Organizational Equilibrium with Capital

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Question

- Time inconsistency is a pervasive issue
  - government policy, sovereign debt, consumption-saving problem, ...
- Benchmark outcome: Markov perfect equilibrium
- Can agents do better than Markov equilibrium?
  - Yes, with trigger strategies (Chari and Kehoe, 1990)
- We show agents do better than Markov equilibrium without trigger strategy

This paper

Propose an equilibrium concept for economies with state variables where

- Initial agents can make proposals for the future
- Future agents can follow the proposal, or
  1. copy the proposal
  2. wait for the next agent to make the same proposal
  3. make their own proposal
- Equilibrium: a proposal that all future agents are willing to follow

Organizational Equilibrium (OE) (Prescott and Rios-Rull, 2000) is a special case

This paper

- Propose organizational equilibrium for economies with state variables
- Apply organizational equilibrium to quasi-geometric discounting model
    - large welfare improvement compared with Markov equilibria
- Extend the organizational equilibrium to government policy problem

Equilibrium Properties in Time-Inconsistent Problems

- Common issue: thank you for the idea, I will do it myself
  - if a proposal favors the initial, the next wants to copy the idea and restart
  - well defined notion of "thank you for the idea": proposals independent of capital
  - history matters for action, but only pay-off relevant variable matters for payoff
  - history can be abolished, no trigger strategy
- Good will has to be built gradually
  - deviation from Markov is like making a sacrifice
  - proposal starts with low sacrifice, otherwise waiting for the next to propose
  - more sacrifice is made overtime, knowing the next will make more sacrifice
- The long-run outcome is close to the case with commitment

Plan

1. An example: a growth model with quasi-geometric discounting.
2. General Organizational equilibrium for "separable" economies
3. Approximating any economy with a "separable" economy
4. A government taxation problem

Part I: A Growth Model

The Environment

- Preferences: quasi-geometric discounting

$$U_t = u(c_t) + \delta \sum_{\tau=1}^{\infty} \beta^{\tau} u\left(c_{t+\tau}\right)$$

  - period utility function $u(c) = \log c$
  - $\delta = 1$ is the time-consistent case

- Technology

$$f(k_t) = k_t^{\alpha}, \qquad k_{t+1} = f(k_t) - c_t.$$

Benchmark I: Markov Perfect Equilibrium

- Consider the differentiable Markov equilibrium with GEE (Krusell and Smith, 2003)
- Recursive formulation: $V(k;g) = \max_{k'} u[f(k) - k'] + \delta\beta\Omega(k';g)$

  Continuation value: $\Omega(k;g) = u[f(k) - g(k)] + \beta\Omega[g(k);g]$
- The Generalized Euler Equation (GEE)

$$u'(f(k) - g(k)) = \beta u'\Big(f[g(k)] - g[g(k)]\Big)\Big[\delta f'[g(k)] + (1-\delta)\, g'[g(k)]\Big]$$

- The equilibrium features a constant saving rate

$$k' = \frac{\delta\alpha\beta}{1 - \alpha\beta + \delta\alpha\beta}\, k^{\alpha} = s^{M}\, k^{\alpha}$$

Benchmark II: Ramsey Allocation with Commitment

- First period: $\max_{k_1} u[f(k_0) - k_1] + \delta\beta\Omega(k_1)$

  Continuation value: $\Omega(k) = \max_{k'} u[f(k) - k'] + \beta\Omega(k')$

- The sequence of saving rates is given by

$$s_t = \begin{cases} s^M = \frac{\alpha\delta\beta}{1-\alpha\beta+\delta\alpha\beta}, & t = 0 \\ s^R = \alpha\beta, & t > 0 \end{cases}$$

- The Markov equilibrium saving is lower than the Ramsey

$$k^M = \left(\frac{\alpha\delta\beta}{1-\alpha\beta+\delta\alpha\beta}\right)^{\frac{1}{1-\alpha}} < k^R = (\alpha\beta)^{\frac{1}{1-\alpha}}$$

Optimal Constant Saving Rate with Quasi-Geometric Utility

- Value function with constant saving rate s

$$V(k;s) = u[(1-s)f(k)] + \delta\beta\,\Omega[sf(k);s]$$

Continuation value

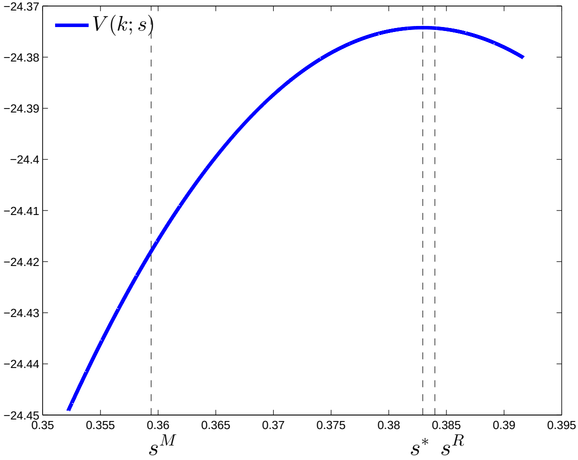
$$\Omega(k;s) = u[(1-s)f(k)] + \beta\,\Omega(k;s)$$

- By guess-and-verify, the value function is

$$V(k;s) = \frac{\alpha(1-\alpha\beta+\delta\alpha\beta)}{1-\alpha\beta}\log k + \left(1+\frac{\beta\delta}{1-\beta}\right)\log(1-s) + \frac{\delta\alpha\beta}{(1-\alpha\beta)(1-\beta)}\log(s)$$

- Which constant saving rate is the optimal one?

How does $V(k;s)$ look like?



$$s^M = \frac{\alpha\delta\beta}{1-\alpha\beta+\delta\alpha\beta} < s^* = \frac{\delta\alpha\beta}{(1-\beta+\delta\beta)(1-\alpha\beta)+\delta\alpha\beta} < s^R = \alpha\beta$$

Towards the Organizational Equilibrium: Can s^* be achieved?

- Imagine the proposal is to choose s^* all the time
  1. $\surd$ the next agent has no incentive to copy the proposal (it is a constant)
  2. $\times$ all agents have incentive to choose s^M, and wait the next to propose
  3. $\surd$ no other proposals yield higher utility for all
- Constant s^* proposal cannot be implemented, violating no waiting condition
- But, something else can be implemented, which converges to s^*
- For this, we need to proceed to define the organizational equilibrium

Proposals and Value Function

- A proposal is a sequence of saving rates $\{s_0, s_1, s_2, \ldots\}$
- Given an initial capital k_0, the proposal induces a sequence of capital

$$k_1 = s_0 k_0^\alpha$$
$$k_2 = s_1 k_1^\alpha = k_0^{\alpha^2} s_1 s_0^\alpha$$
$$\vdots$$
$$k_t = k_0^{\alpha^t} \Pi_{j=0}^{t-1} s_j^{\alpha^{t-j-1}}$$

- The lifetime utility for the agent who makes the proposal is

$$\begin{aligned}
& U(k_0, s_0, s_1, \ldots) \\
=& \log[(1-s_0)k_0^\alpha] + \delta \sum_{j=1}^{\infty} \beta^j \log[(1-s_j)k_j^\alpha] \\
=& \frac{\alpha(1-\alpha\beta+\delta\alpha\beta)}{1-\alpha\beta} \log k_0 + \log(1-s_0) + \delta \sum_{j=1}^{\infty} \beta^j \log[(1-s_j)\Pi_{\tau=0}^{j-1} s_\tau^{\alpha^{j-\tau}}] \\
=& \frac{\alpha(1-\alpha\beta+\delta\alpha\beta)}{1-\alpha\beta} \log k_0 + \Phi(s_0, s_1, \ldots)
\end{aligned}$$

Proposals and Value Function

- A proposal is a sequence of saving rates $\{s_0, s_1, s_2, \dots\}$
- Given an initial capital k_0, the proposal induces a sequence of capital

$$k_t = k_0^{\alpha^t} \Pi_{j=0}^{t-1} s_j^{\alpha^{t-j-1}}$$

- The lifetime utility for agent at period t is

$$U(k_t, s_t, s_{t+1}, \dots) = \frac{\alpha(1 - \alpha\beta + \delta\alpha\beta)}{1 - \alpha\beta} \log k_t + \Phi(s_t, s_{t+1}, \dots)$$

- Crucially, capital k_t and saving rates $\{s_t, s_{t+1}, s_{t+2}, \dots\}$ are separable

Organizational Equilibrium in Quasi-Geometric Discounting Growth Model

Definition

A sequence of saving rates $\{s_\tau\}_{\tau=0}^{\infty}$ is organizationally admissible if

- $\Phi(s_t, s_{t+1}, s_{t+2}, \ldots)$ is (weakly) increasing in t
- The first agent has no incentive to delay the proposal.

$$\Phi(s_0, s_1, s_2, \ldots) \geq \max_s \Phi(s, s_0, s_1, s_2, \ldots)$$

Within organizationally admissible sequences, any sequence that attains the maximum of $\Phi(s_0, s_1, s_2, \ldots)$ is an *organizational equilibrium*.

The admissible sequence of saving rates and the maximum of Φ make sure

1. no incentive to copy previous agent's proposal
2. no incentive to wait for the next agent to propose
3. no incentive to start a new proposal

Remarks on Organizational Equilibrium

- Separability between capital and saving rates allows a global maximum

- Payoff only depends on capital, not a trigger strategy

$$U(k_t, s_t, s_{t+1}, \dots) = \frac{\alpha(1-\alpha\beta+\delta\alpha\beta)}{1-\alpha\beta}\log k_t + \Phi(s_t, s_{t+1}, \dots)$$
$$= \frac{\alpha(1-\alpha\beta+\delta\alpha\beta)}{1-\alpha\beta}\log k_t + \Phi^*$$

- Organizational equilibrium is not a Markov equilibrium
  - agents need to know previous agents' action

Construct the Organizational Equilibrium

- An organizational equilibrium is a sequence of saving rates $\{s_0, s_1, \ldots\}$
- To prevent copying the idea, every generation obtain the same $\overline{\Phi}$

$$\Phi(s_t, s_{t+1}, \ldots) = \Phi(s_{t+1}, s_{t+2}, \ldots) = \overline{\Phi}$$

  which induces the following difference equation

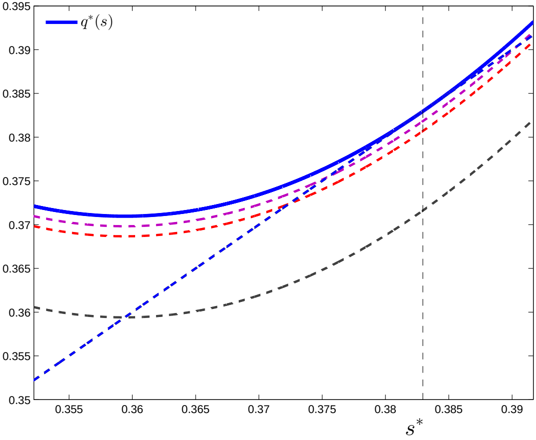
$$\beta(1-\delta)\log(1-s_{t+1}) = \frac{\delta\alpha\beta}{1-\alpha\beta}\log s_t + \log(1-s_t) - (1-\beta)\overline{\Phi}$$

- We call this difference equation as the proposal function

$$s_{t+1} = q(s_t)$$

- The maximal $\overline{\Phi}$ and an initial s_0 are needed to determine $\{s_\tau\}_{\tau=0}^{\infty}$

Determine Φ^*



- As $\overline{\Phi}$ increases, the proposal function $q(s)$ move upwards
- The highest $\overline{\Phi} = \Phi^*$ is achieved when $q(s)$ is tangent with 45 degree line at s^*

Determine the Initial Saving Rate s_0

- The first agent should have no incentive to delay the proposal

$$\Phi(s_0, s_1, s_2, \ldots) \geq \max_s \Phi(s, s_0, s_1, s_2, \ldots)$$

- Given the next agent will propose the organizational equilibrium

$$\max_s \Phi(s, s_0, s_1, s_2, \ldots) = \Phi(s^M, s_0, s_1, s_2, \ldots)$$

- s_0 has to be such that

$$\Phi^* = \Phi(s_0, s_1, s_2, \ldots) \geq \Phi(s^M, s_0, s_1, s_2, \ldots)$$
$$\longrightarrow s_0 \leq q^*(s^M)$$

- We select $s_0 = q^*(s^M)$, which yields the highest welfare during the transition

Organizational Equilibrium in Quasi-Geometric Discounting Growth Model

Proposition

The organizational equilibrium $\{s_\tau\}_{\tau=0}^{\infty}$ is given recursively by the proposal function q^*

$$s_t = q^*(s_{t-1}) = 1 - \exp\left\{ \frac{-(1-\beta)\Phi^* + \frac{\delta\alpha\beta}{1-\alpha\beta}\log s_{t-1} + \log(1-s_{t-1})}{\beta(1-\delta)} \right\}$$

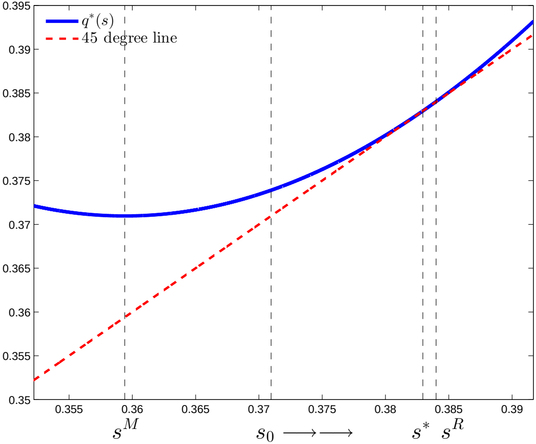
where the initial saving rate s_0, the steady state s^*, and Φ^* are given by

$$s_0 = q^*\left(s^M\right)$$

$$s^* = \frac{\delta\alpha\beta}{(1-\beta+\delta\beta)(1-\alpha\beta)+\delta\alpha\beta}$$

$$\Phi^* = \frac{1-\beta+\delta\beta}{1-\beta}\log(1-s^*) + \frac{\alpha\delta\beta}{(1-\beta)(1-\alpha\beta)}\log s^*$$

Transition Dynamics



- The equilibrium starts from s_0, and monotonically converges to s^*.

Remarks

1. To solve proposal function, no agent can treat herself specially, $\Phi_t = \Phi_{t+1}$

   Thank you for the idea, I will do it myself

2. To determine the initial saving rate, the agent starts from low saving rate

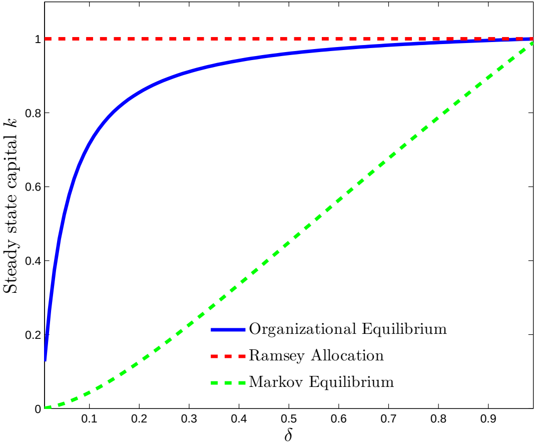
   Good will has to be built gradually

3. We will show how the outcome compared with the Markov and Ramsey

   We do much better than Markov equilibrium

Comparison: Steady State

- The organizational equilibrium is very like the Ramsey allocation



Parameter values: $\alpha = 0.4$, $\beta = 0.96$

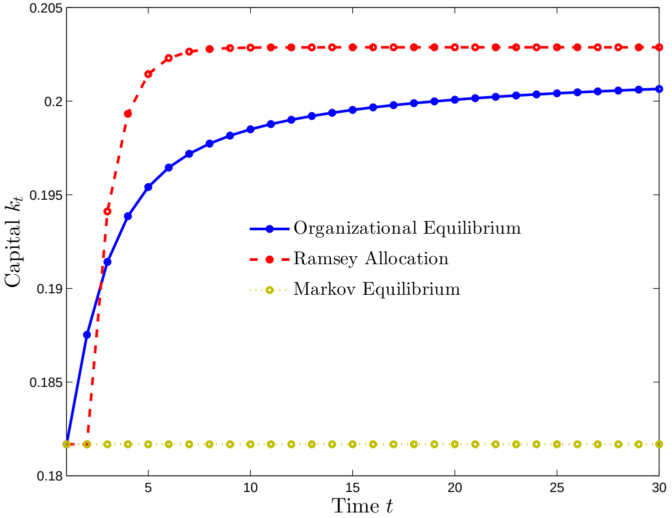
Comparison: Transition

- Recall that the lifetime utility is given by

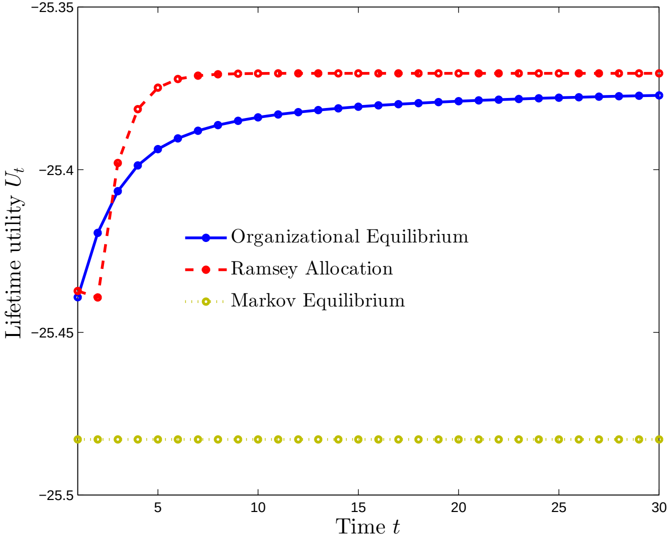
$$U(k_t, s_t, s_{t+1}, \ldots) = \frac{\alpha(1-\alpha\beta+\delta\alpha\beta)}{1-\alpha\beta} \log k_t + \Phi(s_t, s_{t+1}, \ldots)$$

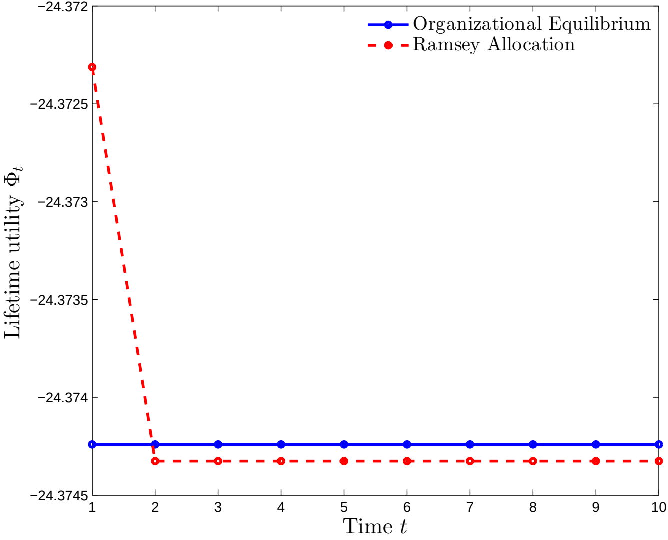
- For the initial agent, starting with the same capital
  - Ramsey allocation: highest $\Phi(s_0, s_1, s_2, \ldots)$
  - Markov equilibrium: lowest $\Phi(s_0, s_1, s_2, \ldots)$
  - Organizational equilibrium: close to but smaller than Ramsey allocation

- For subsequent agents,
  - Ramsey allocation: lower $\Phi(s_t, \ldots)$, highest capital
  - Markov equilibrium: lowest $\Phi(s_t, \ldots)$ and capital
  - Organizational equilibrium: highest $\Phi(s_t, \ldots)$, much higher capital than Markov



Parameter values: $\alpha = 0.4$, $\beta = 0.96$, and $\delta = 0.9$


−25.35
−25.4
−25.45
−25.5
5
10
15
20
25
30
Lifetime utility U_t
Time t
Organizational Equilibrium
Ramsey Allocation
Markov Equilibrium


Organizational Equilibrium
Ramsey Allocation
−24.372
−24.3725
−24.373
−24.3735
−24.374
−24.3745
Lifetime utility Φ_t
1
2
3
4
5
6
7
8
9
10
Time t

Next

- In this example, saving rate permits the lifetime utility to be separable between actions and state variables
  - any monotonic transformation of saving rate yields the same allocation

- We define organizational equilibrium for the class of economy that permits the separable property between action and states

- We then show how to approximate a non-separable economy with a separable economy locally

Part II: Organizational Equilibrium for Weakly Separable Economy

General Setup

- An infinite sequence of decision makers is called to act
  - state variable $k \in K$, action $x \in X(k)$
  - preference: $\Psi(k_t, x_t, x_{t+1}, x_{t+2}, \ldots)$

Assumption

There exists a set A and a function $\gamma : A \times K \to \mathcal{A}$ with the following properties

1. *The action can be recalled:*

$$X(k) = \{a : \exists a \in A \ \textit{such that} \ x = \gamma(a, k)\}$$

2. *Define preferences over rescaled actions as follows:*

$$U(k, a_t, a_{t+1}, a_{t+2}, \ldots) := \Psi(k, x_t, x_{t+1}, x_{t+2}, \ldots),$$

where for $t \geq 0$, x_t is computed recursively as

$$x_t(k) = \gamma(a_t, k_t(k)),$$
$$k_{t+1}(k) = F(k_t(k), x_t).$$

and U is weakly separable in k and in $\{a_\tau\}_{\tau=t}^{\infty}$, i.e., there exist v and V

$$U(k, a_t, a_{t+1}, a_{t+2}, \ldots) = v(k, \Phi(a_t, a_{t+1}, a_{t+2}, \ldots)).$$

Organizational Equilibrium

Definition

A sequence of actions $\{a_\tau\}_{\tau=0}^{\infty}$ is organizationally admissible if

1. $\Phi(a_t, a_{t+1}, a_{t+2}, \ldots)$ is (weakly) increasing in t
2. The first agent has no incentive to delay the proposal.

$$\Phi(a_0, a_1, a_2, \ldots) \geq \max_{a \in \mathcal{A}} \Phi(a, a_0, a_1, a_2, \ldots)$$

Within organizationally admissible sequences, any sequence that attains the maximum of $V(a_0, a_1, a_2, \ldots)$ is an *organizational equilibrium*.

Recursive Structure and Proposal Function

Assumption

There exist functions $w_0 : A \times \mathbb{R} \to \mathbb{R}$, $w : A \times \mathbb{R} \to \mathbb{R}$ *and* $W : A^\infty \to \mathbb{R}$, *s.t.*

$$V(a_0, a_1, a_2, \dots) = w_0\left(a_0, W(a_1, a_2, \dots)\right)$$
$$W(a_t, a_{t+1}, a_{t+2}, \dots) = w\left(a_t, W(a_{t+1}, a_{t+2}, \dots)\right), \quad t \geq 1$$

for all sequences $\{a_0\}_{t=0}^{\infty}$.

Proposition

Assume w *and* w_0 *are smooth functions, then an organizational equilibrium* $\{a_t\}_{t=0}^{\infty}$ *satisfies the following recursive equation*

$$a_{t+1} = q(a_t)$$

Moreover, if A *is bounded, then there exists a fixed point* a^*, *s.t.*

$$a^* = q(a^*)$$

Part III: Approximated Organizational Equilibrium for Non-separable Economy

First Order Approximation

- Original economy: $\Psi_t = u(k_t, x_t) + \delta \sum_{\tau=1}^{\infty} \beta^{\tau} u\left(k_{t+\tau}, x_{t+\tau}\right)$

  where $k_{t+1} = F(k_t, x_t)$

- Use the following separable economy to approximate the original economy

$$\widehat{\Psi}_t = \widehat{u}(k_t, x_t) + \delta \sum_{\tau=1}^{\infty} \beta^{\tau} \widehat{u}\left(k_{t+\tau}, x_{t+\tau}\right)$$

where
$$\widehat{u}(k, x) = h(k) + m(x)$$
$$h(k') = \alpha h(k) + g(x)$$

- Around any $(\overline{x}, \overline{y})$, this approximation matches the level and first derivative

Example

- Original economy
  - Preferences: $U_t = \log(c_t) + \delta \sum_{\tau=1}^{\infty} \beta^{\tau} u(c_{t+\tau})$
  - Technology: $k_{t+1} = k_t^{\alpha} + (1-d)k_t - c_t$

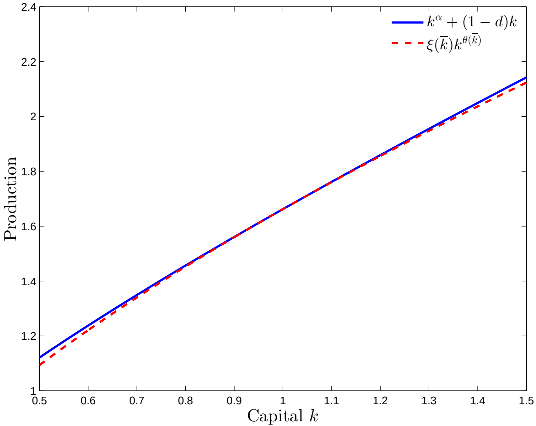
- Approximate the resource constraint around $\overline{k}$ by

$$k^{\alpha} + (1-d)k = \xi(\overline{k})k^{\theta(\overline{k})}$$

- The new "separable" economy is
  - Preferences: $U_t = \log(c_t) + \delta \sum_{\tau=1}^{\infty} \beta^{\tau} u(c_{t+\tau})$
  - Technology: $k_{t+1} = \xi(\overline{k})k^{\theta(\overline{k})} - c_t$

- Additional requirement: $\overline{k}$ is the steady state in organizational equilibrium

Approximated Production Function



- The approximation works well around the steady state

Part IV: Government Policy Problem

Government Policy Problem

- A strategic government v.s. a continuum of competitive agents
- Government cannot commit to a policy rule at time 0
- Apply organizational equilibrium to this environment
- Significant welfare improvement over Markov equilibrium, but gradually

An Optimal Taxation Problem: Environment

- Preference: $\sum_{t=0}^{\infty} \beta^t [\log c_t + \gamma \log g_t]$
- Technology: $f(k_t) = k_t^{\alpha}, \qquad k_{t+1} = f(k_t) - c_t - g_t.$
- Consumers' budget constraint: $c_t + k_{t+1} = (1 - \tau_t) r_t k_t + \pi_t$
- Prices: $r_t = f_k(k_t) \qquad \pi_t = f(k_t) - r_k k_t$
- Government budget constraint: $g_t = \tau_t r_t k_t$

Government Problem

- Government proposal: a sequence of capital tax rates $\{\tau_t\}_{t=0}^{\infty}$
- Given $\{\tau_t\}_{t=0}^{\infty}$, allocation in competitive equilibrium is unique

$$
\begin{aligned}
k_t^{\alpha} &= c_t + k_{t+1} + g_t \\
g_t &= \alpha \tau_t k_t^{\alpha} \\
\frac{1}{c_t} &= \beta \frac{1}{c_{t+1}} (1 - \tau_{t+1}) \alpha k_{t+1}^{\alpha - 1}
\end{aligned}
$$

- Government preference can be written as

$$
U(k, \{\tau_t\}_{t=0}^{\infty}) = \sum_{t=0}^{\infty} \beta^t [\log c_t + \gamma \log g_t] = \frac{\alpha(1+\gamma)}{1-\alpha\beta} \log k + \Phi(\{\tau_t\}_{t=0}^{\infty})
$$

Organizational Equilibrium in Government Taxation Problem

Definition

A sequence of tax rates $\{\tau_t\}_{t=0}^{\infty}$ is organizationally admissible if

- $\Phi(\tau_t, \tau_{t+1}, \tau_{t+2}, \ldots)$ is (weakly) increasing in t
- Government has no incentive to delay the proposal.

$$\Phi(\tau_0, \tau_1, \tau_2, \ldots) \geq \max_{\tau} \Phi(\tau, \tau_0, \tau_1, \tau_2, \ldots)$$

Within organizationally admissible sequences, any sequence that attains the maximum of $\Phi(\tau_0, \tau_1, \tau_2, \ldots)$ is an *organizational equilibrium*.

Optimal Constant Saving Rate with Quasi-Geometric Utility

- Value function with constant saving rate s

$$V(k; s) = u[(1-s)f(k)] + \delta\beta\,\Omega(k; s)$$

Continuation value

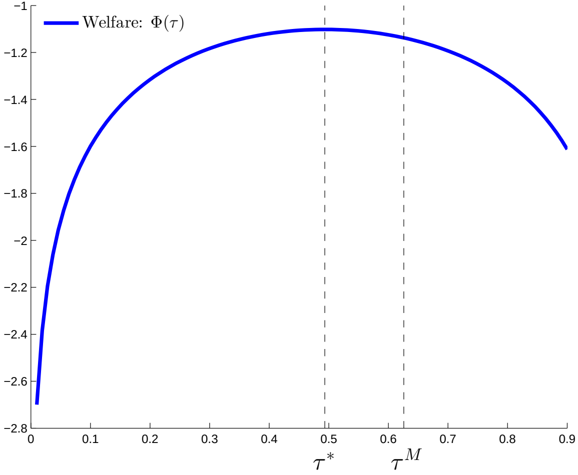
$$\Omega(k; s) = u[(1-s)f(k)] + \beta\,\Omega(k; s)$$

- By guess-and-verify, the value function is

$$V(k; s) = \frac{\alpha(1-\alpha\beta+\delta\alpha\beta)}{1-\alpha\beta}\log k + \left(1+\frac{\beta\delta}{1-\beta}\right)\log(1-s) + \frac{\delta\alpha\beta}{(1-\alpha\beta)(1-\beta)}\log(s)$$

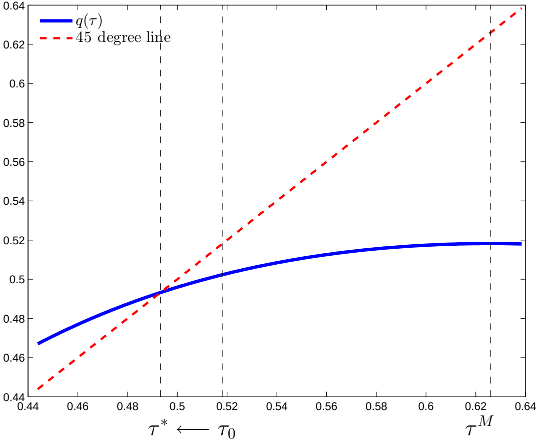
- Which constant saving rate is the optimal one?

Optimal Constant Tax Rate



- Taxation in Markov equilibrium is too high
- Constant τ^* cannot be implemented directly

Transition Dynamics in Organizational Equilibrium



- The equilibrium starts from τ_0, and monotonically converges to τ^*.

Conclusion

- Propose organizational equilibrium for economy with state variables
- Three properties
  - No one is special: thank you for the idea, I will do it myself
  - Good will has to be build gradually
  - Outcome is close to Ramsey allocation, much better than Markov equilibrium
- The equilibrium concept can also be applied to government policy problem