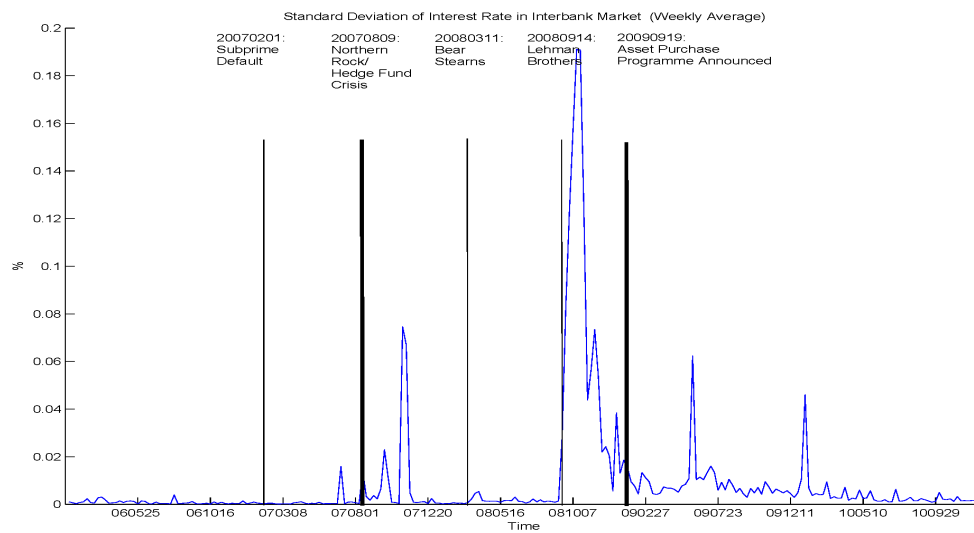


Figure 14: Cross-sectional dispersion of interbank rates.



**Table A1: Full Spatial Error Model Estimation**

|   | <i>Period 1</i>     | <i>Period 2</i>     | <i>Period 3</i>     |
|---|---------------------|---------------------|---------------------|
| $\hat{\phi}$  | 0.8137*<br>(21.47)  | 0.3031*<br>(1.90)   | -0.1794*<br>(-4.96) |
| $1/(1 - \hat{\phi})$                                | 5.3677*<br>(4.92)   | 1.4349*<br>(4.37)   | 0.8479*<br>(32.61)  |
| <i>Macro Control Variables</i>                      |                     |                     |                     |
| $rK_{t-1}$  | 0.1845<br>(1.30)    | 0.0084<br>(0.55)    | -0.0032*<br>(-3.88) |
| $\ln VolPay_{t-1}$                                  | -0.4451<br>(-1.00)  | 0.0308<br>(1.17)    | 0.0291<br>(1.72)    |
| $TOR_{t-1}$   | 0.0166<br>(1.80)    | 0.0007<br>(0.69)    | 0.0018<br>(1.75)    |
| LIBOR   | 0.2378<br>(0.27)    | 0.0928<br>(1.28)    | 0.5800*<br>(2.52)   |
| Interbank Rate Premium                              | 3.8845<br>(1.61)    | -0.0405<br>(-0.33)  | 0.6973*<br>(3.00)   |
| <i>Bank Characteristics/Micro Control Variables</i> |                     |                     |                     |
| Interbank Rate                                      | -0.2081<br>(-0.98)  | -0.0473<br>(-1.03)  | -0.0880<br>(-1.92)  |
| $\ln LevPay_{i,t-1}$                                | -0.0235<br>(-0.62)  | 0.0802*<br>(3.29)   | 0.0808*<br>(5.09)   |
| $rK_{i,t-1}^{in}$                                   | 0.0010<br>(0.14)    | -0.0086<br>(-0.63)  | 0.0045<br>(1.03)    |
| $rK_{i,t-1}^{out}$                                  | 0.0090<br>(0.92)    | 0.0320*<br>(3.62)   | -0.0061<br>(-1.32)  |
| $\ln VolPay_{i,t-1}$                                | 0.0129*<br>(4.59)   | 0.0039<br>(1.92)    | 0.0196*<br>(5.96)   |
| $\ln LU_{i,t-1}$                                    | -0.0038*<br>(-2.86) | -0.0039*<br>(-3.41) | -0.0027*<br>(-3.79) |
| $\frac{Repo\ Liability}{Assets}$                    | -5.5625*<br>(-3.61) | 0.0282<br>(0.43)    | -0.3057<br>(-1.45)  |
| Total Assets (log)                                  | 1.2590*<br>(5.39)   | 0.6328*<br>(10.31)  | 1.0170*<br>(18.92)  |
| $\frac{\Delta Deposit}{Assets}$                     | -0.0014<br>(-0.20)  | 0.0149*<br>(5.15)   | 0.0481*<br>(11.76)  |
| Total Lending and Borrowing (log)                   | -0.1882*<br>(-5.57) | 0.0612*<br>(2.95)   | -0.0025<br>(-1.27)  |
| CDS (log)   | 0.0051<br>(0.13)    | -0.1212*<br>(-6.61) | -0.0383*<br>(-4.00) |
| Stock Return (Inc. Dividend)                        | -0.5667<br>(-0.88)  | 0.1927<br>(1.49)    | 0.2574<br>(1.88)    |
| $R^2$   | 66.01%              | 92.09%              | 91.53%              |

Estimation results of equations (21) and (22). Periods 1, 2 and 3, correspond, respectively, to before the Northern Rock/Hedge Fund Crisis, after Hedge Fund Crisis but before the Asset Purchase Programme, and after the Asset Purchase Programme announcement. The  $t$ -statistics are reported in parentheses under the estimated coefficients, where \* denotes statistically significant estimates at a 10% or higher confidence level. Standard errors are computed via block bootstrap. For the average network multiplier,  $1/(1 - \hat{\phi})$ , the delta method is employed.