Capital Misallocation and Unemployment*

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Abstract

The recent recession was associated not only with a marked disruption in the credit market, but also a sharp deterioration in labor market conditions, as evidenced by high unemployment rates and an outward shift in the Beveridge curve. Motivated by such co-movements of the credit market and the labor market, I develop a tractable dynamic model with heterogeneous entrepreneurs, credit constraints, and labor-search frictions. In this framework, the misallocation of capital across firms has an adverse effect on the matching efficiency in the labor market. I then quantify the importance of capital misallocation for understanding the behavior of unemployment rate. I find that the credit crunch was the key driving force behind the outward shift in the Beveridge curve during and after the Great Recession. More broadly, I find that credit market frictions and labor search frictions almost equally contributed to unemployment over all business cycles between 1951 and 2011.

Key Words: Credit Crunch, Capital/Labor Misallocation, Beveridge Curve, Jobless Recovery.

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1 Introduction

The 2007-2008 financial crisis was accompanied by a marked increase in unemployment and a serious disruption in credit markets. First, the ratio of external funding to non-financial assets, a key measure used in the literature to characterize the functioning of the credit market, shrank significantly, as demonstrated in the right panel of Figure (1.1).[1] Second, as the left panel of Figure (1.1) shows, not only did the unemployment rate increase significantly over time, but the Beveridge curve also shifted outward beginning in the last quarter of 2008. Motivated by such co-movements of the credit market and the labor market, I develop a tractable dynamic model with heterogeneous entrepreneurs, credit constraints, and labor-search frictions. I find that the credit crunch was the key driving force behind the outward shift in the Beveridge curve during and after the Great Recession. More broadly, I find that credit market frictions and labor search frictions almost equally contributed to unemployment over all business cycles between 1951 and 2011.

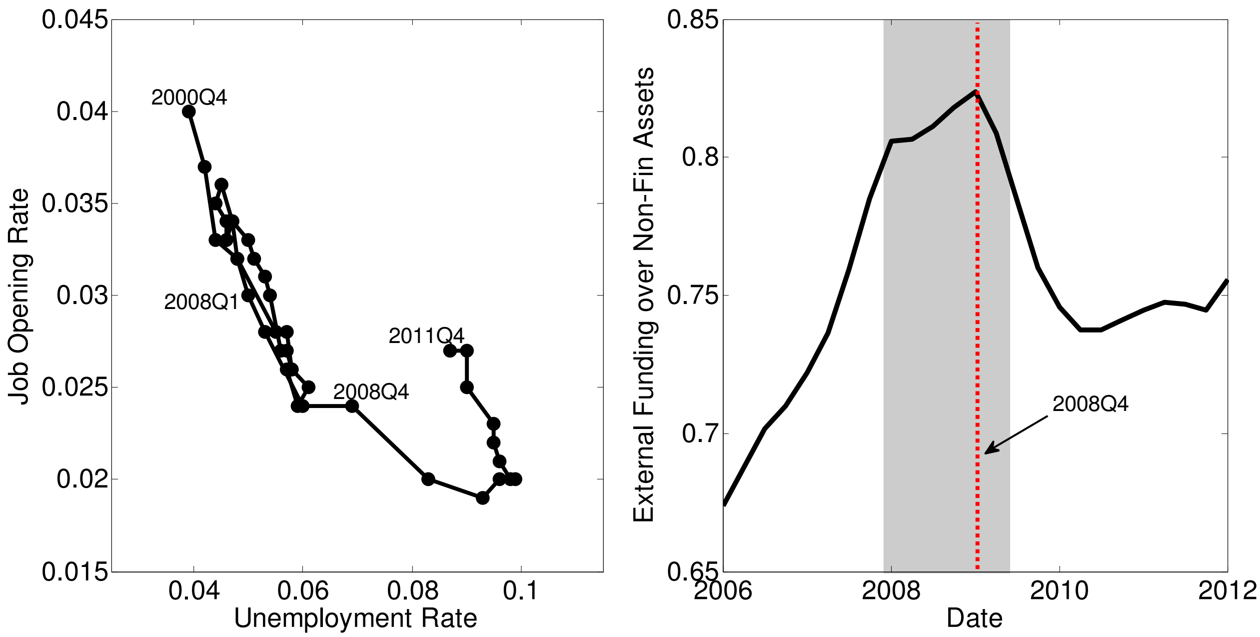


Figure 1.1: **Left Panel: Beveridge Curve, Job Openings and Labor Turnover Survey (JOLTS); Right Panel: External Funding over Non-Financial Assets of Non-Financial Business, Flow of Funds Accounts**

[1]The measure is considered in Buera and Moll (2013) and Buera, Fattal-Jaef and Shin (2013). Both non-financial corporate and non-financial non-corporate business in the Flow of Funds Accounts are considered. Details are documented in Appendix A.

I employ two layers of frictions to model the relationship between credit and labor markets. On the one hand, I introduce credit frictions by using a collateral constraint, which is a powerful tool to characterize credit crunches. On the other hand, I use competitive search to model equilibrium unemployment. Recent empirical findings by Davis, Faberman and Haltiwanger (2013) show that job-filling rates vary significantly across firms. However, a direct implication of random search is that job-filling rate is independent of firm's heterogeneous characteristics. As will be shown in our model, the prediction of competitive search is in line with the empirical regularity.

Entrepreneurs are heterogeneous in two dimensions, net worth and productivity. The former is endogenous and the latter is an exogenous stochastic process. There are three sources of aggregate shocks: i) a credit shock, *i.e.*, the tightening of collateral constraints in the credit market; ii) a matching shock, *i.e.*, the decrease of matching efficiency in the labor market; and iii) an aggregate productivity shock. When a credit crunch occurs, the collateral constraint tightens and more capital would have to be used by relatively unproductive entrepreneurs. The key theoretical contribution of this paper is that capital misallocation worsens labor misallocation, even though it is not accompanied by an adverse matching shock that directly disrupts the labor market.[2] Therefore credit imperfections contribute to endogenous matching efficiency in equilibrium and thus to shifts in the Beveridge curve. In addition to analytically illustrating the effect of capital misallocation on labor misallocation, I also show that equilibrium TFP is determined by the interaction between credit and labor frictions.[3]

The key transmission mechanism proceeds as follows. Although workers are homogeneous, the marginal value of being matched with labor increases with an entrepreneur's productivity. Therefore, entrepreneurs with heterogeneous productivity have an incentive to post different wage offers. I use competitive search to implement this idea. Entrepreneurs with higher productivity tend to post higher wage positions with more workers queuing for those jobs. Thus the job-filling rate will be higher for more productive entrepreneurs. In equilibrium, wage dispersion for homogeneous workers emerges with an endogenous set of segmented labor markets, as in standard competitive search models.

If there is a negative shock to the credit market, *i.e.*, the collateral constraint tightens,

[2]Since our model involves capital misallocation, it belongs to the recently burgeoning literature on misallocation, which mainly includes Hsieh and Klenow (2009), Restuccia and Rogerson (2008), Bartelsman et al. (2012), and a recent discussion by Hopenhayn (2013), among others. Moreover, there has been extensive discussion on capital misallocation due to financial frictions, such as Buera, Kaboski and Shin (2011), Azariadis and Kaas (2012), Moll (2012), Wang and Wen (2012), Bigio (2013), Buera and Moll (2013), Cui (2013), Khan and Thomas (2013), and Liu and Wang (2013).

[3]Lagos (2006) develops a model of TFP with labor search frictions. Our work contributes to this line of literature by incorporating both credit and labor search frictions into an otherwise standard RBC model.

then capital misallocation worsens, since the interest rate decreases and more capital is used by relatively unproductive entrepreneurs. Since the job-filling rate in active sub-labor markets increases with an entrepreneur's productivity, the redistribution of capital from high-productivity to low-productivity firms decreases the total number of matched workers. In addition to the direct effect imposed on unemployment, capital misallocation also generates an indirect and offsetting effect in general equilibrium such that workers also move from labor markets with high productivity to those with lower productivity. Therefore, the job-filling rates as well as equilibrium wage dispersion in all sub-labor markets responds to credit crunches in general equilibrium. However, the concavity of the matching function in each active sub-labor market implies that job destruction by high-productivity entrepreneurs will outweigh job creation by low-productivity ones. Therefore those indirect general-equilibrium effects are dominated by the direct effect described above. In sum, this is how credit crunches contribute to the outward shift in the Beveridge curve.[4]

In each period, the collateral constraint is not necessarily binding for all heterogeneous entrepreneurs. An infinite-horizon model with this setup is potentially complicated. Moreover, I allow for capital accumulation with both financial frictions in the credit market and search frictions in the labor market. Our model is highly tractable because of the linearity of individual policy functions, which is driven by the linearity of the capital revenue in equilibrium. The analytical solution is beneficial in making transparent the mechanism through which capital and labor misallocation interact with each other.

The unemployment effect of capital misallocation is not only of theoretical interest, but also offers a new channel for amplification and propagation in our quantitative analysis. A negative credit shock not only creates capital misallocation and works at the intensive margin, but also affects the extensive margin by lowering matching efficiency. Therefore, even in the absence of the price effect in Kiyotaki and Moore (1997), credit frictions have an amplification effect with a new channel through which capital misallocation worsens labor misallocation. When it comes to the unemployment effect, credit crunches lower endogenous matching efficiency in the labor market. Additional, the new amplification effect of credit crunches dampens capital accumulation and thus further increases unemployment and lowers output in the next period. This is a dynamic implication of credit crunches for aggregate variables of interest.

I then move on to quantify the unemployment effect of credit imperfections as well as that of labor search frictions. In particular, I explore how much credit and labor frictions

[4]Complementary to our work, Mehrotra and Sergeyev (2012) develop a multi-sector model with labor search to characterize conditions under which sector-specific shock, such as in the construction sector, can decreases aggregate matching efficiency and generate an outward shift in the Beveridge curve.

explain unemployment. Moreover, does the credit crunch contribute to the outward shift in the Beveridge curve in the recent financial crisis? Three insights are gained from the quantitative exercise. First and most importantly, the counter-factual analysis shows that the credit crunch serves as a driving force behind the outward shift in the Beveridge curve in the recent financial crisis. I present a preview in Figure (1.2). The left panel indicates that the Beveridge curve predicted by our model fit well with the data. The right panel illustrates that, if there had been no credit crunch in the last quarter of 2008, the predicted unemployment would continue to rise with the negative shocks to aggregate productivity and to the matching efficiency in the labor market. However, in the absence of the credit crunch, the predicted Beveridge curve would not shift outward, but instead would move along with the original curve prior to the financial crisis.

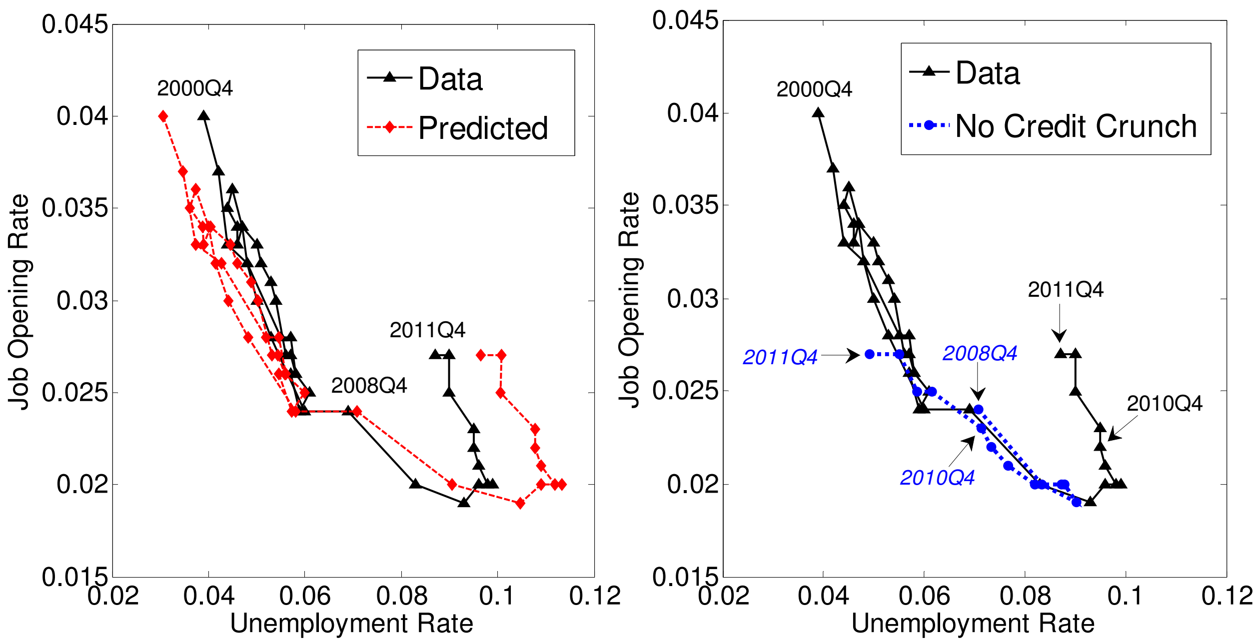


Figure 1.2: **Left Panel: Data and Model-Predicted for the Beveridge Curve; Right Panel: Data and Model-Predicted without the Credit Crunch in 2008**

The second finding of our quantitative exercise shows that the shocks to the credit or labor markets generate a co-movement on output and unemployment. This prediction is in line with the data prior to the recent three recessions. In contrast, the shock to aggregate productivity generates a gap between output and unemployment recovery. This is what happened in the past three recessions. This phenomenon is called a jobless or sluggish recovery and has spawned a large literature; see Berger (2012), among others. Most of the literature assumes a frictionless labor market and only addresses the recovery gap between output and employment numbers.

Therefore previous studies cannot explain the persistently high unemployment rates of the past recessions.[5] Finally, I also find that the shock to the credit market and the shock to the labor market increases and decreases respectively the power of credit imperfections in explaining unemployment. Since both credit and labor shocks are procyclical, the contribution of credit imperfections to unemployment could be ambiguous in theory. Confronting the model with data after a calibration to the US economy indicates that the explanatory power of credit imperfections is procyclical. That is, the labor market itself receives a relatively larger negative shock in recessions. The decomposition exercise suggests credit imperfections account for around 46% of unemployment over all cycles.

In addition to investigating the aggregate implications of three shocks of interest, tractability also offers a transparent discussion on the different micro-level implications of these shocks. I test the predictions of different shocks with micro-level empirical findings. Credit shocks are seemingly most essential in explaining the widening productivity dispersion as well as the disproportional employment loss of firms with different sizes. I generalize the transmission mechanism through which capital misallocation worsens labor misallocation. I begin by introducing a general tax scheme upon capital revenue, which treats the baseline as a special case. I then put an additional constraint on working capital to our model, which generates a non-trivial labor wedge in equilibrium. Finally, I show that endogenizing firm's search effort amplifies the transmission channel in the baseline.

The recent financial crisis has spawned a large volume of research on the role financial shocks play in output fluctuation, following the works of Williamson (1987), Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Carlstrom and Fuerst (1997), and Bernanke, Gertler and Gilchrist (1999). Jermann and Quadrini (2012) and Khan and Thomas (2013) are two such recent studies. However, very few papers connect financial frictions and unemployment.[6] Wasmer and Weil (2004) adopt matching functions with random search to model frictions in both credit and labor markets.[7] They then use the general-equilibrium interaction between these two markets to illustrate the workings of a financial accelerator. Monacelli, Quadrini and Trigari (2011) discuss the role of credit frictions in unemployment by introducing the strategic use of debt by firms with limited enforcement.[8] They build the model to explain

[5]Jaimovich and Siu (2013) are an exception. They investigate the empirical relationship between jobless recoveries and job polarization, and then set up a labor search model with equilibrium unemployment.

[6]Merz (1995) and Andolfatto (1996) were among the first to introduce labor search frictions in the RBC framework, which admits capital accumulation but is subject to no financial frictions. See Shimer (2010) for a survey on the recent development of quantitative analysis for labor search.

[7]A quantitative extension is done by Petrosky-Nadeau and Wasmer (2013), among others. Meanwhile, see Carrillo-Tudela, Graber, and Waelde (2013) for a recent related theoretical model.

[8]Garin (2013) and Blanco and Navarro (2013) extend the work of Monacelli, Quadrini and Trigari (2011)

why firms lower labor demand after a credit contraction even though there is no shortage of funds for hiring. Miao, Wang and Xu (2013) integrate an endogenous credit constraint into a model with random search. They show that the collapse of the bubble, one of the self-fulfilling equilibria, tightens the credit constraint, and in turn decreases labor demand. Liu, Miao and Zha (2013) incorporate the housing market and the labor market in a DSGE model with credit and search frictions. They then make a structural analysis of the dynamic relationship between land prices and unemployment. All of the aforementioned papers focus on the connection between firm-side credit imperfections and unemployment, while Bethune, Rocheteau and Rupert (2013) emphasize the relationship between household credit and unemployment.

Our paper complements the work of Buera, Fattal-Jaef and Shin (2013). Both papers quantify the effect of a credit crunch on unemployment in a heterogeneous-entrepreneurs model with credit frictions and employment frictions. However, our papers differ in several important dimensions. First, their analysis is largely quantitative while the linear property of our model generates tractability and makes transparent the new channel contributed by our paper. Second, we use different modeling strategies for equilibrium unemployment. They specify a Walrasian labor market with a unique and publicly displayed price. To sustain equilibrium unemployment, they assume only a fraction of unemployed workers can enter the centralized hiring market in a given period. I instead use competitive search by following Shimer (1996) and Moen (1997). Finally, they focus on the recent credit crunch while I take into account the historical business cycles as well as the recent recession.

The rest of the paper is organized as follows. Sections 2 describes the model setup. Section 3 characterizes general equilibrium. Section 4 presents a quantitative analysis. Section 5 addresses the disaggregate implications of our model with recent micro-level empirical findings. Section 6 concludes. Appendix A provides the data definition, description and calculation. Appendix B offers a simplified and static model. Appendix C considers model extension. Appendix D includes all omitted proofs.

2 Model

This section describes the model setup by introducing agents and specifying frictions in credit and labor markets.

by allowing for capital accumulation and by introducing flexible number of employees and equilibrium default, respectively.

2.1 Demography and Timing

Time is discrete and goes from zero to infinity. There is no information asymmetry. The economy is populated by three kinds of infinitely lived players: workers, entrepreneurs and financial intermediaries.[9]

Workers. There is a representative household with measure L of homogeneous household members. Each worker has one unit of indivisible labor. I assume the household has access to neither production skills nor the credit market. If a worker is unemployed, she has no revenue.[10] If a worker is matched with an entrepreneur, she receives labor revenues after production.[11] The household distributes consumption equally to each member by pooling labor revenue at the end of each period. All workers engage in hand-to-mouth consumption. In this paper, the new channel through which capital misallocation affects unemployment is on the side of labor demand. To sharpen our transmission mechanism, I assume labor supply is inelastic.[12]

Entrepreneurs. There is a unit measure of entrepreneurs. Only entrepreneurs have access to the credit market as well as to production skills. Entrepreneurs are heterogeneous in two dimensions: one is net worth a while the other is productivity x. I assume x is the product of aggregate productivity z and individual component φ, *i.e.*, $x = z \cdot \varphi$. The distribution of net worth endogenously evolves over time while that of an idiosyncratic and aggregate productivity shock is exogenous. The distribution of individual productivity is denoted as $F(\cdot)$ with a bounded support $[\underline{\varphi}, \overline{\varphi}]$. In the next period, individual productivity φ is preserved or is re-drawn from some fixed distribution $\widetilde{F}(\cdot)$ with probability ρ and $1-\rho$, respectively. When $\rho = 1$, it is degenerate to the case with *iid* productivity shock. For simplicity, I assume $\widetilde{F}(\cdot)$ coincides with $F(\cdot)$ in the first period. Therefore, the distribution of individual

[9] Our paper does not consider occupational choice. See Wiczer (2012) and Buera, Fattal-Jaef and Shin (2013), among others, for a quantitative discussion on unemployment with occupational choice.

[10] That is, I assume the replacement ratio is zero throughout this paper. As shown soon, I assume a fixed labor supply and focus on the demand side for labor. Thus this assumption of no unemployment compensation does not affect the key channel of our paper. However, as pointed out in the quantitative analysis by Hobijn and Sahin (2012) and Hagedorn, Karahan, Manovskii and Mitman (2013) with a different context of modeling, the extension of unemployment insurance benefits could be quantitatively important in explaining the worsening labor market in the past recession.

[11] There is no constraint on working capital in the baseline model. Appendix C considers the case in which entrepreneurs need to pay part of wage bill before production.

[12] Alternatively, I can explicitly specify the household's utility function as $U_W = \mathbb{E}\left\{\sum_{t=0}^{\infty} \beta^t \cdot \left[log(C_t) - \xi \cdot \frac{L_t^{1+\nu}}{1+\nu}\right]\right\}$, where C and L denotes consumption and labor supply respectively. Since the household has a continuum of workers and does not save, I have $C = W \cdot L$, where W denotes expected labor revenue.The details of labor search and matching is specified very soon in the part of labor market. The log-utility setup, alongside with the first order condition of the intra-period decision on labor supply, implies a fixed labor supply by the household.

productivity is stationary over time.[13] The stochastic process governing z is not essential for our analysis right now. I will return to it in the quantitative analysis. For tractability, I assume productivity shock is independent of net worth. Therefore the joint distribution $H(a, \varphi)$ can be rewritten as the product of $F(\varphi)$ and $G(a)$, the distribution of individual productivity and that of net worth. An entrepreneur's objective function is given by

$$U_E = \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \cdot log(c_t)\right],$$

where c_t denotes consumption.

Financial Intermediary (FI) & Credit Market. The representative financial intermediary is risk neutral and fully competitive. I assume all borrowing and lending between entrepreneurs is intermediated by FI. One of the possible elements to make FI essential is to assume FI can verify an entrepreneur's individual productivity but it is too costly for entrepreneurs themselves if they directly contact each other. FI herself does not own, produce or use capital.[14] I model credit imperfections by assuming productive entrepreneurs cannot borrow as much as they want.

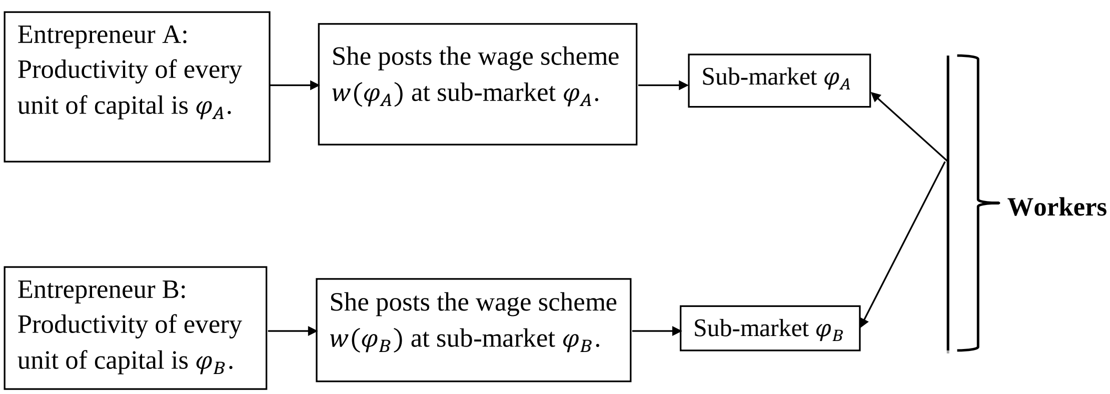


Figure 2.1: **Wage Posting by Active Entrepreneurs**

Labor Market. I use competitive search, which is also called directed search, to model equilibrium unemployment. As is standard in the literature, the production function is Leontief: only after one unit of capital from entrepreneur-(a, φ) is matched with one unit of labor can φ units of consumption goods be realized. Entrepreneur-(a, φ) could either borrow and

[13]In general, I have $F_{t+1}(\cdot) = \rho \cdot F_t(\cdot) + (1-\rho) \cdot \widetilde{F}(\cdot)$.

[14]Dong and Wen (2013) address a case in which FI not only intermediates borrowing and lending, but also produces capital goods with a linear transformation technology.

produce by posting a wage contract $w(\varphi)$ in sub-market φ, or lend to other entrepreneurs in the credit market.[15] The opportunity cost of running capital is the endogenous interest rate r.[16] Therefore, not all entrepreneurs choose to produce. If a worker goes to sub-market φ and gets matched, she obtains wage $w(\varphi)$. Workers self-select into active sub-markets $\varphi \in \Phi^A \subseteq \Phi$. See Figure (2.1). Only matched workers receive revenues. The household pools all the labor income together and distributes it equally to all members. Each household member engages in hand-to-mouth-consumption. The borrower entrepreneurs receive capital revenue, part of which they pay back to lender entrepreneurs via the financial intermediary. All entrepreneurs make decisions about consumption and saving.

State Variables and Timing. I assume all matched relationships between firms and workers are terminated as the end of every period. This assumption simplifies our analysis. If I use a long-term contract, then entrepreneurs would be heterogeneous in three dimensions in each period: net worth, productivity, and numbers of employed workers. In turn, the theoretical analysis would be deprived of tractability.[17] Therefore I make the above assumption.[18] Consequently, the idiosyncratic state variable is two dimensional, (a, φ), the net worth and productivity. The aggregate state is denoted as $X = (z, \lambda, \eta, H(a, \varphi))$, where z is aggregate productivity shock, λ is the shock to the credit market, η is the matching efficiency in every sub-labor market, and $H(a, \varphi)$ is the joint distribution of net worth and productivity. Given our assumption about the productivity shock, the aggregate state can be rewritten as $X = (\lambda, \eta, F(x), G(a))$, where $F(\varphi)$ and $G(a)$ denote the distribution of productivity and that of net worth, respectively, and the product yields their joint distribution. Finally, I present the time-line in Figure (2.2).

2.2 Labor Market

As is standard in the literature, the matching function $m(v(\varphi), l(\varphi))$ in all sub-markets $\varphi \in \Phi$ is homogeneous of degree one, and increases with both arguments, where $v(\varphi)$ and $l(\varphi)$ denote, respectively, the measure of capital and labor with market tightness $\theta(\varphi) \equiv l(\varphi)/v(\varphi)$. Then

[15] The framework of competitive search implies $w(\varphi)$ has nothing to with productivity distribution. This in turn helps preserve model tractability.

[16] Since there is no entry and exit, I assume for simplicity that there is no explicit cost of wage posting.

[17] Schaal (2012) characterizes and quantifies a search model with heterogeneity in productivity and labor use. However, there is heterogeneity in net worth since there is no capital use and capital accumulation. As noted at the end of Schaal (2012), it is promising and challenging to consider financial frictions after introducing capital accumulation. Complementary to his work, our paper considers heterogeneity in productivity and capital.

[18] However, this assumption immediately implies the ratio of job destruction to total employment is 100%. To solve this problem, I use the net flow to measure job destruction and job creation. See more details in Section 4.

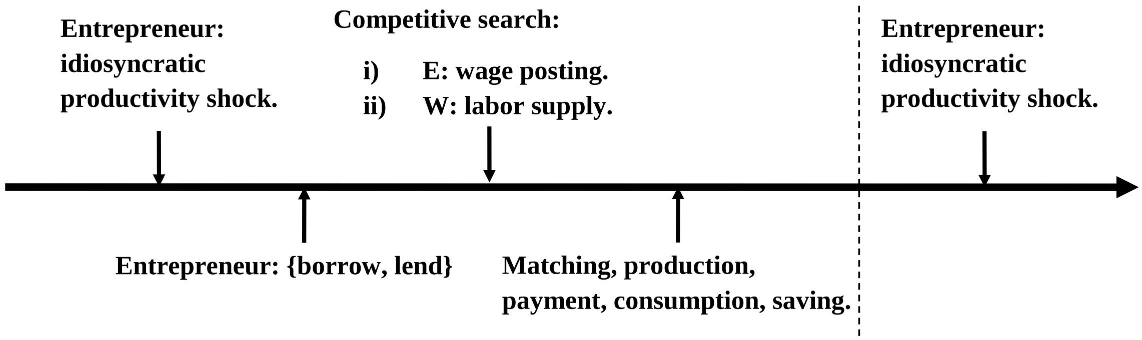


Figure 2.2: **Time-line**

the job-filling rate and job finding rate, $q(\theta(\varphi))$ and $p(\theta(\varphi))$, have the following property: $q' > 0$, $q'' < 0$, $p' < 0$ and $p'' > 0$, where

$$q(\theta(\varphi)) \equiv \frac{m(v(\varphi), l(\varphi))}{v(\varphi)} = m(1, \theta(\varphi))$$
$$p(\theta(\varphi)) \equiv \frac{m(v(\varphi), l(\varphi))}{l(\varphi)} = m\left(\frac{1}{\theta(\varphi)}, 1\right) = \frac{q(\theta(\varphi))}{\theta(\varphi)}.$$

I assume throughout the paper that the matching function is Cobb-Douglas, *i.e.*, $m(v(\varphi), l(\varphi)) = \eta \cdot v(\varphi)^{\gamma} \cdot l(\varphi)^{1-\gamma}$ with $\gamma \in (0,1)$, where η denotes matching efficiency and is exogenously given.[19] Due to search frictions and heterogeneity in capital productivity, there exists no unique wage such that labor supply equals demand. Instead, I only have the following constraint on labor supply.

$$\int_{\Phi} l(\varphi) \cdot d\varphi = L. \tag{2.1}$$

I formulate $\pi(\varphi, W)$, the expected revenue of one unit of capital in sub market-φ, as below.

$$\pi(\varphi, W) \equiv \max_{\{\theta(\varphi,W), w(\varphi,W)\}} \{q(\theta(\varphi, W)) \cdot (\varphi - w(\varphi, W))\}, \tag{2.2}$$

subject to

$$p(\theta(\varphi, W)) \cdot w(\varphi, W) = W, \tag{2.3}$$

where $W(\varphi) = W(\varphi') \equiv W$ denotes the expected wage revenue by going to sub-market $\varphi, \varphi' \in \Phi^A \subseteq \Phi$, where Φ_A denotes the set of entrepreneurs active in production. I characterize Φ^A in Section 2.4, and right now treat it as given. I now characterize the endogenous wage offer in active sub markets Φ_A.

[19] Motivated by recent empirical findings, Appendix C endogenizes firms' recruiting efforts, which amplifies the transmission mechanism in the baseline.

Proposition 1. (Wage Scheme)

1. Given W, the market tightness in any active sub-market $\varphi \in \Phi_A$ is determined by

$$q'(\theta(\varphi)) = \frac{W}{\varphi}. \tag{2.4}$$

2. The wage scheme and expected capital revenue obtained from sub-market $\varphi \in \Phi_A$ is given by

$$\begin{aligned} w(\varphi, W) &= \frac{W}{p(\theta(\varphi))} \\ \pi(\varphi, W) &= [q(\theta(\varphi)) - \theta q'(\theta(\varphi))] \cdot \varphi. \end{aligned} \tag{2.5}$$

3. Comparative statics:

$$\frac{\partial \pi(\varphi, W)}{\partial \varphi} > 0, \ \frac{\partial \pi(\varphi, W)}{\partial W} < 0, \ \frac{\partial \theta(\varphi, W)}{\partial \varphi} > 0, \ \frac{\partial \theta(\varphi, W)}{\partial W} < 0, \ \frac{\partial q(\theta(\varphi, W))}{\partial \varphi} > 0, \ \frac{\partial q(\theta(\varphi, W))}{\partial W} < 0.$$

The marginal value of being matched with labor increases with the productivity. Therefore, the wage scheme increases with productivity. In turn, entrepreneurs with higher productivity enjoy a higher job-filling rate. Thus, high-productivity entrepreneurs are more efficient in both extensive and intensive margins. This observation is the key to understanding the general-equilibrium effect of capital misallocation on unemployment in the next section. Finally, Proposition 1 shows the expected capital revenue increases with productivity. This property, like that in Melitz (2003), delivers a cut-off point for active entrepreneurs and greatly simplifies our analysis in Section 2.4.

2.3 Entrepreneur's Constrained Optimization

At the beginning of each period, entrepreneurs rely on two pieces of public information to decide whether or not to be active in production. One is the individual state variable, which includes net worth a and productivity φ. The other one is the aggregate state variable $X = (\lambda, \eta, z, F(\varphi), G(a))$. Assume some entrepreneur uses k units of capital for production. Then I use $b \equiv k - a$ to denote the external funding. That $b < 0$ means net lending. Since the production function is Leontief, active entrepreneurs posts their wage scheme $w(\varphi)$ for every unit of capital at sub market $\varphi \in \Phi_A$. For notational ease, I replace $\pi(\varphi, W)$ with $\pi(\varphi)$ in the rest of the paper. Assume the law of large numbers holds here. Then the total capital revenue is $\Pi(k, \varphi) = \pi(\varphi) \cdot k$ for the entrepreneur with productivity φ and using k units of capital for production. I model credit frictions with the simplest collateral constraint, *i.e.*,

$k \leq \lambda \cdot a$, where k and a denotes the total capital available and own net worth, respectively, and λ the exogenous financial shock to the credit market. If $\lambda = 1$, the credit market collapses and entrepreneurs are in autarky. If $\lambda = \infty$, the credit market is complete since the collateral constraint would never be binding. Finally, the constrained optimization of entrepreneur-(a, φ) is formulated as below.

$$V(a, \varphi; X) = max \left\{ log(c) + \beta \cdot \mathbb{E}\left[V(a', \varphi'; X') \,|X\right] \right\} \tag{2.6}$$

subject to

$$\begin{aligned} r \cdot b + c + i &= \Pi(k, \varphi) = \pi(\varphi) \cdot k & (2.7) \\ a' &= (1 - \delta) \cdot a + i & (2.8) \\ b &= k - a & (2.9) \\ k &\leq \lambda \cdot a & (2.10) \\ k &\geq 0 & (2.11) \end{aligned}$$

Equation (2.7) is the budget constraint with $\Pi(k, \varphi)$ being the capital revenue, $r \cdot b$ the debt repayment, c the consumption and i the investment for next period. Equation (2.8) is the accounting identity on investment, net worth and the total capital obtained for production. Equation (2.9) is the definition on external funding b. Equation (2.10) is a collateral constraint, in which the maximum available capital is proportional to the entrepreneur's own net worth. The collateral constraint $k \leq \lambda \cdot a$ implies the leverage ratio is the same across heterogeneous entrepreneurs, and has nothing to do with the interest rate r. This is purely for tractability.[20] As emphasized by Moll (2012), it is the linearity of collateral constraint that guarantees tractability. Equation (2.11) denotes a no-short-selling constraint.

I use the simplest form of collateral constraint. Unlike Kiyotaki-Moore (1997), I eliminate the price effect. As shown in Section 3, this simplification will illustrate the unemployment effect of capital misallocation in a transparent way. Moreover, I can anticipate that the additional consideration of price effect would strengthen the new channel proposed there. Second, credit imperfections are characterized by the above collateral constraint in a reduced-form way. There are several alternatives with micro-foundation to support the linear form of collateral constraint. In addition to the limited liability proposed by Kiyotaki and Moore (1997), I can also obtain the linearity by considering costly state verification by Williamson (1987) and Bernanke and Gertler (1989), or moral hazard by Holmstrom and Tirole (1997).

[20] I also tried a complicated version in which the collateral constraint is related to interest rate and productivity heterogeneity. The result is still tractable at both micro and aggregate levels. It is available upon request.

Finally, our baseline only takes into account credit frictions and labor search frictions. This helps us focus on the unemployment effect of worsening capital misallocation in the simplest and most clear way.

2.4 Credit Market

I use this part to characterize the conditions under which the collateral constraint is binding for entrepreneurs heterogeneous in net worth and productivity. Denote $\Pi(k,\varphi)$ as the capital revenue by entrepreneurs with productivity φ and using k units of capital for production. Based on Proposition 1 and assuming the law of large number applies, I know the capital revenue is linear in k, and

$$\Pi(k,\varphi) = \pi(\varphi)\cdot k = kq(\varphi)\varphi - k\theta(\varphi)W, \tag{2.12}$$

Then the constrained optimization by entrepreneur-(a,φ) can be simplified as below.

$$V(a,\varphi;X) = max\left\{log(c) + \beta\cdot\mathbb{E}\left[V(a',\varphi';X')\,|X\right]\right\}$$

subject to

$$\begin{aligned} c + a' &= [r + (1-\delta)]\cdot a + max\{\pi(\varphi) - r,\, 0\}\cdot k \\ k &\in [0,\, \lambda\cdot a]\,, \quad \lambda \in (1,\infty) \end{aligned}$$

The entrepreneur-(a,φ) can always receive the capital revenue $[r+(1-\delta)]\cdot a$ by making a deposit to the financial intermediary. Additionally, if the entrepreneur uses k units of capital for production, then the net gain is $\pi(\varphi) - r$, where $\pi(\varphi)$ and r denotes the expected revenue and the opportunity cost of using one unit of capital for production. Therefore, the option value for each unit of capital held by an entrepreneur with productivity φ is $max\{\pi(\varphi) - r,\, 0\}$. In turn, I follow Buera and Moll (2013) to define the return premium as $RP \equiv \mathbb{E}\left[max\left(\pi(\varphi) - r,\, 0\right)\right]$. If there is no credit friction or no productivity heterogeneity, then the return premium is simply zero. Given the individual capital demand $k(\varphi,a)$, the clearing condition in the credit market is then obtained by

$$\int\int k(\varphi,a)\cdot h(\varphi,a)d\varphi da = \int\int a\cdot h(\varphi,a)d\varphi da. \tag{2.13}$$

I then use the following lemma to characterize the individual capital demand.

Lemma 1. (Capital Demand and Cash Holding) Capital demand by entrepreneur-(a,φ)

conforms to a corner solution, *i.e.*,

$$k(\varphi, a) = \begin{cases} 0 & \textit{if } \varphi \in [\underline{\varphi}, \widehat{\varphi}] \\ \lambda \cdot a & \textit{if } \varphi \in [\widehat{\varphi}, \overline{\varphi}] \end{cases},$$

where the cut-off value $\widehat{\varphi}$ is determined by

$$\pi(\widehat{\varphi}) = r, \tag{2.14}$$

and the ratio of cash holding to assets is $\lambda \cdot [1 - q(\varphi)]$.

Denote the aggregate net worth as $K \equiv \int a \cdot dG(a)$. The above lemma suggests the measure of capital in sub market φ is

$$v(\varphi) = \left[\int k(\varphi, a)\, dG(a) \right] \cdot f(\varphi) \cdot \mathbf{1}_{\{\varphi \geq \widehat{\varphi}\}} = \lambda K f(\varphi) \cdot \mathbf{1}_{\{\varphi \geq \widehat{\varphi}\}}. \tag{2.15}$$

Entrepreneurs with high enough productivity produce and hit a binding collateral constraint. The rest prefer lending in the credit market. The property of choosing corner solutions is due to the linearity of capital gains. Besides, this lemma immediately reveals that the set of active entrepreneurs is $\Phi_A = \{\varphi | \varphi \geq \widehat{\varphi}\}$. It is worth noting that, although active entrepreneurs want to borrow as much as they want with a binding collateral constraint, the equilibrium leverage ratio used for production is $\lambda \cdot q(\varphi)$ rather than λ in the presence of labor search frictions. Consequently, cash holding emerges in equilibrium. The ratio of cashing hold to assets decreases with productivity. This is determined by the use of capital with labor search frictions, which is illustrated as follows.

Corollary 1. (Double Selection on Capital Use) The productivity distribution of active entrepreneurs and that of matched entrepreneurs are

$$F^A(\varphi) = \frac{F(\varphi) - F(\widehat{\varphi})}{1 - F(\widehat{\varphi})}, \quad F^M(\varphi) = \frac{\int_{\widehat{\varphi}}^{\varphi} q(\varphi') \cdot dF(\varphi')}{\int_{\widehat{\varphi}}^{\overline{\varphi}} q(\varphi') \cdot dF(\varphi')},$$

and $F^M(\varphi) < F^A(\varphi) < F(\varphi)$.

It is worth noting that the equilibrium productivity distribution is $F^M(\varphi)$ rather than $F^A(\varphi)$. The latter is the truncated distribution in the first step. As proved in Proposition 1, the job-filling rate of active entrepreneurs increases their individual productivity. As a result, the equilibrium productivity distribution is obtained after the selection in the second step, which reflects in the weight $q(\varphi)$ in the above equation of $F^M(\varphi)$. I illustrate the relationship of these three distributions in Figure (2.3). In the end, I obtain the policy function of entrepreneur-(a, φ) in partial equilibrium.

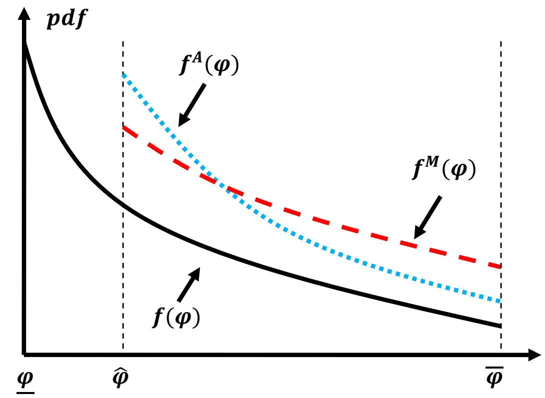


Figure 2.3: **Double Selection of Capital Use**

Corollary 2. (Individual Policy Function) Given the aggregate state variable X, the consumption and saving by entrepreneur-(a, φ) is linear with her own net worth.

$$\begin{aligned} a_{t+1}(a_t, \varphi_t) &= \beta \cdot \Psi_t(\varphi) \cdot a_t \\ c_t(a_t, \varphi_t) &= \Psi_t(\varphi) \cdot a_t - a_{t+1}(a_t, \varphi_t), \end{aligned}$$

where $\Psi_t(\varphi) \equiv \lambda_t \cdot max\{\pi_t(\varphi) - r_t, 0\} + [r_t + (1-\delta)]$.

The linearity of policy function admits a tractable aggregation.[21] Therefore, I can keep track of the endogenous evolution of the distribution without resorting to purely numerical work like Krusell and Smith (1998). The linear property of policy function makes it easy for us to connect with recent literature on credit frictions. For example, Wang and Wen (2012) develop an incomplete credit market model with heterogeneity in investment efficiency as well as with partial irreversibility such that $a' \geq \lambda_I \cdot (1-\delta) \cdot a$. Notice that $\lambda_I = 0$ and $\lambda_I = 1$ denote the cases with perfect reversibility and complete irreversibility, respectively. Based on the above corollary, the individual policy function is still tractable with the additional constraint of partial investment irreversibility upon our framework. In this scenario, the intertemporal decision would be adjusted as

$$a_{t+1}(a_t, \varphi_t) = max\{\beta \cdot \Psi_t(\varphi), \lambda_I \cdot (1-\delta)\} \cdot a_t.$$

[21] In the presence of partial irreversibility, the policy function is adjusted as $a_{t+1}(a_t, \varphi_t) = max\{\beta \cdot \Psi_t(\varphi), \lambda_{I,t} \cdot (1-\delta)\} \cdot a_t$. Thus the linearity property is preserved.

3 Equilibrium

I have so far addressed the decisions of all agents in partial equilibrium. I summarize the key results in Figure (3.1).

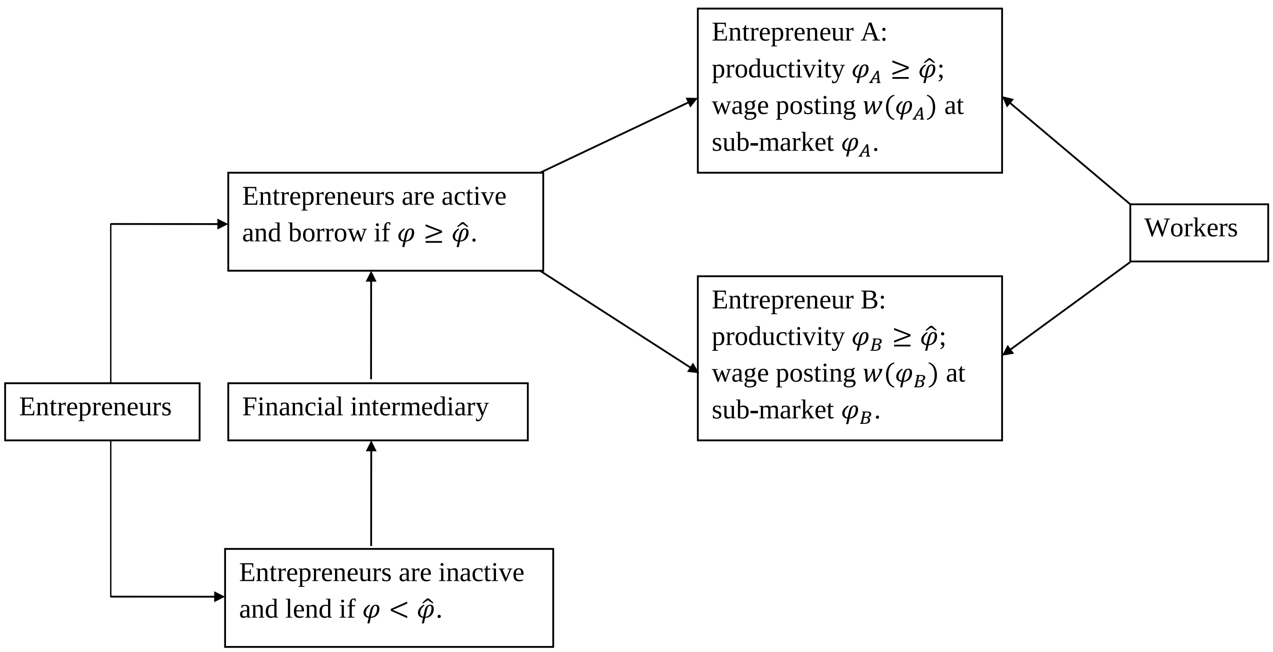


Figure 3.1: **Decision Rules of All Agents**

This section is devoted to exploring the general equilibrium of our model with heterogeneous entrepreneurs, and with credit and labor search frictions. I characterize not only the equilibrium in each period, but also the transition dynamics. I start with defining the recursive competitive equilibrium as below.

Definition 1. (Recursive Competitive Equilibrium) A recursive competitive equilibrium consists of

1. labor supply $l(\varphi)$, capital $v(\varphi)$ and market tightness $\theta(\varphi)$ at active sub-market $\varphi \in \Phi_A$,

2. a set of price functions, including the interest rate r, the wage scheme $w(\varphi)$ and the expected labor gain from sub-market $W(\varphi)$ in active sub-market $\varphi \in \Phi_A$,

3. a set of individual policy functions, including consumption c, debt b, and net worth for next period a',

4. the value function $V(a, \varphi)$,

5. the law of motion for the aggregate state variable $X = (z, \lambda, \eta, F(\varphi), G(a))$, such that,

- given X and W the market tightness $\theta(\varphi) = l(\varphi)/v(\varphi)$ is determined by Equation (2.4), $v(\varphi)$ by Equation (2.15) and wage $w(\varphi)$ by Equation (2.5),
- given X, the cut-off point, $\widehat{\varphi}$, the interest rate r, and the expected wage revenue W are jointly determined by Equations (2.14), (2.13), and (2.1),
- $c(a, X)$ and $a'(a, X)$ is the solution to the entrepreneur's dynamic optimization, and the value function $V(a, X)$ is obtained with $c(a, X)$ and $a'(a, X)$,
- the credit market clears as in Equation (2.13).

3.1 Equilibrium Wedges

I first address the social planner's problem. More specially, there is only labor search friction in the benchmark. Then the problem is formulated as below.

$$Y^* = \max_{\{v(\varphi), l(\varphi)\}} \int_{\Phi} z \cdot \varphi \cdot m(v(\varphi), l(\varphi)) d\varphi$$

subject to

$$\begin{aligned} \int_{\Phi} v(\varphi) d\varphi &\leq K \equiv \int\int a \cdot h(\varphi, a) d\varphi da \\ \int_{\Phi} l(\varphi) d\varphi &\leq L \\ v(\varphi),\, l(\varphi) &\geq 0, \end{aligned}$$

where $v(\varphi)$ and $l(\varphi)$ denotes the measure of capital and labor in sub-labor market φ. I summarize the key results below.

Lemma 2. (Benchmark) If the matching function is constant return to scale, the most efficient allocation is that all capital and labor are assigned to the most productive entrepreneurs, *i.e.*, $v^*(\varphi) = K \cdot \mathbf{1}_{\{\varphi = \overline{\varphi}\}}$, $l^*(\varphi) = L \cdot \mathbf{1}_{\{\varphi = \overline{\varphi}\}}$, $Y^* = z \cdot \overline{\varphi} \cdot m(K, L)$, $N^* = m(K, L)$, $u = 1 - \frac{N}{L}^*$, and $ALP^* \equiv \frac{Y^*}{N^*} = z \cdot \overline{\varphi}$.

First, the efficient allocation can be realized if all firms have to post a unique wage. The Bertrand competition would then drive up the wage to $z \cdot \overline{\varphi}$. Second, the benchmark results on allocation have a caveat. If I use the span-of-control model by Lucas (1978), then it is not necessarily true that all resources should be used by the most productive firms.

In the rest of this section, I characterize the equilibrium allocation of the decentralized economy. To start with, I make the below assumption.

Assumption 1. $\Upsilon(\widetilde{\varphi}) \equiv \frac{\mathbb{E}_F\left(\varphi^{\frac{1-\gamma}{\gamma}} \mid \varphi \in [\widetilde{\varphi}, \overline{\varphi}]\right)}{\left[\mathbb{E}_F\left(\varphi^{\frac{1}{\gamma}} \mid \varphi \in [\widetilde{\varphi}, \overline{\varphi}]\right)\right]^{1-\gamma}}$ strictly increases with $\widetilde{\varphi} \in \left(\underline{\varphi}, \overline{\varphi}\right)$ for $\gamma \in (0,1)$

This assumption is reasonable in the sense that it is held with Uniform distribution, Power distribution, and Upper Truncated Pareto distribution, all of which are frequently used in the literature.[22] As emphasized in Section 2, I assume the upper bound of productivity distribution is less than infinity. I did not consider Pareto distribution in the theoretical or quantitative parts of our paper. On the one hand, the boundedness of $\overline{\varphi}$ is of theoretical importance. When the credit market is complete, *i.e.*, $\lambda \to \infty$, only the most productive entrepreneurs would take over the production. Models with a Pareto distribution would not be well defined in the extreme scenario, as emphasized by Moll (2012) and Wang and Wen (2013), who address heterogeneity in productivity and investment efficiency, respectively, with an incomplete financial market. On the other hand, our key channel through which credit imperfections affect unemployment would heavily depend on the above assumption. However $\Upsilon(\widetilde{\varphi})$ would be purely constant if I adopt a Pareto distribution, and thus the transmission mechanism would be shut down in equilibrium. Therefore, I instead use a Power distribution with a normalized support $[0,1]$ in the coming quantitative analysis.[23]

Following the literature on business cycle accounting, such as Chari, Kehoe and McGrattan (2007), I characterize allocation and wedges of the decentralized economy in general equilibrium as below.

Proposition 2. (Wedges in General Equilibrium) Given the aggregate state variable X,

1. the cut-off point $\widehat{\varphi}$ increases with λ such that $\lim_{\lambda \to 1} \widehat{\varphi} = \underline{\varphi}$ and $\lim_{\lambda \to \infty} \widehat{\varphi} = \overline{\varphi}$.

2. the aggregate output and the total matched workers are

$$
\begin{aligned}
Y &= (1-\tau_y) \cdot Y^* = (1-\tau_y) \cdot \overline{\varphi} \cdot m(K, L) \\
N &= (1-\tau_n) \cdot N^* = (1-\tau_n) \cdot m(K, L)
\end{aligned}
$$

[22] As shown in Appendix D, the above assumption is equivalent to assuming, for all $\widetilde{\varphi} \in \left(\underline{\varphi}, \overline{\varphi}\right)$, we have

$$
\mathbb{E}_F\left[\left(\frac{\varphi}{\widetilde{\varphi}}\right)^{\frac{1}{\gamma}} | \varphi \in (\widetilde{\varphi}, \overline{\varphi})\right] \cdot \left\{1 - \left(\frac{1}{\gamma}\right) \cdot \left[\frac{1-F(\widetilde{\varphi})}{\widetilde{\varphi} \cdot f(\widetilde{\varphi})}\right]\right\} \leq 1.
$$

[23] Uniform distribution is a special case of Power distribution. I use uniform distribution as an example in our theoretical analysis since it is a perfect candidate to exercise mean preserving spread. I then calibrate the parameters of Power distribution in the quantitative part. I also tried the Upper Truncated Pareto distribution.

where

$$1-\tau_y = \Lambda(\lambda) \equiv \mathbb{E}_F\left[\left(\frac{\varphi}{\overline{\varphi}}\right)^{\frac{1}{\gamma}} | \varphi \in [\widehat{\varphi}, \overline{\varphi}]\right]^{\gamma} \in (0,1)$$

$$1-\tau_n = \Omega(\lambda) \equiv \frac{\mathbb{E}_F\left(\varphi^{\frac{1-\gamma}{\gamma}} | \varphi \in [\widehat{\varphi}, \overline{\varphi}]\right)}{\left[\mathbb{E}_F\left(\varphi^{\frac{1}{\gamma}} | \varphi \in [\widehat{\varphi}, \overline{\varphi}]\right)\right]^{1-\gamma}} \in (0,1).$$

both of which increases with λ, and $\lim_{\lambda\to\infty}\tau_y = \lim_{\lambda\to\infty}\tau_n = 0$.

3. the average labor productivity, $ALP \equiv \frac{Y}{N}$, and unemployment, $u \equiv 1-\frac{N}{L}$, is[24]

$$ALP = (1-\tau_{alp}) \cdot ALP^* = \mathbb{E}_{F_M}(\varphi)$$

$$u \equiv (1+\tau_u) \cdot u^* = u^* + \tau_n \cdot (1-u^*) \quad (3.1)$$

where

$$1-\tau_{alp} = \frac{1-\tau_y}{1-\tau_n} = \Upsilon(\lambda) \equiv \frac{\mathbb{E}_{F^i}\left[\varphi_i^{\frac{1}{\gamma}} | \varphi_i \in [\widehat{\varphi}_i, \overline{\varphi}_i]\right]}{\mathbb{E}_{F^i}\left[\left(\frac{\overline{\varphi}_i}{\varphi_i}\right) \cdot \varphi_i^{\frac{1}{\gamma}} | \varphi_i \in [\widehat{\varphi}_i, \overline{\varphi}_i]\right]} \in (0,1)$$

$$1+\tau_u = 1+\tau_n \cdot \left(\frac{1-u^*}{u^*}\right) \in (1,\infty).$$

4. the wedge to the expected labor revenue is zero, *i.e.*, $W = \frac{\partial Y}{\partial L}$ while the wedge to the interest rate is

$$r = (1-\tau_r) \cdot \left(\frac{\partial Y}{\partial K}\right)$$

where $1-\tau_r \equiv \frac{1}{\mathbb{E}_{F^i}\left[(\varphi_i/\widehat{\varphi}_i)^{\frac{1}{\gamma}} | \varphi_i \in [\widehat{\varphi}_i, \overline{\varphi}_i]\right]}$, which increases with λ, and $\lim_{\lambda\to\infty}\tau_r = 0$.

5. the equilibrium labor supply and the corresponding wage offer in sub market φ is

$$\frac{l(\varphi)}{L} = \left[\frac{\varphi}{\Lambda(\lambda)}\right]^{\frac{1}{\gamma}} \cdot \left[\frac{v(\varphi)}{Kf(\varphi)}\right]$$

$$w(\varphi) = (1-\gamma) \cdot \varphi \cdot \mathbf{1}_{\{\varphi \geq \widehat{\varphi}(\lambda)\}},$$

and the cumulative distribution is $F_w(\omega) \equiv Pr\{w \leq \omega\} = F_M\left(\frac{\omega}{1-\gamma}\right)$, where $F_M(\cdot)$ denotes the equilibrium productivity distribution of the capital they are matched with labor.

[24] I use $\lambda = \infty$ as the limit case for our theoretical analysis. If I use some $\overline{\lambda} < \infty$ instead as the limit scenario, then the formula between u and u^* is adjusted as $u = \overline{u} + \left[1 - \frac{\Omega(\lambda)}{\Omega(\overline{\lambda})}\right] \cdot (1-\overline{u})$, where $\overline{u} \equiv 1 - \Omega\left(\overline{\lambda}\right) \cdot m\left(\frac{K}{L}, 1\right)$.

First, both ALP and N increase with λ. Therefore, credit imperfections affect the output not only through lowering capital misallocation, *i.e.*, the decrease of ALP, but also by alleviating labor misallocation, *i.e.*, the increase of employment. The former and latter denote the intensive and extensive margins, respectively. Therefore our model offers a new channel through which a credit crunch generates an amplification effect on output. I further illustrate this result in the quantitative exercise in Section 4.

Second, given $\varphi \geq \widehat{\varphi}$, both $v(\varphi)$ and $l(\varphi)$ increases with λ. However, as shown in the above proposition, $l(\varphi)$ does not increase as much as $v(\varphi)$ does. Therefore, the market tightness $\theta(\varphi) \equiv l(\varphi)/v(\varphi)$ and the associated job-filling rate $q(\varphi)$ decreases with λ in general equilibrium. That is, as more capital is concentrated at the top end, the market tightness tends to be less favorable to firms.

Third, Proposition 2 provides a micro-foundation for the Cobb-Douglas aggregation. In turn, equilibrium TFP is defined as

$$TFP(\lambda, \eta, z) \equiv \frac{Y}{K^{\gamma} L^{1-\gamma}} = [ALP(\lambda, z)] \cdot [\Omega(\lambda) \cdot \eta], \tag{3.2}$$

which is determined by aggregate productivity and frictions to credit and labor markets. Therefore, credit imperfections affect equilibrium TFP at intensive margin (capital misallocation) as well as extensive margin (employment). I can also characterize TFP wedge as $TFP \equiv (1 - \tau_{tfp}) \cdot TFP^{*}$ and thus $\tau_{tfp} = \tau_{y}$. Moreover, following Lagos (2006), I can alternatively use the finally matched capital and labor, *i.e.*, $L_M = K_M = N$ to measure equilibrium TFP. Then I have $\widetilde{TFP} \equiv \frac{Y}{K_M^{\gamma} L_M^{1-\gamma}} = \frac{Y}{N} = ALP$, which is affected by both z and λ. However, it is independent of η since matching efficiency only affects matched capital and labor.

I have characterized at the end of Section 2 the intertemporal decision of individual entrepreneurs. I close this part by characterizing the aggregate transition dynamics.

Corollary 3. (Aggregate Transition Dynamics)

$$\begin{aligned} K_t &= \beta \cdot [\gamma \cdot Y_t + (1-\delta) \cdot K_t]. \\ G_{t+1}(a) &= \int G_t \left(\frac{a}{\beta \cdot \Psi_t(\varphi)} \right) \cdot dF_t(\varphi). \end{aligned}$$

The evolution of aggregate capital stock behaves like a Solow model in which output is subject to a tax rate $(1-\gamma)$ and the saving rate is constant. On the one hand, the Cobb-Douglas matching function in all sub-labor markets suggests a fixed split of output between entrepreneurs and workers. Since I assume workers cannot have access to the credit market, only entrepreneurs make intertemporal decisions. On the other hand, I use log-utility, which exactly cancels income and substitution effects and implies a fixed saving rate.

3.2 The Unemployment Effect of Credit Imperfections

The key theoretical contribution of this paper is to show that a credit crunch, *i.e.*, a decrease in λ, lowers aggregate matching efficiency. I use this section to present the details of this new transmission mechanism. As shown in the proof of Proposition 2, equilibrium employment can be formulated as

$$N = \mathbb{E}_F \left[q(\varphi) | \varphi \geq \widehat{\varphi} \right] \cdot K, \tag{3.3}$$

where K denotes the aggregate capital supply and $q(\varphi)$ the job-filling rate in sub-labor market φ. In turn I obtain the employment effect of credit imperfections as below.

$$\frac{\partial N}{\partial \lambda} = \left\{ \left(\frac{\partial \mathbb{E}_F \left[q(\varphi) | \varphi \geq \widehat{\varphi} \right]}{\partial \widehat{\varphi}} \right) \cdot \left(\frac{\partial \widehat{\varphi}}{\partial \lambda} \right) + \mathbb{E}_F \left[\frac{\partial q(\varphi)}{\partial \lambda} | \varphi \geq \widehat{\varphi} \right] \right\} \cdot K \geq 0 \tag{3.4}$$

First, the increase of λ drives up the interest rate r and thus the cut-off value $\widehat{\varphi}$. Then more capital is redistributed from low-productivity to high-productivity entrepreneurs. As proved in Section 2.2, the entrepreneur's job-filling rate $q(\varphi)$ increases with φ. Therefore, the direct effect, which is shown in the first item of the right hand of Equation (3.4), is that the employment increases. I call it the *selection effect*. However, holding everything else unchanged, when more capital is concentrated in the hands of high-productivity entrepreneurs, the job-filling rate of all active entrepreneurs tends to decrease. Although labor supply responds to the increase of λ, the concavity of the matching function suggests $l(\varphi)$ does not change as much as $v(\varphi)$ and thus $q(\varphi)$ decreases with λ in general equilibrium. This can be verified from the above proposition. This indirect general equilibrium effect is labeled as the *congestion effect*, which is shown in the second item of the right hand of Equation (3.4). As proved in Appendix D, the selection effect dominates the congestion effect under Assumption 1.

As suggested by Equation (3.1), productivity heterogeneity with an incomplete credit market does matter for matching efficiency in the labor market. Such an effect cannot be obtained in a standard framework with a representative firm and worker. For example, the seminal work by Wasmer and Weil (2004) introduces credit frictions into an otherwise standard Diamond-Mortensen-Pissarides model. They model credit frictions with a matching function between a representative firm and bank. When credit frictions worsen, which could be driven by the decrease of matching efficiency between firms and banks, this affects equilibrium unemployment

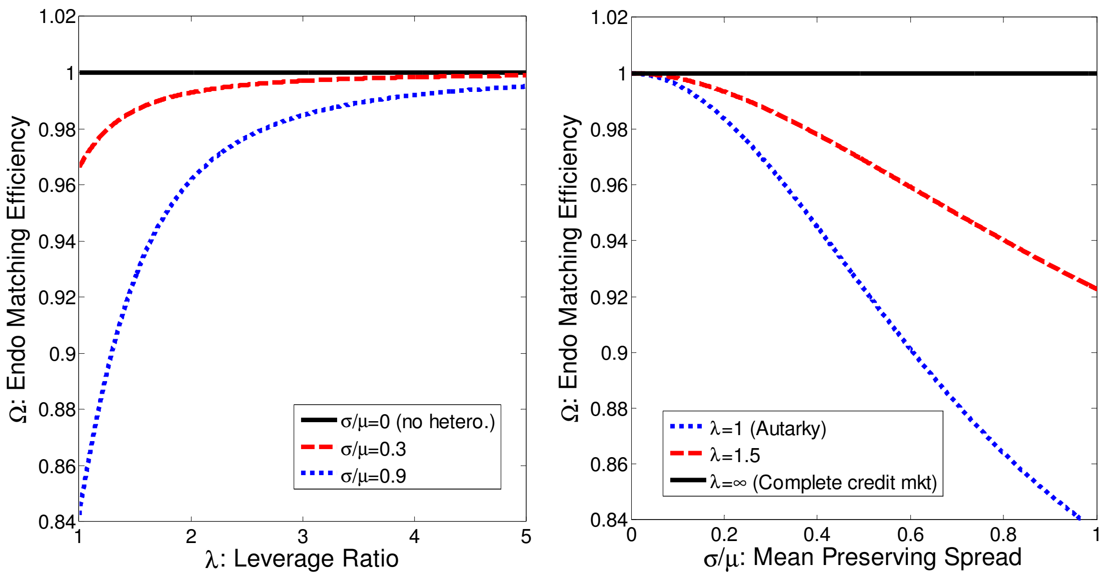


Figure 3.2: **Left Panel:** (Ω, λ)**; Right Panel:** $\left(\Omega, \frac{\sigma}{\mu}\right)$**;** $\varphi \overset{U}{\sim} [\mu - \sigma, \mu + \sigma]$.

in the steady state. However, unlike our heterogeneity model, the Beveridge curve in their work does not move with such kind of disruption in the credit market.

Finally, endogenous matching efficiency contributed by credit imperfections, Ω, is affected not only by λ, but also by the productivity distribution. Given any distribution $F(\cdot)$, I have shown Ω increases with λ. I close this section by addressing the implications of an MPS (mean preserving spread) of $F(\cdot)$ for Ω. The general discussion is beyond the scope of this paper. I instead use a special case to illustrate the idea by assuming $F(\cdot)$ is a Uniform distribution with support $[\mu - \sigma, \mu + \sigma]$ and $\sigma \in [0, \mu]$. I use a uniform distribution since it is a perfect candidate to perform MPS. More specifically, given any λ, I can check the effect of $\frac{\sigma}{\mu}$ on Ω. The right panel of Figure (3.2) implies an MPS increases unemployment. Our exercise with MPS is related to the recent literature on the relationship between adverse selection and output fluctuation; see Kurlat (2012) and Bigio (2013), among others. I show that an MPS depresses the output. There are mainly two key differences. First, information asymmetry is indispensable in their work while I perform the MPS using complete information. Second, they assume a frictionless labor market while I assume labor search frictions and an MPS drives up unemployment.

3.3 Unemployment Decomposition

Motivated by the channel through which credit imperfections affect the labor market, I make a theoretical decomposition for unemployment in this section. In particular, I explore how much credit imperfections and the classic labor search frictions add to unemployment.

Steady State

Using Corollary 3 reaches steady-state unemployment as below.

$$u_{ss} \quad = \quad 1 - \left[\Omega\left(\lambda_{ss}\right) \cdot \eta_{ss}\right]^{\frac{1}{1-\gamma}} \cdot \left[\frac{\gamma \cdot ALP\left(\lambda_{ss}, z_{ss}\right)}{1/\beta - 1 + \delta}\right]^{\frac{\gamma}{1-\gamma}}, \tag{3.5}$$

As indicated by Equation (3.5), the credit friction λ plays two roles in determining unemployment in the steady state. On the one hand, the increase of λ contributes to a higher TFP, which in turn suggests a higher capital stock in the steady state. Therefore, unemployment tends to decrease. On the other hand, given any level of capital stock, the endogenous matching efficiency would also increase with λ and thus lower unemployment. In the end, I reach the general equilibrium effect of credit imperfections on unemployment in the steady state as below.

$$\left(u_{ss} - u_{ss}^*\right) = \left(u_{ss} - \widetilde{u}\right) + \left(\widetilde{u} - u_{ss}^*\right),$$

where u_{ss} and u_{ss}^* denote, respectively, the steady state unemployment with a steady state λ and with a "high enough" λ. The difference between u_{ss} and u_{ss}^* is defined as unemployment contributed by credit imperfections in the steady state. Furthermore, $\widetilde{u}$ is denoted as unemployment implied by a higher λ, but the matching efficiency is held constant. That is, $\widetilde{u}$ is the steady state unemployment with a higher capital stock implied by an improvement of capital reallocation, but the efficiency of labor reallocation is held unchanged. I have formulated u_{ss} in Equation (3.5). In turn, u_{ss}^* and $\widetilde{u}$ are given as below.

$$\begin{aligned} u_{ss}^* &\equiv 1 - \left[\Omega\left(\lambda^*\right) \cdot \eta_{ss}\right]^{\frac{1}{1-\gamma}} \cdot \left[\frac{\gamma \cdot ALP\left(\lambda^*, z_{ss}\right)}{1/\beta - 1 + \delta}\right]^{\frac{\gamma}{1-\gamma}} \\ \widetilde{u} &\equiv 1 - \left[\Omega\left(\lambda_{ss}\right) \cdot \eta_{ss}\right]^{\frac{1}{1-\gamma}} \cdot \left[\frac{\gamma \cdot ALP\left(\lambda^*, z_{ss}\right)}{1/\beta - 1 + \delta}\right]^{\frac{\gamma}{1-\gamma}}, \end{aligned}$$

where λ^* denotes a "high" financial development. I have two alternative candidates for λ^*, one is ∞ while the other one is $max\left\{\lambda_t\right\}$. The former is mainly of theoretical interest. As proved in Proposition 2, endogenous matching efficiency by credit imperfections would converge to the maximum level when λ^* approaches infinity. The latter is instead used for the quantitative analysis presented below.

Non-Steady State

In every period I have $u = 1 - \Omega(\lambda) \cdot m\left(\frac{K}{L}, 1\right)$. That is, the total matching efficiency is the product of that contributed by financial friction and that by labor search frictions, *i.e.*, $\widehat{\eta} = \Omega(\lambda) \cdot \eta$. Then I have

$$u \quad = \quad u^{**} + \left(\lim_{\lambda \to \lambda^*} u - u^{**}\right) + \left(u - \lim_{\lambda \to \lambda^*} u\right) \equiv u^{**} + u^{\eta} + u^{\lambda},$$

where $u^{**} = max\left\{1 - \frac{K}{L}, 0\right\}$ and $\lim_{\lambda \to \lambda^*} u = 1 - \Omega\left(\lambda^*\right) \cdot m\left(\frac{K}{L}, 1\right)$ denote, respectively, the efficient unemployment and the unemployment without credit imperfections. First, data on $\frac{K}{L}$ suggest $u^{**} = 0$. Then I break down unemployment into two parts: one is due to the classic search friction while the other is due to credit imperfections. I denote them as u^{η} and u^{λ}, respectively. In turn, I define the explanatory power of credit imperfections on unemployment as $\chi \equiv \frac{u^{\lambda}}{u}$. Given K, aggregate productivity shock z does not directly affect unemployment since z has nothing to do with the equilibrium aggregate matching efficiency. Therefore, the decomposition exercise does not involve z. However, z exerts a dynamic effect on unemployment because aggregate productivity shock plays a role in equilibrium TFP, which in turn influences the speed of capital accumulation.

Finally, I get that $\frac{\partial \chi}{\partial \lambda} < 0$, $\frac{\partial \chi}{\partial \eta} > 0$, $\frac{\partial^2 \chi}{\partial \lambda \partial \eta} < 0$ and $\frac{\partial \chi}{\partial \lambda^*} > 0$. The increase of λ suggests an amelioration of capital misallocation, and thus the role of credit imperfections in explaining unemployment decreases. As a duality, I have $\frac{\partial(1-\chi)}{\partial \eta} < 0$, which immediately translates into $\frac{\partial \chi}{\partial \eta} > 0$. Furthermore, there exists an interaction effect. These properties turn out to be helpful in interpreting the results, mainly Figure (4.6), in Section 4.3. I illustrate the key results on χ in Figure (3.3).

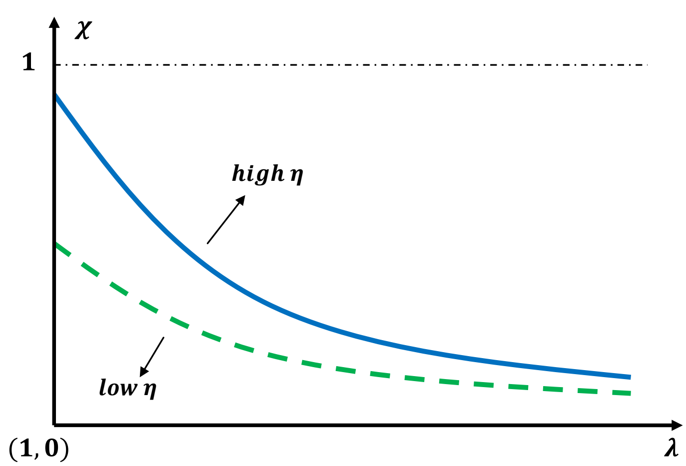


Figure 3.3: **Explanatory Power of Credit Imperfections for Unemployment**

3.4 The Relationship to A Model with Only Credit Frictions

I have finished the specification and characterization of our model with credit and labor-search frictions. Comparing the equilibrium of the decentralized economy with the benchmark with only search frictions delivers Proposition 2. I address the wedges to productivity, employment, average labor productivity, unemployment rate and factor prices there.

I use this section to propose an alternative benchmark in which there is no labor search friction but just credit friction. More specifically, I compare our model with Moll (2013). I establish the connection as below.

Proposition 3. (Comparison with a Model with Only Credit Frictions) The heterogeneous-entrepreneurs model with both search frictions in the labor market with matching function $m(l(\varphi), v(\varphi)) = \eta \cdot v(\varphi)^{\gamma} l(\varphi)^{1-\gamma}$, and credit frictions in the form of a collateral constraint $k \leq \lambda \cdot a$ delivers the same output aggregation and transition dynamics on $F(\varphi)$, $G(a)$ and K with the model with the following characteristics:

1. The production function by entrepreneur-(a, φ) is $y(\varphi, a) = \varphi \cdot m(k(\varphi, a), l(\varphi, a))$.

2. The labor market is frictionless in each period, *i.e.*, there exists a unique and publicly displayed wage w such that labor supply equals demand. Unemployment is then zero by definition.

3. The credit market is subject to a collateral constraint, *i.e.*, $k \leq \lambda \cdot a$.

Moreover, I use the model with only financial friction to recover unemployment in the model with dual frictions as $u = 1 - \frac{Y}{L \cdot \mathbb{E}_{F_M}(\varphi)}$.

The key message from this proposition is that, our model with two layers of frictions behaves as if there exist only credit imperfections, and the Leontief production function is replaced by the Cobb-Douglas. On the other hand, I interpret the heterogeneous model with only credit frictions as a model with both credit and labor search frictions, and the Cobb-Douglas production function is decomposed into a Leontief production and an associated Cobb-Douglas matching function.

3.5 Job Destruction and Firm Growth

I have assumed throughout the paper that all matched relationships between firms and workers are terminated after production. This assumption greatly simplifies our analysis since entrepreneurs are heterogeneous in only two dimensions. The associated cost is that, the ratio of job destruction to total employment is 100% at the end of each period. To partially fix this problem, I redefine job destruction in terms of net flow.

$$N_{t+1} = N_t - JD_{t+1} + JC_{t+1}, \tag{3.6}$$

where

$$\begin{aligned} N_t &\equiv \int\int \widetilde{l}_t h_t(\varphi, a) d\varphi da = \Omega(\lambda_t) \cdot m_t(K_t, L_t) \\ JD_{t+1} &\equiv \int\int max\left\{\widetilde{l}_t - \widetilde{l}_{t+1},\, 0\right\} h_t(\varphi, a) d\varphi da \\ JC_{t+1} &\equiv \int\int max\left\{\widetilde{l}_{t+1} - \widetilde{l}_t,\, 0\right\} h_t(\varphi, a) d\varphi da \end{aligned}$$

and $\widetilde{l}_t$ denotes the finally matched workers, and $h(\varphi, a)$ the joint distribution of productivity and net worth. Given N_t,N_{t+1} and JD_{t+1}, I can then calculate JC_{t+1} from Equation (3.6). Moreover, in each period, given the aggregate state variable X_t, I can pin down N_t. Therefore, it remains for us to characterize JD_{t+1}. I summarize the key results below.

Corollary 4. (Job Creation and Destruction) In each period, job destruction can be formulated as

$$JD_{t+1} = \left[\int max\left(\Delta_{t,t+1}(\varphi_t),\, 0\right) \cdot dF(\varphi_t)\right] \cdot K_{t+1}.$$

where

$$\Delta_{t,t+1}(\varphi_t) \equiv \lambda_t \cdot \mathbf{1}_{\{\varphi_t \geq \widehat{\varphi}_t\}} q_t(\varphi_t) - \lambda_{t+1}\beta\Psi_t(\varphi_t) \cdot \left[\rho \cdot \mathbf{1}_{\{\varphi_t \geq \widehat{\varphi}_{t+1}\}} q_{t+1}(\varphi_t) + (1-\rho) \cdot \int_\Phi \mathbf{1}_{\{\varphi \geq \widehat{\varphi}_{t+1}\}} q_{t+1}(\varphi) \cdot dF(\varphi)\right].$$

Using the corollary immediately suggests that job creation and job destruction in the steady state is

$$JD = JC = \left[\int_{\widehat{\varphi}}^{\overline{\varphi}} max\left\{1 - \beta\Psi(\varphi) \cdot \left[\rho \cdot q(\varphi) + (1-\rho) \cdot \left(\frac{N}{\lambda K}\right)\right],\, 0\right\} dF(\varphi)\right] \cdot \lambda K.$$

where K and N in the steady state are

$$\begin{aligned} K &= \left[\frac{\gamma \cdot TFP(\lambda, \eta, z)}{1/\beta - (1-\delta)}\right]^{\frac{1}{1-\gamma}} \cdot L \\ N &= \Omega(\lambda) \cdot m(K, L). \end{aligned}$$

I close the theoretical section with a discussion on the implications of this model for firm-level growth.

Corollary 5. (Firm Size and Growth Rate)

1. The firm size of entrepreneur-(a, φ), measured by capital holding k and employee numbers n, are

$$\begin{aligned} k(a,\varphi) &= \lambda a \cdot \mathbf{1}_{\{\varphi \geq \widehat{\varphi}\}} \\ n(a,\varphi) &= \lambda q(\varphi) a \cdot \mathbf{1}_{\{\varphi \geq \widehat{\varphi}\}}. \end{aligned}$$

2. The growth rate of capital and employment is,

$$\begin{aligned} \mathbb{E}\left[\frac{k_{t+1}}{k_t}|(k_t,\varphi_t;X_t)\right] &= \beta \cdot \left(\frac{\Psi_t(\varphi_t)}{\lambda_t}\right) \cdot \mathbb{E}\left\{\left[\rho \cdot \mathbf{1}_{\{\varphi_t \geq \widehat{\varphi}_{t+1}\}} + (1-\rho)\cdot\left(1-F\left(\widehat{\varphi}_{t+1}\right)\right)\right] \cdot \lambda_{t+1}|(\varphi_t,X_t)\right\} \\ \mathbb{E}\left[\frac{n_{t+1}}{n_t}|(n_t,\varphi_t;X_t)\right] &= \beta \cdot \Psi_t(\varphi_t)\left(\frac{\lambda_{t+1}}{\lambda_t}\right)\left[\rho \cdot \left(\frac{q_{t+1}(\varphi_t)}{q_t(\varphi_t)}\right) + (1-\rho)\cdot\left(\frac{\int_{\widehat{\varphi}_{t+1}}^{\overline{\varphi}} q(\varphi_{t+1})dF(\varphi_{t+1})}{q_t(\varphi_t)}\right)\right]. \end{aligned}$$

Our model predicts that the growth rate has nothing to do with capital or employment itself. This is purely because of the linearity of the policy function of individual entrepreneurs. Moreover, the heterogeneous growth rate connects our paper with the empirical and theoretical works on the volatility of firm growth rate, such as that by Wang and Wen (2013).

4 Quantitative Analysis

In this section I confront the model with data by quantifying the unemployment effect of capital misallocation due to credit imperfections. I calibrate the model on US economy and then recover the realization of three pieces of aggregate shocks. Then I estimate the stochastic process of these three shocks and do the impulse response exercise. Furthermore, I decompose unemployment into two parts, one which is due to credit imperfections and the other which is due to the classic labor search frictions. Unemployment decomposition is considered not only for the business cycles between 1951Q4 and 2011Q4, but also for the recent financial crisis. Finally, I discuss which shocks are most essential in terms of their aggregate and disaggregate implications.

4.1 Calibration

Data are of quarterly frequency. Details on the criterion of data use, data description and calculation are documented in Appendix A. The date horizon ranges between 1951Q4 to 2011Q4.[25]

Calibration

Assume the individual productivity component follows a Power distribution, *i.e.*, $F(\varphi) = \varphi^{\frac{1}{\varepsilon}}$ with the support $[\underline{\varphi}, \overline{\varphi}] = [0, 1]$ and $\varepsilon > 0$. Notice that the lower bound is truncated at zero since productivity is non-negative by definition. Besides, I normalize the upper bound by one.[26] The empirical literature reveals the probability density of the function of the productivity distribution decreases at the right tail. Therefore, I should have $\frac{1}{\varepsilon} < 1$. The calibration in the following Table (1) confirms the empirical regularity.

Parameter	**Note**
$\beta = 0.99$	discount factor
$\delta = 0.025$	depreciation rate
$[\underline{\varphi}, \overline{\varphi}] = [0, 1]$	support of the Power distribution
$\gamma = 0.28$	matching elasticity *and* revenue share of capital
$\eta = 0.61$	matching efficiency of labor market
$\lambda = 1.4$	collateral constraint
$\rho = 0.91$	persistence of individual productivity
$z = 6.4$	aggregate productivity
$\varepsilon = 1.68$	parameter in the Power distribution

Table 1: **Calibration**

I target the annual interest rate as 4% by setting the quarterly discount factor as $\beta = 0.99$. As suggested in Bigio (2013), the combination of $\beta = 0.99$ and log utility function are quantitatively similar to that of $\beta = 0.97$ and the coefficient of risk aversion being 2. Therefore, I use $\beta = 0.97$ throughout the quantitative analysis. As standard in the literature, I set the depreciation rate as 2.5%. TFP is defined as $Y/\left(K^{\gamma} \cdot L^{1-\gamma}\right)$. In turn, TFP_{ss} is a function of γ.

[25] The data start with 1951Q4 since it is the earliest date in the Flow of Funds Account such that the data on external funding on non-financial assets are available. The date ends with 2011Q4 since this is the last date in which the quarterly data on non-financial private investment are obtainable. Although the annual data on capital are available until 2012, I have to use both the annual data on capital and the quarterly data on investment to recover the quarterly data on capital.

[26] In general, I can specify the distribution as $F(\varphi) = (\varphi/\overline{\varphi})^{\frac{1}{\varepsilon}}$. In this case, I can show that $\widehat{\varphi}$ is homogeneous of degree one with respect to $\overline{\varphi}$. Therefore, without loss of generality, I can normalize $\overline{\varphi} \equiv 1$.

Moreover, as shown in Section 3.3, the capital per capita in the steady state is $\left(\frac{\gamma \cdot TFP_{ss}}{1/\beta - 1 + \delta}\right)^{\frac{1}{1-\gamma}}$, which is also related to γ. I therefore recover $\gamma = 0.281$ from the average capital per capita. On the one hand, γ denotes the elasticity of the matching function, which was estimated between 0.28 by Shimer (2005) and 0.5 by Petrongolo and Pissarides (2001). On the other hand, γ stands for the revenue share of capital, which was set as 0.25 by Cui (2013), 0.28 by Gomme and Rupert (2007), and 0.33 by Buera, Fattal-Jaef and Shin (2013), among others. Our estimation $\gamma = 0.28$ falls into the intervals of values used by previous research.

Using data from JOLTS, I construct quarterly data on job-filling rates and market tightness. I follow Michaillat (2012) to use OLS to estimate η. I get the steady state λ by averaging the ratio of external funding over non-financial assets and using the relationship that $\frac{D}{K} = 1 - \frac{1}{\lambda}$. In turn, I obtain ρ, the persistence coefficient of individual productivity, by using 10%, the annual exit rate of establishment as below. On the one hand, by definition I have $1 - \{\rho + (1-\rho) \cdot [1 - F(\widehat{\varphi})]\}^4 = 0.1$. On the other hand, the clearing condition in the credit market suggests $\lambda \cdot [1 - F(\widehat{\varphi})] = 1$. Therefore, I get ρ as $\rho = \left(0.9^{\frac{1}{4}} - \frac{1}{\lambda}\right) / \left(1 - \frac{1}{\lambda}\right) = 0.91$. Alternatively, I can use $\frac{JD}{N}$, the ratio of job destruction to total employment, to pin down ρ. The data on JD and N are available from the Bureau of Labor Statistics, which suggest $\frac{JD}{N} = 1.5\%$ on average. In turn, I have $\rho = 0.99$.[27]

When it comes to the estimation on the distribution parameter ε, I can first construct the series of average labor productivity (ALP) by dividing the output by the employment. Additionally, using the theoretical results in Section 3 suggests that $\Omega = TFP / (ALP \cdot \eta)$, which delivers the steady state of matching efficiency by credit imperfections. Meanwhile, notice that Ω is a function of the distribution parameter ε and that of the steady-state λ. Thus I recover $\varepsilon = 1.684$, which suggests that $1/\varepsilon < 1$. Therefore, the pdf decreases with productivity and thus is in line with empirical regularity. Finally, I reach the steady-state value of z from TFP.

Backing Out Shocks

There are three aggregate shocks in our model, $\{\lambda_t, \eta_t, z_t\}$, which are not directly observable from the data. I recover these shocks from certain observable time series by following Michaillat (2012). On the one hand, I have data on i) $\frac{D_t}{K_t}$, the external funding over non-financial assets, ii) ALP_t, the average labor productivity, and iii) TFP_t. On the other hand, all three variables of interest are functions of $\{\lambda_t, \eta_t, z_t\}$ in our model, *i.e.*,

[27] Since the entrepreneur may lose her productivity with probability $(1-\rho)$ and then may redraw a very low productivity, she may stop hiring then. Consequently, $\frac{JD}{N}$ is a function of ρ in steady state.

$$
\begin{aligned}
\frac{D}{K} &\equiv \frac{\int_0^\infty \int_{\underline{\varphi}}^{\overline{\varphi}} max\left\{k(\varphi,a)-a,\,0\right\} h(\varphi,a)d\varphi da}{K} = \left(\frac{D}{K}\right)(z,\lambda,\eta) \\
ALP &\equiv \frac{Y}{N} = ALP(z,\lambda,\eta) \\
TFP &\equiv \frac{Y}{K^\gamma L^{1-\gamma}} = TFP(z,\lambda,\eta).
\end{aligned}
$$

Furthermore, I can verify the diagonal property holds such that i) $\frac{D}{K} = \left(\frac{D}{K}\right)(\lambda)$, ii) $ALP = ALP(z,\lambda)$, and iii) $TFP = TFP(\eta,z,\lambda)$.[28] Therefore, in each period I can first use $\left(\frac{D}{K}\right)_t$ to recover λ_t. Then z_t could be inferred by jointly using ALP_t and the already recovered λ_t. Finally, I use TFP_t alongside the pairwise estimated value on (λ_t, z_t) to retrieve η_t. In turn, I obtain their corresponding HP filter in Figure (4.1).

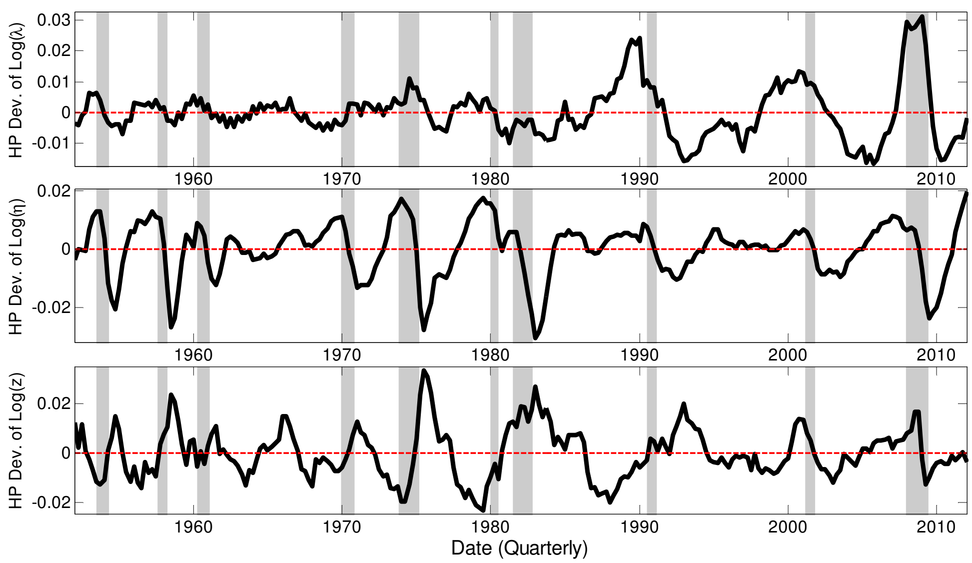


Figure 4.1: **HP Deviation of Three Shocks**

All these shocks are procyclical in general.[29] First, all three aggregate shocks were significantly negative in the recent financial crisis, especially for λ, the shock to the credit market. Moreover, both λ and η decreases in recessions over the cycles. Notice that I have adopted in a reduced-form way to model the credit and labor search frictions. On the one hand, as discussed in Section 2.3, the decrease of λ in recessions may originate from a worsening condition in adverse selection, moral hazard, costly state verification or limited enforcement. On

[28]Notice that the ratio of external funding to non-financial asset can be simplified as $\frac{D}{K} = 1 - \frac{1}{\lambda}$.

[29]The correlation between output and (λ,η,z), after HP filtering, is 0.44, 0.64 and 0.21, respectively. Besides, after HP filtering, $corr\,(\lambda,\eta) = 0.21$, $corr\,(\lambda,z) = -0.16$ and $corr\,(\eta,z) = -0.52$.

the other hand, the negative shock to η may be due to the decrease of aggregate matching efficiency, which is in turn caused by some sector-specific shocks, as shown by Mehrotra and Sergeyev (2012). Alternatively, the decrease in η may stem from the job polarization proposed by Jaimovich and Siu (2013). The results are mixed when it comes to the cyclicality of z, aggregate productivity shock. The shocks to z were also negative in the past three recessions, but just opposite for the previous recessions. Our quantitative exercise is in line with their findings. It is worth noting that, although aggregate productivity z increased in some recessions, it is not necessarily true that equilibrium TFP also increased correspondingly. As shown in Figure (4.2), TFP is procyclical with a correlation of 0.91 with the output.[30]

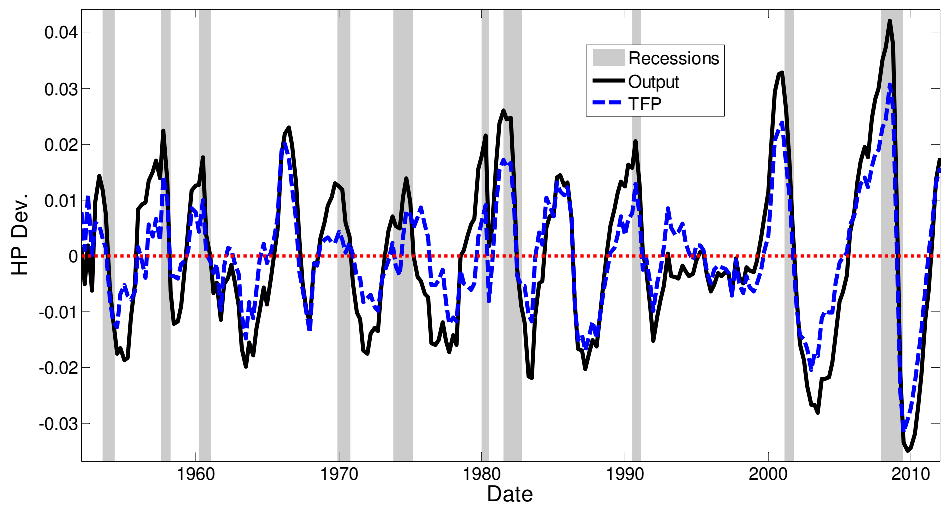


Figure 4.2: **HP Deviation of TFP**

4.2 Impulse Response Exercise and Jobless Recovery

Now I investigate the implications of the aggregate shocks for output and unemployment. I also address their effects on unemployment decomposition.

Impulse Response without Correlation or Persistence

I assume these three shocks decrease 1%, but with no persistence or correlation. I summarize the impulse response of output and unemployment in the first row of each panel in Figure (4.3).

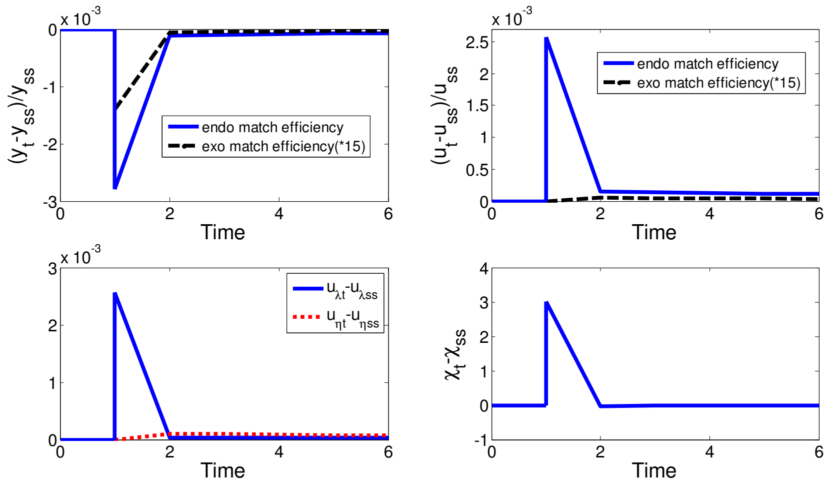
[30]There is seemingly no consensus on the movement of TFP for the recent recession. Petrosky-Nadeau (2012) proposes a model to explain why TFP increased in this recession. However, as shown in our calculation, TFP, along with the output, suffered a significant decrease in the past financial recession. This may be due to different measurement methods.

On the one hand, the path of output driven by different shocks shares a similar pattern. On the other hand, the implications of the shocks are different when it comes to unemployment. The credit and the labor market shocks exerts a large and immediate response for unemployment. On the contrary, aggregate productivity affects unemployment one period later and generates a relatively slow recovery.

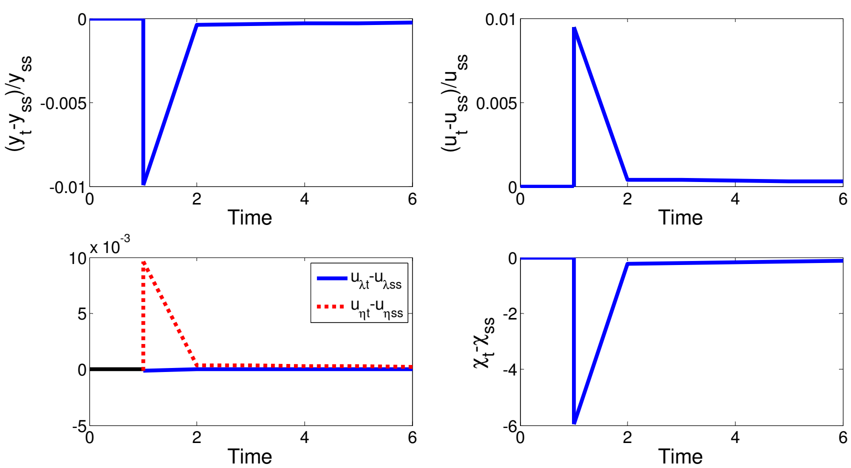
Since I mainly focus on the connection between credit and labor markets, I devote more analysis in this line. As illustrated in Section 3.2, credit imperfections lowers aggregate matching efficiency. In the upper panel of Figure (4.3), I compare the effect of credit crunches in two scenarios, one with endogenous matching efficiency and the other with exogenous matching efficiency. As shown in the upper panel, endogenous matching efficiency due to credit imperfections amplifies the effect of a credit crunch for output as well as for unemployment. Moreover, the amplification is quantitatively important.

Now I address the implications of the shocks for unemployment decomposition $\left(u^{\lambda}, u^{\eta}\right)$ in the second row of each panel in Figure (4.3).[31] First, although credit and labor shocks have similarity in their effect on output and unemployment, their predictions on χ, the explanatory power of credit imperfections on unemployment, are opposite. These results corroborate the theoretical predictions in Section 3.3, *i.e.*, $\frac{\partial \chi}{\partial \lambda} < 0$ and $\frac{\partial \chi}{\partial \eta} > 0$. On the one hand, a negative credit shock, *i.e.*, a credit crunch, boosts the importance of credit imperfections in explaining the associated increasing unemployment. On the other hand, a negative labor shock, *i.e.*, the matching efficiency decreases, puts more emphasis on the responsibility of the labor market itself in worsening unemployment. Moreover, these two shocks exert a qualitatively asymmetric effect on $\left(u^{\lambda}, u^{\eta}\right)$. The former increases both unemployment compositions while the latter increases u^{η} while decreases with u^{λ}. Finally, as predicted by Section 3.4, aggregate productivity shock does not directly affect unemployment in the current period. It works through capital accumulation and thus affects unemployment in the next period. The decrease of capital stock in turn attenuates the importance of credit imperfects while increases the importance of labor search frictions in explaining the rising unemployment. However, relative to the previously two shocks, aggregate productivity shock does not affect χ significantly when it comes to the quantitative concern.

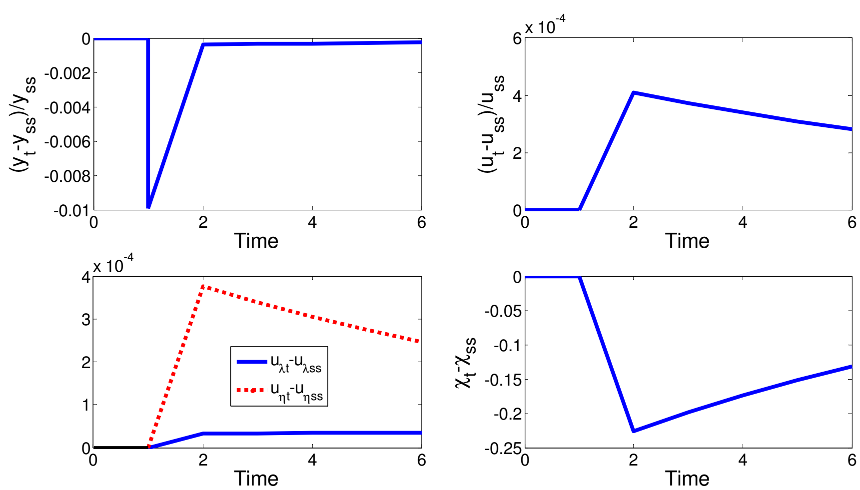
[31] The theory formulated in Section 3.3 proposes an unemployment decomposition, *i.e.*, $u = u^{\eta} + u^{\lambda}$, where $u^{\eta} \equiv \lim_{\lambda \to max\{\lambda_t\}} u$ and u^{λ} denotes unemployment contributed by the labor search frictions and credit imperfections respectively. In turn, I define the explanatory power by credit imperfections to unemployment as $\chi \equiv u^{\lambda}/u$.



(a) λ-shock



(b) η-shock



(c) z-shock

Figure 4.3: **Impulse Response of Shocks (1%) without Correlation or Persistence**

VAR Estimation and Jobless Recovery

I have so far demonstrated the effects of the various shocks. Using the HP deviations of these three shocks delivers the estimation of the VAR process on (λ_t, η_t, z_t). I document the estimated coefficient and variance matrix as below,

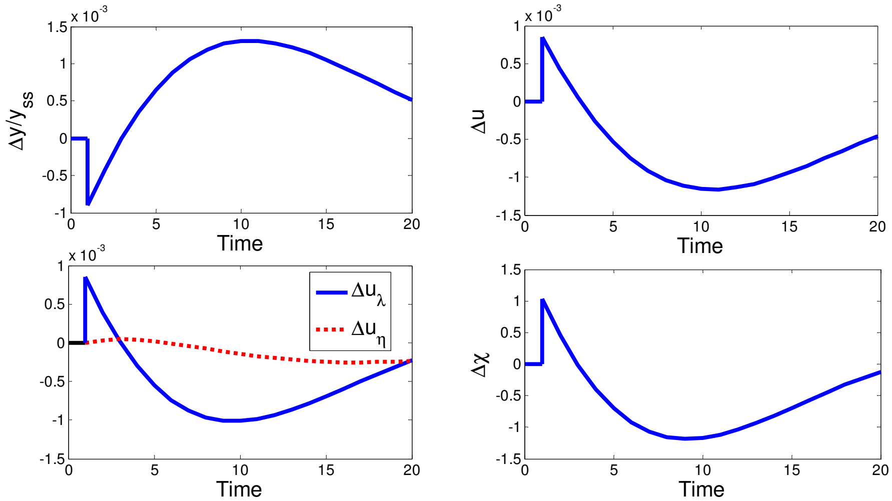
$$\begin{pmatrix} \beta_{\lambda\lambda} & \beta_{\lambda\eta} & \beta_{\lambda z} \\ \beta_{\eta\lambda} & \beta_{\eta\eta} & \beta_{\eta z} \\ \beta_{z\lambda} & \beta_{z\eta} & \beta_{zz} \end{pmatrix} = \begin{pmatrix} 0.894 & 0.147 & 0.054 \\ -0.120 & 0.905 & -0.004 \\ -0.009 & -0.003 & 0.865 \end{pmatrix}, \ \Sigma = \begin{pmatrix} \sigma_\lambda^2 & \rho_{\lambda z}\cdot\sigma_\lambda\sigma_z & \rho_{\lambda\eta}\cdot\sigma_\lambda\sigma_\eta \\ \rho_{\lambda z}\cdot\sigma_\lambda\sigma_z & \sigma_z^2 & \rho_{z\eta}\cdot\sigma_z\sigma_\eta \\ \rho_{\lambda\eta}\cdot\sigma_\lambda\sigma_\eta & \rho_{z\eta}\cdot\sigma_z\sigma_\eta & \sigma_\eta^2 \end{pmatrix},$$

where β_{ij} denotes the effect of shock-j on shock-i, and

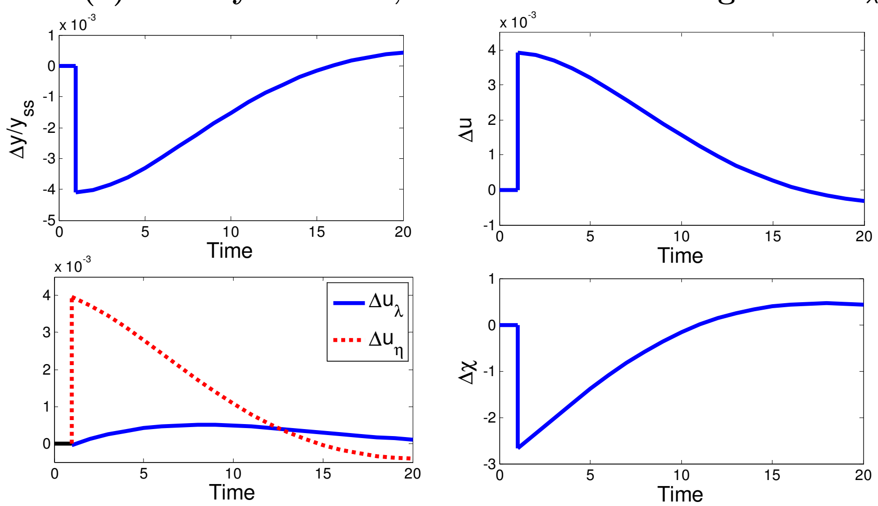
$\sigma_\lambda = 0.0032$	$\sigma_\eta = 0.0041$	$\sigma_z = 0.0051$
$\rho_{\lambda z} = -0.0644$	$\rho_{\lambda\eta} = 0.0899$	$\rho_{z\eta} = -0.4322$

Based on $\{\beta_{ij}\}$ and Σ, I revisit the impulse response exercise with correlations on the shocks. In each exercise, the level of the initial shock is set as σ_i with $i \in \{\lambda, \eta, z\}$. Then the shocks proceed with the VAR matrix $\{\beta_{ij}\}$. I document the key results in Figure (4.4). As illustrated in the upper and the middle panel, the shocks to the credit and labor markets generates co-movement of the recovery on output and employment. Therefore $\left(\frac{y_t - y_{ss}}{y_{ss}}, u_t - u_{ss}\right)$ is characterized with a perfectly negatively relationship in the associated figures. In contrast, the shock to aggregate productivity produces a gap (4 quarters) between the recovery of output and that of employment, which is demonstrated in the lower panel.

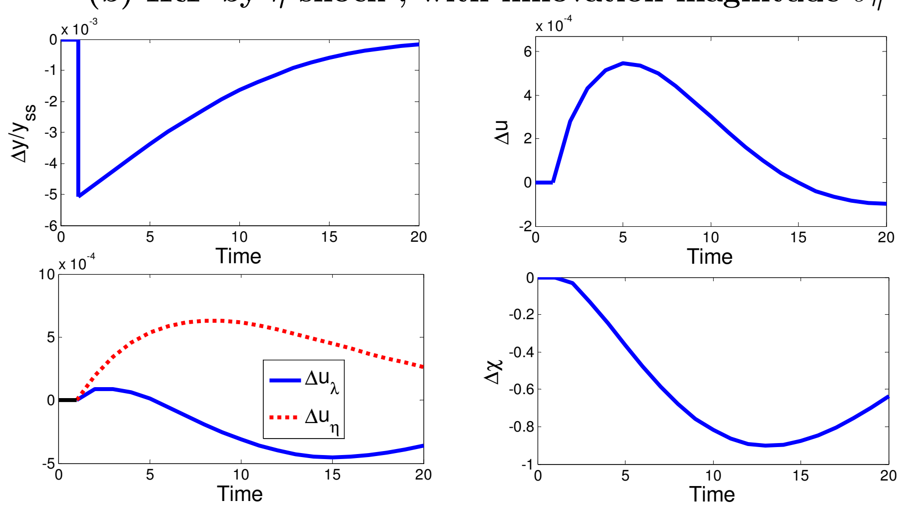
When it comes to recessions before the 1990s, the recovery on output and employment/unemployment occurred at the same pace. However, as shown in Figure (4.5), there exist gaps between output and employment/unemployment recovery in the past three recessions. It is typically labeled as jobless or sluggish recovery in the literature. The key message from Figure (4.4) is that the aggregate productivity shock is more likely to generate recovery gaps and thus more responsible for explaining the jobless or sluggish recovery in the recent three recessions. The shocks to the credit and labor markets, on the contrary, synchronize the recovery of output and employment/unemployment, and are in line with recessions prior to the 1990s. The intuition is, λ and η affect TFP and the equilibrium matching efficiency at the same time, which in turn determines output and unemployment. However, z only exerts an influence on TFP, but does not directly relate to the labor market. Consequently, the recovery of z immediately



(a) IRF by λ-shock , with innovation magnitude σ_λ



(b) IRF by η-shock , with innovation magnitude σ_η



(c) IRF by z-shock , with innovation magnitudeσ_z

Figure 4.4: **Impulse Response of Shocks with Correlations**

improves output while it alleviates unemployment only through increasing capital in the next few periods. In turn the z-shock produces a gap between output and employment recovery.

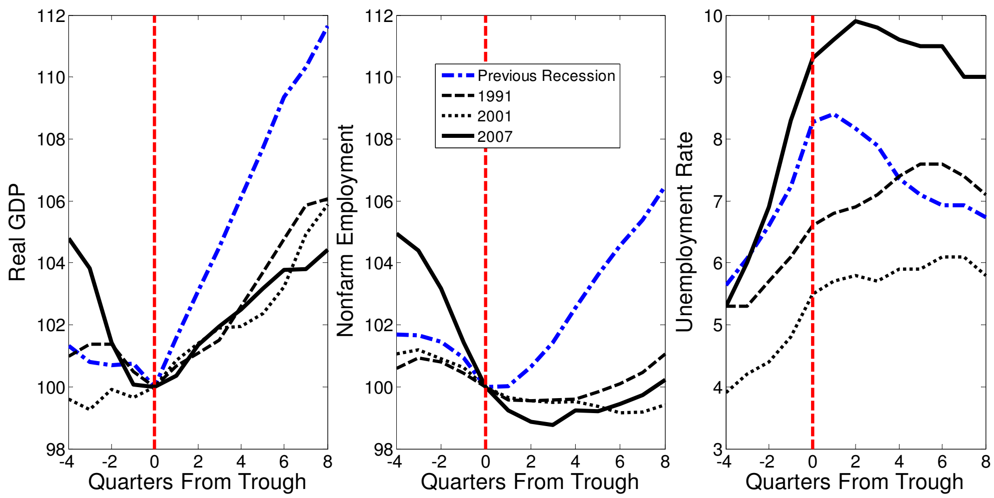


Figure 4.5: **Real GDP, Non-farm Private Employment and Unemployment Rate. Data Source: Bureau of Economic Analysis and Bureau of Labor Statistics**

In the end, taking into account the correlation between shocks still confirms the key insights obtained from the previous exercise on impulse response. That is, the shocks to the credit and labor markets increase and decrease the importance of credit imperfections in explaining a rising unemployment. The aggregate productivity shock exerts a lagged influence on unemployment compositions and helps emphasize the role of labor search frictions themselves in affecting unemployment. The patterns are more non-linear purely because of the correlations and persistence of these shocks.

4.3 Unemployment Decomposition over the Cycles

Now I initiate the unemployment decomposition. This subsection presents an analysis over all cycles between 1951Q4 and 2011Q4. The next part engages in a discussion for the recent financial crisis.

Regressions

In the spirit of Fujita and Ramey (2010) and Hobijn *et al.* (2012), I start our analysis with regressions. On the one hand, I have data on unemployment u_t. On the other hand, I have recovered data on the shocks (λ_t, η_t, z_t). I summarize the results in Table (2).

	u_t	$log\,(u_t)$	$log\left(u_t^{\text{predicted}}\right)$
$log(\lambda_t)$	-0.213^{**}	-4.122^{**}	-5.204^{**}
$log(\eta_t)$	-0.551^{**}	-11.048^{**}	-13.696^{**}
$log(z_t)$	-0.179^{**}	-3.615^{**}	-4.525^{**}
$Adjusted - R^2$	0.901	0.875	0.886
$^{**}:\ p < 0.05,\ \ ^{**}:\ p < 0.01$; constant is controlled.			

Table 2: **OLS Regression of Unemployment on Three Shocks**

The first and second columns use u and $log(u)$ as the dependent variables, respectively. Thus the regression results denote the semi-elasticity and the elasticity of the shocks to unemployment, respectively. First, both regressions suggest a negative relationship between unemployment and the shocks. For example, the 1% increase of λ decreases unemployment by 0.0021, or by 4.1%. Second, the coefficient of η is largest since it directly relates to the matching efficiency in the labor market by definition. The effect of λ is larger than that of z since the former directly affects the matching efficiency while the latter does not. I return to the results in the third column after finishing the decomposition over the cycles in the next section.

Decomposition over the Cycles

The negative correlation between u and λ in Table (2) offers us a quick glimpse of contribution by credit imperfections to unemployment. To sharpen the analysis, I further explore the relationship between χ and the shocks. As already shown in the impulse response, I can recover unemployment compositions $\left\{u_t^{\lambda}, u_t^{\eta}\right\}$ and thus obtain $\chi_t \equiv u_t^{\lambda}/u_t$. In turn I reach Figure (4.6), a scatter plot immediately suggesting a negative and positive relationship between χ and (λ, η) respectively. Moreover, χ is negatively related to z, but not as significantly as with λ. This observation is consistent with the impulse response in the lower panels of Figures (4.3) and (4.4).

Moreover, like the regressions in Table (2), I obtain Table (3) by regressing χ on these shocks. The contribution of credit imperfections to unemployment rate does increase with η while it decreases with λ. This is true and is always statistically significant under all the robustness checks in the table. The findings are in line with the theoretical predictions in Sections 3.2 and 3.3. However, the coefficient of z is not robust, as shown in the third and the fifth columns. What is reassuring is the effect of z on χ is relatively small, which is also consistent with the impulse response exercise in the previous parts.

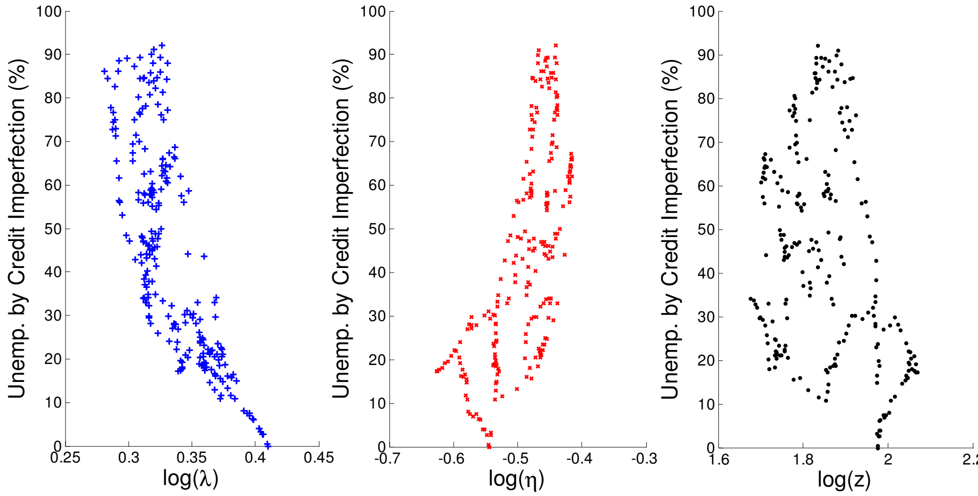


Figure 4.6: **Log-explanation of Credit Imperfections** $log(\chi)$ **Versus Shocks**

In sum, the negative shock to the credit market increases the role of credit imperfections in explaining unemployment. It is just the opposite in the presence of a negative shock to the labor market or to aggregate productivity. Given the back-out series of (λ, η, z), I demonstrate u^{λ} and χ, respectively, in the left and right panels of Figure (4.7). As implied in Figure (4.1), all the shocks are procyclical, in particular for λ and η. Consequently, it could be ambiguous whether χ is procyclical or not. I illustrate this argument in Figure (4.7). In the upper panel, I present the data, the predicted series and the predicted unemployment without credit imperfections. The lower panel in turn documents the series of χ over the cycles and suggests that χ is counter-cyclical with the real data. That is, even though credit crunches tend to increase χ, the simultaneous worsening labor market itself in recessions dampens the competing explanatory power of credit imperfections. Finally, averaging $\{\chi_t\}$ over the cycles suggests credit imperfections and the labor search frictions contribute around 46% and 54%, respectively, to unemployment.

Decomposition in the Steady State

I have so far engaged in unemployment decomposition for each period over the cycles. I now move on to decompose unemployment in the steady state. As a review, I list the theoretical results as below.

$log(\chi_t)$: log-explanation by credit imperfections for unemployment					
$log(\lambda_t)$	-22.46^{**}			-18.25^{**}	-16.27^{**}
$log(\eta_t)$		9.77^{**}		5.24^{**}	10.36^{**}
$log(z_t)$			-2.92^{**}		2.59^{**}
$Adjusted-R^2$	0.710	0.433	0.156	0.810	0.833
$*p<0.05$; $**: \ p<0.01$; constant is controlled					

Table 3: **OLS Regression of** $log\,(\chi_t)$ **on** $\{log\,(\lambda_t)\,, log\,(\eta_t)\,, log\,(z_t)\}$

$$
\begin{aligned}
u_{ss} &= 1-\left[\Omega\left(\lambda_{ss}\right)\cdot\eta_{ss}\right]^{\frac{1}{1-\gamma}}\cdot\left[\frac{\gamma\cdot ALP\left(\lambda_{ss},z_{ss}\right)}{1/\beta-1+\delta}\right]^{\frac{\gamma}{1-\gamma}} \\
u_{ss}^{*} &= 1-\left[\Omega\left(\lambda^{*}\right)\cdot\eta_{ss}\right]^{\frac{1}{1-\gamma}}\cdot\left[\frac{\gamma\cdot ALP\left(\lambda^{*},z_{ss}\right)}{1/\beta-1+\delta}\right]^{\frac{\gamma}{1-\gamma}}
\end{aligned}
$$

Replacing λ_{ss} with $\overline{\lambda}=max\left\{\lambda_t\right\}$ suggests that, the contribution of credit imperfections to unemployment, $\frac{u_{ss}-u_{ss}^{*}}{u_{ss}}$, is 52%. I can further decompose the explanatory power by credit imperfections into the extensive and intensive margins, which are 98% and 2% respectively. Notice the explanatory power in the steady state, 52%, is larger than the average explanatory power over the cycles mentioned above (46%). This is due to the fact that I treat capital as given when calculating χ_t over the cycles while capital accumulation is endogenous and is positively related with λ in the steady state. If I use ∞ rather than $max\left\{\lambda_t\right\}$ as the limit case, then the importance of credit imperfections in unemployment would be higher.

4.4 Unemployment Decomposition in the Recent Financial Crisis

As shown in the left panel of Figure (4.9), the last quarter of 2008 experienced a significant credit crunch. I use our model to address the consequence of the credit crunch for the labor market. More specifically, how much does the credit crunch add to the outward shift in the Beveridge curve in the recent financial crisis? I illustrate the controlled experiment in Figure (4.8). The solid lines in these three panels denote the recovered shocks since the last quarter of 2008. The dashed line in the left panel denotes the counter-factual shock to the credit market, which is held constant at the level just before the credit crunch. Equivalently, I show the controlled shock to the credit market in the left panel of Figure (4.9).

Then I simulate the economy with capital accumulation being endogenous, which is governed by the transition dynamics in Section 3. I summarize the counter-factual dynamics of unemployment in the middle panel of Figure (4.9). The difference between the data and the counter-factual is interpreted as unemployment contributed by the credit crunch. Taking the

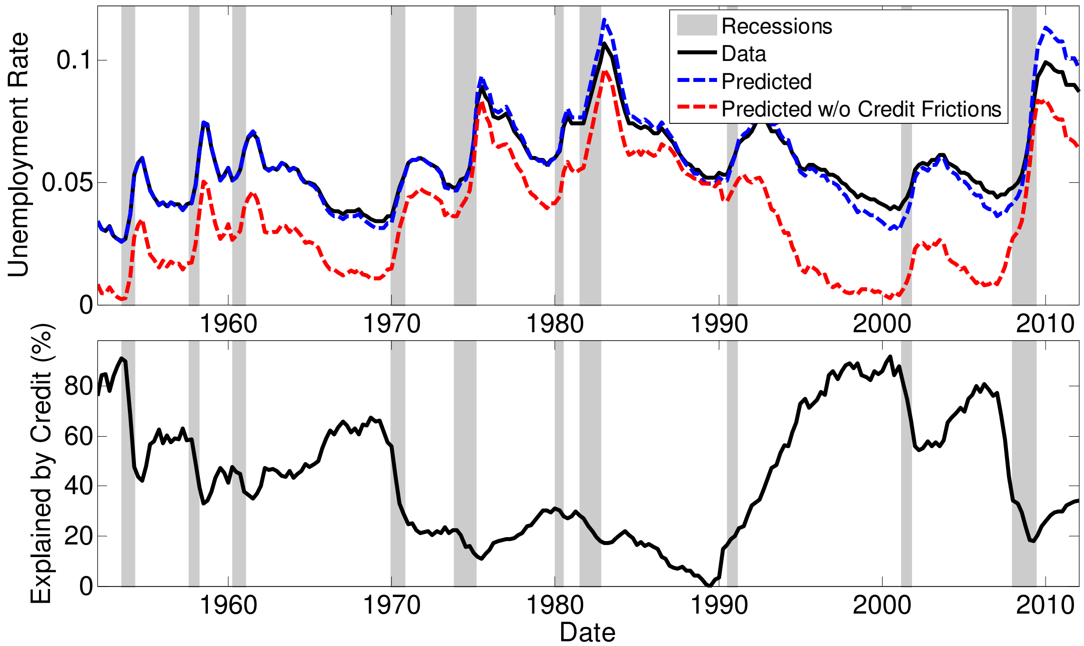


Figure 4.7: **Unemployment: Data, the Model-Predicted, and that without Credit Frictions**

average yields that the credit crunch contributes around 26.7% of unemployment in this financial crisis. Notice that the number is lower than that over the cycles. I have shown $\frac{\partial \chi}{\partial \eta} < 0$ in Section 3.3. Meanwhile, the middle panel of Figure (4.8) reveals that in the past recession η_t also decreases, *i.e.*, the labor market itself has also received a negative shock. It in turn attenuates the power of negative credit shocks in explaining unemployment in recessions.

Alternatively, I can use the results already established in unemployment decomposition over the cycles. In particular, I focus on the decomposition since the last quarter of 2008 in Figure (4.7), which is documented in the right panel of Figure (4.9). A calculation suggests that credit imperfections accounts for around 27.4% for the recent recession. Notice that the quantitative results are very similar to each other. Therefore, I claim that the credit crunch contributes 27% (=(27.4%+26.7%)/2) for the recent financial crisis.

Finally, I reach Figure (4.10) by combining the model-predicted unemployment with vacancy data in JOLTS. The left panel suggests that data and the model fit well with each other. The right panel describes the path of the Beveridge curve if there was no credit crunch in the past financial recession. Notice that unemployment in the right panel continues to rise although I unplug the negative shock to the credit market. Unemployment in the counterfactual analysis is then purely driven by the negative shocks to the labor market and to

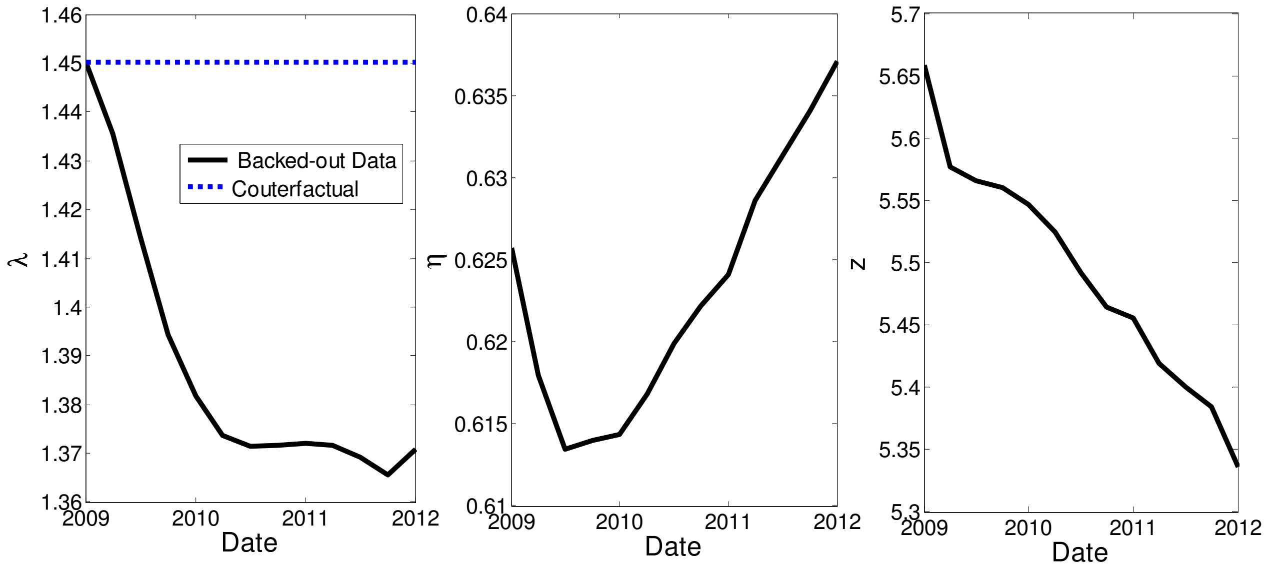


Figure 4.8: **Corresponding Shocks for the Counter-factual Analysis for the 2008 Financial Crisis**

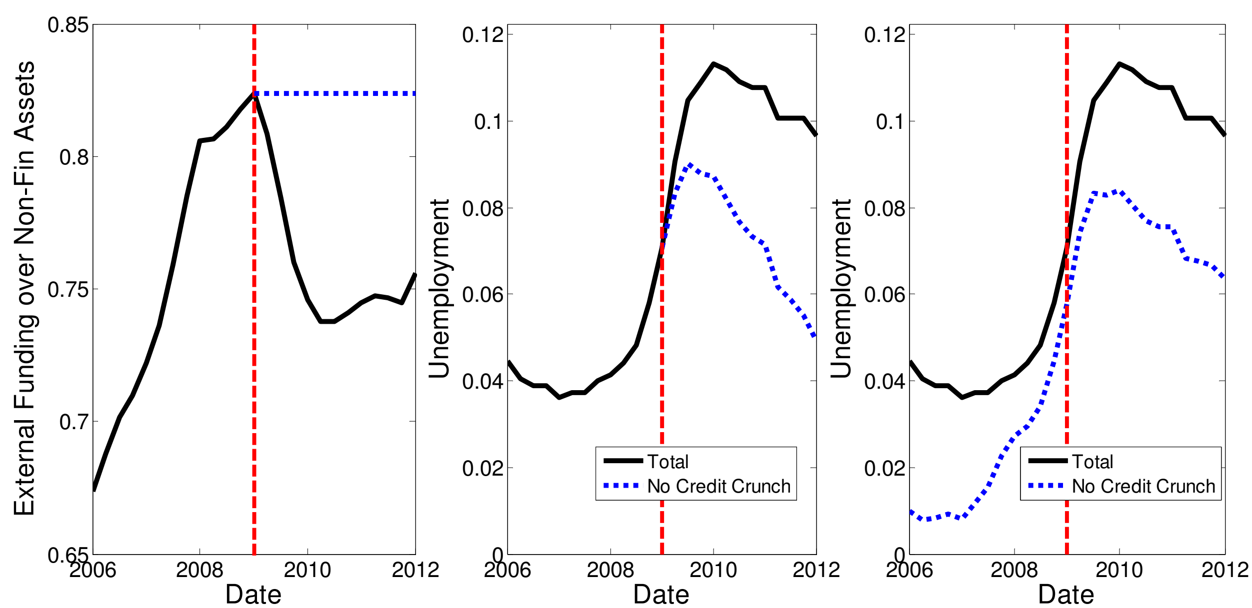


Figure 4.9: **Counter-factual Analysis for the Recent Financial Crisis**

aggregate productivity, as indicated by the middle and right panels of Figure (4.9). The most intriguing finding is that the counter-factual Beveridge curve does not shift outward, but instead moves alongside the original curve prior to the financial crisis. Therefore, the credit crunch seems to be mainly responsible for the outward shift in the Beveridge curve in the recent financial crisis.

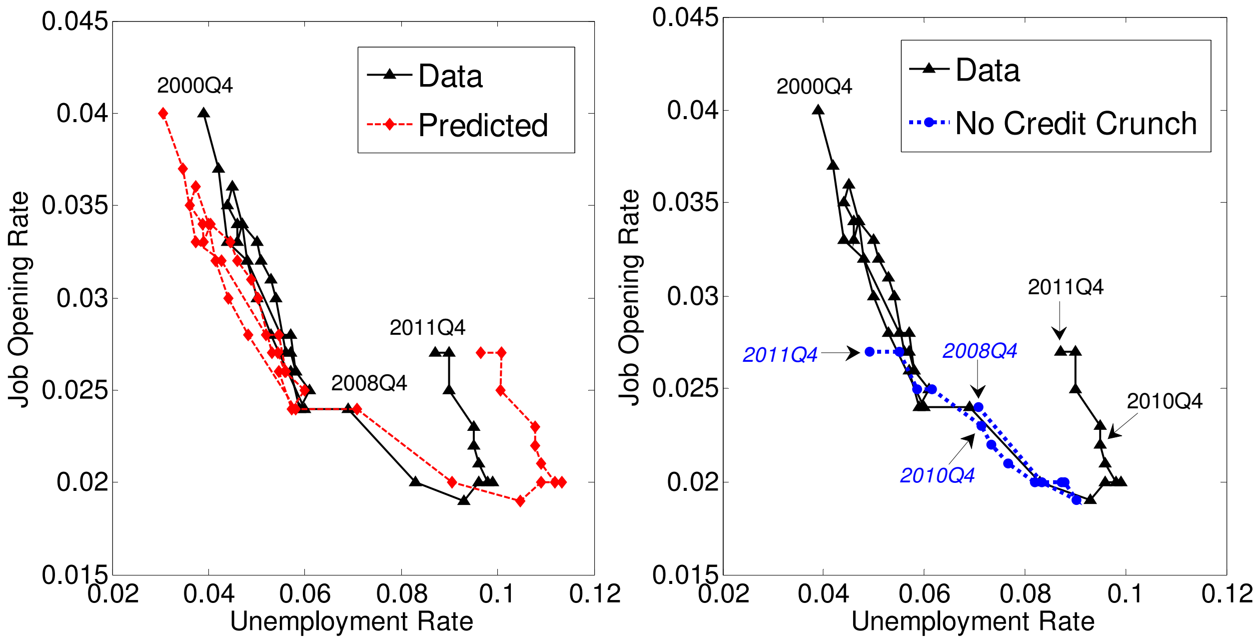


Figure 4.10: **Left Panel: Data and Model-Predicted Beveridge Curve; Right Panel: Data and Model-Predicted Beveridge without Credit Crunch in 2008**

5 Which Shocks Are Most Essential

I have so far exclusively addressed the aggregate implications of the model, especially for the unemployment rate. However, the transmission mechanism works through the heterogeneous agents at the micro level. Therefore, this section is devoted to discussing the heterogeneous treatment effect of the aggregate shocks on firms. In particular, I confront the disaggregate implications of these three shocks with micro-level empirical findings.

Employment Distribution

Moscarini and Postel-Vinay (2012) suggest that large firms have a more significant response to employment than do small firms in recessions. In our paper, the share of employment by firms with individual productivity no smaller than $\varphi^{\omega} = x^{\omega}/z$ is given as follows,

where φ^{ω} is denoted as the upper-ω percentile, $i.e.$, $1-F(\varphi^{\omega}) = \omega$. The employment share by firms with productivity no smaller than φ^{ω} is illustrated in Figure (5.1). I can check that $\Gamma(\varphi^{\omega})$ has nothing to do with matching efficiency η or aggregate productivity z, but only increases with financial friction λ. I illustrate the effect of λ on $\Gamma(\varphi^{\omega})$ in Figure (5.1). Alternatively, the job-filling rate $q(\varphi)$ increases with φ. Therefore, the employment loss increases with φ when a credit crunch occurs.

$$\Gamma(\varphi^{\omega}) = \frac{\int_{max\{\varphi^{\omega},\widehat{\varphi}\}}^{\overline{\varphi}} q(\varphi) \cdot dF(\varphi)}{\int_{\widehat{\varphi}}^{\overline{\varphi}} q(\varphi) \cdot dF(\varphi)} = 1 - F^{M}(\varphi^{\omega}),$$

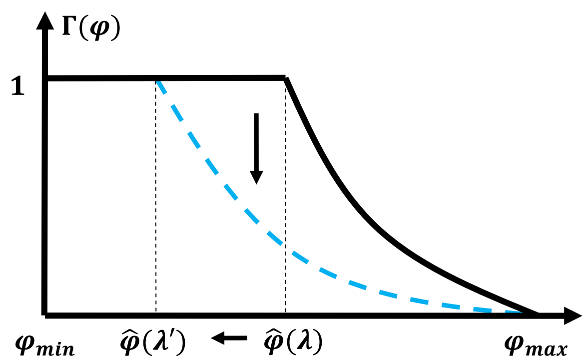


Figure 5.1: **Employment Shares**

Productivity Dispersion

The empirical research by Kehrig (2011) suggests the productivity dispersion widens in recessions. Firm productivity in our paper is the product of aggregate productivity and the individual component, $i.e.$, $x = z \cdot \varphi$. I can verify that the shock to the labor market (η) is related to neither the productivity distribution of the incumbents nor that of the new-entry firms. Instead, the negative shock to the credit market (λ) and that to aggregate productivity (z) widens and shrinks the productivity distribution of the incumbents and that of the new-entry firms, respectively.

	$z\downarrow$	$\eta\downarrow$	$\lambda\downarrow$
employment share of productive firms	$\rightarrow$	$\rightarrow$	$\downarrow$
productivity dispersion	$\downarrow$	$\rightarrow$	$\uparrow$

Table 4: **Aggregate Implications of These Shocks**

I summarize the disaggregate implications of the aggregate shocks in Table (4). It seems the prediction by credit shock coincides with the empirical regularity set by Kehrig (2011) and Moscarini and Postel-Vinay (2012). Moreover, the aggregate shocks have different aggregate implications, as shown in Table (5).[32]

	D/K	ALP	TFP	Y	u	χ	RP
$\lambda\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$
$z\downarrow$	$\rightarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\rightarrow$	$\rightarrow$	$\downarrow$
$\eta\downarrow$	$\rightarrow$	$\rightarrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$

Table 5: **Aggregate Implications of These Shocks**

6 Conclusion

I develop a tractable dynamic model with heterogeneous entrepreneurs to explain the interaction between credit and labor markets. The model is used to characterize the implications of capital misallocation due to credit imperfections for unemployment. The marginal value of being matched with labor increases with an entrepreneur's level of productivity. Therefore, high-productivity entrepreneurs offer a higher wage in equilibrium and thus the job-filling rate increases with their productivity. A credit crunch worsens capital misallocation by redistributing capital from high-productivity to low-productivity entrepreneurs. The former group of entrepreneurs is not only more productive given that capital is matched with labor, but also they are better at being matched with labor. Consequently, a credit crunch lowers aggregate matching efficiency in the labor market. This is the key theoretical contribution of this paper.

I then quantify the unemployment effect of credit imperfections after a calibration to the US economy. First, the exercise on unemployment decomposition reveals that credit imperfections and the classic labor search frictions contribute around 46% and 54%, respectively, to unemployment over the cycles between 1951 and 2011. Second, I conduct a counter-factual analysis to show that the credit crunch serves as the key driving force behind the outward shift in the Beveridge curve in the recent recession.

Another quantitative analysis is to addresses the jobless/sluggish recovery. On the one hand, the impulse response after VAR implies that the shock to aggregate productivity is more likely to be a cause for the jobless or sluggish recovery in the recent three recessions.

[32] As defined in Section 2, the return premium is defined as $RP \equiv \mathbb{E}\left[max\left(\pi\left(\varphi\right)-r,\,0\right)\right]$. I can rewrite it as $RP=\left(\frac{\tau(\lambda)}{\lambda}\right)\cdot\left(\frac{\partial Y}{\partial K}\right)$ and thus decreases with λ while increases with η and z.

On the other hand, the shocks to the credit and labor markets seem more responsible for the simultaneous recovery pattern between output and unemployment in recessions prior to the 1990s. Finally, I confront the disaggregate prediction of our model with micro-level empirical findings. Credit shocks are seemingly most essential to explain the widening productivity distribution and the dis-proportional loss of employment in recessions.

I close with several promising directions for further research. First, as documented by Hobijn and Sahin (2012), the outward shift in the Beveridge curve since the Great Recession occurred not only in the US, but also in other OECD countries like Portugal, Spain, and the UK. Like Hsieh and Klenow (2007), and Restuccia, Yang and Zhu (2008), it could be interesting to initiate a cross-country analysis for the quantitative effect of credit crunches on unemployment. Second, I have exclusively focused on the effect of credit imperfections on unemployment. As shown in Section 3.4, I take a preliminary step related to the implications of an MPS of the productivity distribution for unemployment. It may be fruitful to combine our work with that of Bigio (2013). Third, I have used a static wage contract throughout the paper for tractability. It could be challenging but fruitful to address the same problem with a dynamic labor contract as well as with credit imperfections and endogenous capital accumulation. The recent progress on labor search with heterogeneity by Menzio and Shi (2010, 2011), Schaal (2012) and Kaas and Kircher (2013) may serve as a reasonable starting point. Finally, the flexible framework of our paper could offer a tractable tool for us to model and quantify the role of unconventional monetary policy in curing the high rate unemployment. See the recent work by Dong and Wen (2013) for an example.

References

[1] **Andolfatto, David**. "Business cycles and Labor-Market Search." *The American Economic Review* (1996): 112-132.

[2] **Azariadis, Costas,** and **Leo Kaas**. "Capital Misallocation and Aggregate Factor Productivity." Federal Reserve Bank of St. Louis Working Paper Series (2012).

[3] **Bai, Yan, José-Víctor Ríos-Rull** and **Kjetil Storesletten**. "Demand Shocks that Look Like Productivity Shocks." University of Rochester, University of Minnesota and Federal Reserve Bank of Minnesota (2012).

[4] **Bethune, Zach, Guillaume Rocheteau,** and **Peter Rupert.** "Unemployment and Household Unsecured Debt." University of California, Santa Barbara and Irvine, mimeo (2013).

[5] **Bigio, Saki**. "Endogenous Liquidity and the Business Cycle." Columbia University, mimeo (2013).

[6] **Berger, David**. "Countercyclical Restructuring and Jobless Recoveries." Northwestern University, mimeo (2012).

[7] **Bernanke, Ben,** and **Mark Gertler.** "Agency Costs, Net Worth, and Business Fluctuations." *The American Economic Review* (1989): 14-31.

[8] **Bernanke, Ben S., Mark Gertler,** and **Simon Gilchrist.** "The Financial Accelerator in a Quantitative Business Cycle Framework." Handbook of Macroeconomics 1 (1999): 1341-1393.

[9] **Blanco, Julio, and Gaston Navarro.** "Equilibrium Default and Slow Recoveries." NYU, mimeo (2013).

[10] **Buera, Francisco J., Joseph P. Kaboski,** and **Yongseok Shin.** "Finance and Development: A Tale of Two Sectors." *The American Economic Review* 101, no. 5 (2011): 1964-2002.

[11] **Buera, Francisco J.,** and **Benjamin Moll.** "Aggregate Implications of a Credit Crunch." Princeton and UCLA, mimeo (2013).

[12] **Buera, Francisco J., Roberto Fattal-Jaef,** and **Yongseok Shin.** ”Anatomy of a Credit Crunch: from Capital to Labor Markets.” UCLA, IMF, Washington University in St. Louis and Federal Reserve Bank of St. Louis, mimeo (2013).

[13] **Carlstrom, Charles T.,** and **Timothy S. Fuerst.** ”Agency costs, net worth, and business fluctuations: A computable general equilibrium analysis.” *The American Economic Review* (1997): 893-910.

[14] **Carrillo-Tudela, Carlos, Michael Graber,** and **Klaus Waelde.** ”Unemployment and Vacancies Dynamics with Imperfect Financial Markets.” University of Essex, UCL and University of Mainz, mimeo (2013).

[15] **Chari, Varadarajan V., Patrick J. Kehoe,** and **Ellen R. McGrattan.** ”Business Cycle Accounting.” *Econometrica* 75, no. 3 (2007): 781-836.

[16] **Cui, Wei.** ”Delayed Capital Reallocation.” University College London, mimeo (2013).

[17] **Davis, Steven J., R. Jason Faberman,** and **John C. Haltiwanger.** ”Recruiting Intensity during and after the Great Recession: National and Industry Evidence.” No. w17782. National Bureau of Economic Research (2012).

[18] **Davis, Steven J., R. Jason Faberman,** and **John C. Haltiwanger.** ”The Establishment-Level Behavior of Vacancies and Hiring.” *The Quarterly Journal of Economics* 128, no. 2 (2013): 581-622.

[19] **Dong, Feng,** and **Yi Wen.** ”Unemployment under Optimal Exit Strategy.” Washington University and Federal Reserve Bank of St. Louis, work-in-progress mimeo (2013).

[20] **Garin, Julio.** ”Borrowing Constraints, Collateral Fluctuations, and the Labor Market.” University of Georgia, mimeo (2013).

[21] **Gomme, Paul,** and **Peter Rupert.** ”Theory, Measurement and Calibration of Macroeconomic Models.” *Journal of Monetary Economics* 54, no. 2 (2007): 460-497.

[22] **Hagedorn, Marcus, Fatih Karahan, Iourii Manovskii,** and **Kurt Mitman.** ”Unemployment benefits and unemployment in the great recession: the role of macro effects.” Economics working paper, University of Pennsylvania (2013).

[23] **Hobijn, Bart,** and **Aysegül Sahin.** ”Beveridge Curve Shifts across Countries since the Great Recession.” Federal Bank of San Francisco Working Paper 2012-24 (2012).

[24] **Holmstrom, Bengt,** and **Jean Tirole.** ”Financial intermediation, loanable funds, and the real sector.” *The Quarterly Journal of Economics* 112, no. 3 (1997): 663-691.

[25] **Hopenhayn, Hugo.** ”On the Measure of Distortions.” UCLA, mimeo (2013).

[26] **Hosios, Arthur J.** ”On the Efficiency of Matching and related Models of Search and Unemployment.” *The Review of Economic Studies* 57, no. 2 (1990): 279-298.

[27] **Hsieh, Chang-Tai,** and **Peter J. Klenow.** ”Misallocation and Manufacturing TFP in China and India.” *The Quarterly Journal of Economics* 124, no. 4 (2009): 1403-1448.

[28] **Jaimovich, Nir,** and **Henry E. Siu.** ”The Trend is the Cycle: Job Polarization and Jobless Recoveries.” No. w18334. National Bureau of Economic Research (2012).

[29] **Jermann, Urban,** and **Vincenzo Quadrini.** ”Macroeconomic Effects of Financial Shocks.” *The American Economic Review* 102, no. 1 (2012): 238-271.

[30] **Kaas, Leo,** and **Philipp Kircher**. ”Efficient Firm Dynamics in a Frictional Labor Market.” University of Konstanz and University of Edinburgh, mimeo (2013).

[31] **Kehrig, Matthias.** ”The Cyclicality of Productivity Dispersion.” University of Texas at Austin, mimeo (2011).

[32] **Khan, Aubhik,** and **Julia K. Thomas.** ”Credit Shocks and Aggregate Fluctuations in an Economy with Production Heterogeneity.” *Journal of Political Economy*, forthcoming (2013).

[33] **Kiyotaki, Nobuhiro,** and **John Moore.** ”Credit Cycles.” *The Journal of Political Economy* 105, no. 2 (1997): 211-248.

[34] **Krusell, Per,** and **Anthony A. Smith, Jr.** ”Income and Wealth Heterogeneity in the Macroeconomy.” *Journal of Political Economy* 106, no. 5 (1998): 867-896.

[35] **Kurlat, Pablo.** ”Lemons Markets and the Transmission of Aggregate Shocks.” Stanford University, mimeo (2012).

[36] **Lagos, Ricardo.** ”A Model of TFP.” *The Review of Economic Studies* 73, no. 4 (2006): 983-1007.

[37] **Liu, Zheng, Jianjun Miao,** and **Tao Zha.** ”Land Prices and Unemployment.” No. w19382. National Bureau of Economic Research (2013).

[38] **Liu, Zheng,** and **Pengfei Wang.** "Credit Constraints and Self-Fulfilling Business Cycles." *American Economic Journal: Macroeconomics*, forthcoming (2011).

[39] **Lucas Jr, Robert E.** "On the Size Distribution of Business Firms." *The Bell Journal of Economics* (1978): 508-523.

[40] **Melitz, Marc J.** "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity." *Econometrica* 71, no. 6 (2003): 1695-1725.

[41] **Menzio, Guido,** and **Shouyong Shi.** "Block Recursive Equilibria for Stochastic Models of Search on the Job." *Journal of Economic Theory* 145, no. 4 (2010): 1453-1494.

[42] **Menzio, Guido, and Shouyong Shi.** "Efficient Search on the Job and the Business Cycle." *Journal of Political Economy* 119, no. 3 (2011): 468-510.

[43] **Merz, Monika.** "Search in the Labor Market and the Real Business Cycle." *Journal of Monetary Economics* 36, no. 2 (1995): 269-300.

[44] **Miao, Jianjun, Pengfei Wang,** and **Lifang Xu**. "Stock Market Bubbles and Unemployment." Boston University and Hong Kong University of Science and Technology, mimeo (2013).

[45] **Michaillat, Pascal.** "Do Matching Frictions Explain Unemployment? Not in Bad Times." *The American Economic Review* 102, no. 4 (2012): 1721-1750.

[46] **Moen, Espen R.** "Competitive search equilibrium." *Journal of Political Economy* 105, no. 2 (1997): 385-411.

[47] **Moll, Benjamin.** "Productivity Losses from Financial Frictions: Can Self-financing Undo Capital Misallocation?" Princeton University, mimeo (2012).

[48] **Monacelli, Tommaso, Vincenzo Quadrini,** and **Antonella Trigari**. "Financial Markets and Unemployment." No. w17389. National Bureau of Economic Research, 2011.

[49] **Moscarini, Giuseppe,** and **Fabien Postel-Vinay**. "The Contribution of Large and Small Employers to Job Creation in Times of High and Low Unemployment." *The American Economic Review* 102, no. 6 (2012): 2509-2539.

[50] **Mortensen, Dale T**. Wage Dispersion: Why are Similar Workers Paid Differently. MIT Press (2005).

[51] **Mukoyama, Toshihiko, Christina Patterson,** and **Aysegül Sahin**. "Job Search Behavior over the Business Cycle." University of Virginia, Federal Reserve Board and Federal Reserve Bank of New York, mimeo (2013).

[52] **Jaimovich, Nir,** and **Henry E. Siu.** "The Trend is the Cycle: Job Polarization and Jobless Recoveries." Duke and UBC, mimeo (2013).

[53] **Petrongolo, Barbara,** and **Christopher A. Pissarides**. "Looking into the Black Box: A Survey of the Matching Function." *Journal of Economic Literature* 39, no. 2 (2001): 390-431.

[54] **Petrosky-Nadeau, Nicolas.** "TFP during a Credit Crunch." *Journal of Economic Theory* (2013).

[55] **Petrosky-Nadeau, Nicolas,** and **Etienne Wasmer**. "The Cyclical Volatility of Labor Markets under Frictional Financial Markets." *American Economic Journal: Macroeconomics* 5, no. 1 (2013): 193-221.

[56] **Pissarides, Christopher A**. Equilibrium Unemployment Theory. MIT press, 2000.

[57] **Restuccia, Diego,** and **Richard Rogerson**. "Policy Distortions and Aggregate Productivity with Heterogeneous Establishments." *Review of Economic Dynamics* 11, no. 4 (2008): 707-720.

[58] **Restuccia, Diego, Dennis Tao Yang,** and **Xiaodong Zhu**. "Agriculture and Aggregate Productivity: A Quantitative Cross-Country Analysis." *Journal of Monetary Economics* 55, no. 2 (2008): 234-250.

[59] **Schaal, Edouard**. "Uncertainty, Productivity and Unemployment in the Great Recession." Federal Reserve Bank of Minneapolis, mimeo (2012).

[60] **Shimer, Robert.** "Contracts in a Frictional Labor Market. " MIT, mimeo (1996).

[61] **Shimer, Robert.** "Labor Markets and Business Cycles." Princeton University Press (2010).

[62] **Wasmer, Etienne,** and **Philippe Weil**. "The Macroeconomics of Labor and Credit Market Imperfections." *The American Economic Review* (2004): 944-963.

[63] **Wang, Pengfei,** and **Yi Wen**. "Hayashi meets Kiyotaki and Moore: A Theory of Capital Adjustment Costs." *Review of Economic Dynamics* 15, no. 2 (2012): 207-225.

[64] **Wang, Pengfei,** and **Yi Wen**. ”Financial Development and Long-Run Volatility Trends.” Hong Kong University of Science and Technology and Federal Reserve Bank of St. Louis, mimeo (2013).

[65] **Wiczer, David.** ”Long-term Unemployment: Attached and Mismatched?.” University of Minnesota, mimeo (2012).

[66] **Williamson, Stephen D**. ”Financial Intermediary, Business Failures, and Real Business Cycles.” *The Journal of Political Economy* 95, no. 6 (1987): 1196-1216.

Appendix A - Data Sources, Definitions and Calculations

All the data throughout this paper are of quarterly frequency. There are three sources of data used in Section 1 and Section 4. First, I use financial data from the Flow of Funds Accounts (FFA) to construct the ratio of external funding over non-financial assets. I follow exactly Buera, Fattal-Jaef and Shin (2013) for this measurement. On the one hand, the external funding corresponds to the credit market instruments in FFA. It consists of the bank loans of the corporate and non-corporate sectors, and the commercial papers, corporate bonds and municipal securities of the corporate business. On the other hand, non-financial assets include real estate stock, equipment, software and inventories of the corporate and non-financial non-corporate business.

Second, data on employment, unemployment rate and job creation/destruction come from Bureau of Labor Statistics (BLS) while data on the Beveridge curve Job Opening and Labor Turnover (JOLTS). I only consider employment by non-farm private sectors. I use unemployment rate and employment to recover the total labor participation numbers in non-farm private sectors. The Beveridge curve with job opening rate and unemployment rate started with the last of 2000 because that is the starting point of the data in JOLTS.

Finally, National Income and Product Account (NIPA) documents quarterly data on output and investment, and annual data on capital. Output is defined as the sum of private non-durable consumption and private non-residential investment. I use the quarterly data on investment and the annual data on capital to recover the quarterly data on capital.

Appendix B - A Static Simplified Model

I use a static and simplified model to illustrate the key mechanism through which credit misallocation lowers aggregate matching efficiency. Aggregate productivity is simply set as $z = 1$. Each entrepreneur has K units of net worth. The distribution of individual productivity is a simple Binomial, *i.e.*, φ adopts $\varphi_H = \mu + \sigma$ and $\varphi_L = \mu - \sigma$ with equal probability, where $\sigma \in [0, \mu]$. As in the baseline, I model credit and labor frictions by a collateral constraint and competitive search respectively. I first characterize the case with only labor search frictions.

$$Y^* = max\left\{\varphi_H \cdot m\left(v_H, l_H\right) + \varphi_L \cdot m\left(v_L, l_L\right)\right\}$$

subject to

$$\begin{aligned} v_H + v_L &\leq K \\ l_H + l_L &\leq L \\ v_i,\, l_i &\geq 0,\;\; i \in \{L, H\}, \end{aligned}$$

where v_i and l_i denotes respectively the measure of capital and labor in sub-labor market $i \in \{L, H\}$, and $m(\cdot,\cdot)$ a matching technology. The efficient allocation consists of $v_H^* = K$, $v_L^* = 0$, $l_H^* = L$, and $l_L^* = 0$. In turn, aggregate output is $Y^* = \varphi_H \cdot m(K, L)$, employment $N^* = m(K, L)$, average labor productivity $ALP^* \equiv \frac{Y^*}{N^*} = \varphi_H$, and unemployment $u^* \equiv 1 - \frac{N}{L}^*$. Then I reach the equilibrium allocation as below.

Corollary 6. (Equilibrium Wedges under a Simple Binomial Distribution) Denote $\widetilde{F}$ as a Binomial distribution such that φ adopts φ_H and φ_L with probability $\alpha_H(\lambda)$ and $1-\alpha_H(\lambda)$ respectively, where $\alpha_H(\lambda) \equiv min\left\{\frac{\lambda}{2}, 1\right\}$. Then

1. for $i \in \{L, H\}$, the total capital used by type-i entrepreneurs is

$$v_H = min\left\{\frac{\lambda}{2}, 1\right\} \cdot K,\;\; v_L = K - v_H.$$

2. aggregate output and employment is

$$Y \;=\; (1 - \tau_y) \cdot Y^*,\;\; N = (1 - \tau_n) \cdot N^*,$$

where

$$
\begin{aligned}
1-\tau_y &= \Lambda(\lambda) \equiv \frac{\left[\mathbb{E}_{\widetilde{F}}\left(\varphi^{\frac{1}{\gamma}}\right)\right]^{\gamma}}{\varphi_H} = \left[\alpha_H(\lambda) + (1-\alpha_H(\lambda)) \cdot \left(\frac{\varphi_L}{\varphi_H}\right)^{\frac{1}{\gamma}}\right]^{\gamma} \\
1-\tau_n &= \Omega(\lambda) \equiv \frac{\mathbb{E}_{\widetilde{F}}\left(\varphi^{\frac{1-\gamma}{\gamma}}\right)}{\left[\mathbb{E}_{\widetilde{F}}\left(\varphi^{\frac{1}{\gamma}}\right)\right]^{1-\gamma}} = \frac{\alpha_H(\lambda) + (1-\alpha_H(\lambda)) \cdot \left(\frac{\varphi_L}{\varphi_H}\right)^{\frac{1-\gamma}{\gamma}}}{\left[\alpha_H(\lambda) + (1-\alpha_H(\lambda)) \cdot \left(\frac{\varphi_L}{\varphi_H}\right)^{\frac{1}{\gamma}}\right]^{1-\gamma}}.
\end{aligned}
$$

both of which increases with λ, decreases with $\frac{\sigma}{\mu}$, $\lim_{\lambda\to\infty}\tau_y = \lim_{\lambda\to\infty}\tau_n = 0$, and $\lim_{\frac{\sigma}{\mu}\to 0}\tau_y = \lim_{\frac{\sigma}{\mu}\to 0}\tau_n = 0$.

Similar to Proposition 2, a credit crunch increases the wedge of output and employment. Moreover, an MPS of the productivity distribution, *i.e.*, the increase of $\frac{\sigma}{\mu}$, also lowers aggregate matching efficiency. I use Figure 6.1 to illustrate those findings.

The main merit of using a Binomial distribution is a more clear intuition behind the transmission mechanism from credit to labor markets. By definition, employment is $N \equiv m(v_H, l_H) + m(v_L, l_L) = v_H \cdot q_H + v_L \cdot q_L$, where q_i denotes the job-filling rate in sub-labor market i. To make the analysis non-trivial, I assume both sub-labor markets are active, *i.e.*, $v_H > 0$ and $v_L > 0$. Then I have

$$
\frac{\partial N}{\partial \lambda} = (q_H - q_L) \cdot \frac{\partial v_H}{\partial \lambda} + \left(v_H \cdot \frac{\partial q_H}{\partial \lambda} + v_L \cdot \frac{\partial q_L}{\partial \lambda}\right) \geq 0.
$$

I have shown that $w_H > w_L$, $\theta_H > \theta_L$, and $q_H > q_L$. Then as the above decomposition suggests, on the one hand, the increase of λ transfers capital from low-productivity to high-productivity entrepreneurs, which directly implies an increase of employment. On the other hand, the increase of λ makes the use of capital more congested and thus the job-filling rate in the active sub-labor markets decrease. However, the direct effect can be verified to dominate the indirect general-equilibrium effect.

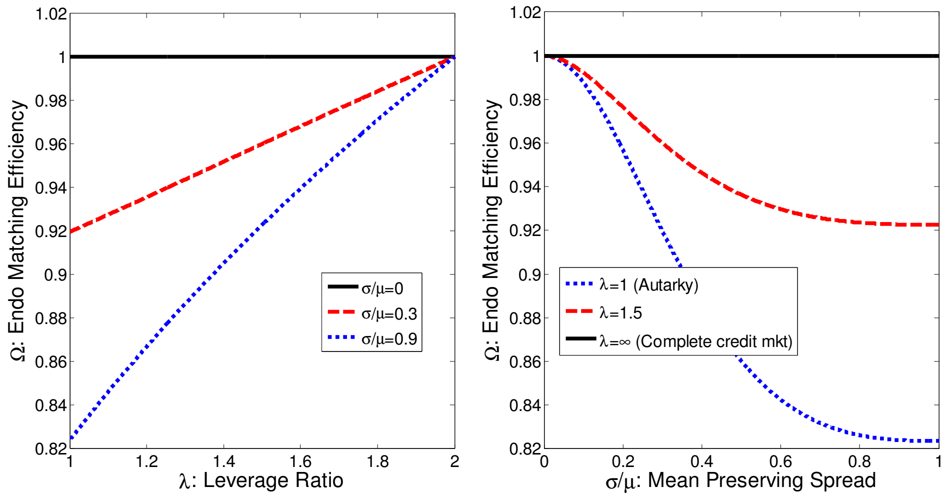


Figure 6.1: **Left Panel:** $\left(\Omega, \frac{\sigma}{\mu}\right)$; **Right Panel:** (Ω, λ)

Appendix C - Model Extension

This section consists of three pieces of model extension.[33] The first two extensions consider other possible sources of capital misallocation. One is to introduce tax on capital revenue while the other is to address the implications of working capital constraint. For space concern, I omit the discussion on transition dynamics. Finally, motivated by recent empirical findings, I endogenize firm's procyclical recruiting effort, which in turn amplifies the transmission mechanism in our baseline.

Tax on Capital Revenue

Motivated by Restuccia and Rogerson (2008), I extend the model with a tax scheme on capital revenue $\{\tau_k(\varphi)\}_{\varphi\in\Phi}$.[34] The expected capital revenue is then adjusted as $\widetilde{\pi}(\varphi) = [1-\tau_k(\varphi)]\cdot\pi(\varphi)$. Meanwhile, the active set is updated as $\Phi_A = \{\varphi|\widetilde{\pi}(\varphi)\geq r\}$ with its associated cumulative distribution as F_A and the lower bound as $\widehat{\varphi} = inf\{\Phi_A\}$. In the baseline, I characterize capital misallocation by the decrease of the cut-off point $\widehat{\varphi}$. I generalize the notion of capital misallocation as follows.

Definition 2. Denote F_A and $F_{A'}$ as two pieces of productivity distribution of active entrepreneurs. $F_{A'}$ causes a worse capital misallocation than F_A if and only if F_A Second-Order Stochastic Dominates (SOSD) $F_{A'}$.

[33]For space concern, I remove all the proofs associated with this section. The proofs are available upon request.

[34]For simplicity, I assume entrepreneurs with the same productivity share the same tax rate.

The employment in Equation (3.4) is now generalized by

$$N\left(F_A\right) = \mathbb{E}_{F_A}\left[q\left(\varphi, W\right)\right] \cdot K = \left[\int_{\Phi} q\left(\varphi, W\left(F_A\right)\right) dF_A\right] \cdot K \tag{6.1}$$

where K denotes the aggregate capital supply, $q\left(\varphi\right)$ the job-filling rate in sub-labor market φ and W the expected labor revenue. As in the baseline, capital misallocation generates two competing effects on employment. I illustrate the generalized version as below.

$$N\left(F_A\right) - N\left(F_{A'}\right) = \left\{\left[\int_{\Phi} q\left(\varphi, W\left(F_A\right)\right) \cdot dF_A - \int_{\Phi} q\left(\varphi, W\left(F_A\right)\right) \cdot dF_{A'}\right] + \left[\int_{\Phi} q\left(\varphi, W\left(F_A\right)\right) \cdot dF_{A'} - \int_{\Phi} q\left(\varphi, W\left(F_{A'}\right)\right) \cdot dF_{A'}\right.\right.$$

To sharpen the analysis, I make an assumption as below, which delivers the generalized version of the unemployment effect of capital misallocation.

Assumption 2. The distribution F is specified such that, if the truncated distribution F_A SOSD $F_{A'}$,

$$\frac{\mathbb{E}_{F_A}\left(\varphi^{\frac{1-\gamma}{\gamma}}\right)}{\left[\mathbb{E}_{F_A}\left(\varphi^{\frac{1}{\gamma}}\right)\right]^{1-\gamma}} > \frac{\mathbb{E}_{F_{A'}}\left(\varphi^{\frac{1-\gamma}{\gamma}}\right)}{\left[\mathbb{E}_{F_{A'}}\left(\varphi^{\frac{1}{\gamma}}\right)\right]^{1-\gamma}}.$$

Corollary 7. (Wedges with Capital Revenue Tax) Under Assumption 2, if F_A SOSD $F_{A'}$,

1. the wedges to aggregate output and employment are

$$\begin{aligned} 1-\tau_y &\equiv \left\{\mathbb{E}_{F_A}\left[\left(\varphi/\overline{\varphi}\right)^{\frac{1}{\gamma}}\right]\right\}^{\gamma} \in [0,1] \\ 1-\tau_n &\equiv \frac{\mathbb{E}_{F_A}\left(\varphi^{\frac{1-\gamma}{\gamma}}\right)}{\left[\mathbb{E}_{F_A}\left(\varphi^{\frac{1}{\gamma}}\right)\right]^{1-\gamma}} \in [0,1] \end{aligned}$$

   where $\left(\tau_y, \tau_n\right)$ are larger with $\Phi_{A'}$.

2. the wedges to ALP and unemployment are

$$\begin{aligned} 1-\tau_{alp} &= \frac{1-\tau_y}{1-\tau_n} = \frac{\mathbb{E}_{F_A}\left(\varphi^{\frac{1}{\gamma}}\right)}{\mathbb{E}_{F_A}\left[\left(\frac{\overline{\varphi}}{\varphi}\right) \cdot \varphi^{\frac{1}{\gamma}}\right]} \in [0,1] \\ 1+\tau_u &= 1+\tau_n \cdot \left(\frac{1-u^*}{u^*}\right) \in (1,\infty), \end{aligned}$$

3. the wedge to the expected wage revenue is zero while that to the interest rate is

$$1-\tau_r = \frac{1-\tau_k\left(\widehat{\varphi}\right)}{\mathbb{E}_{F_A}\left[\left(\frac{\varphi}{\widehat{\varphi}}\right)^{\frac{1}{\gamma}}\right]} \in [0,1]$$

On the one hand, if $\tau_K(\varphi) \equiv 0$, then the active sets reduces to that in the baseline. On the other hand, if $\tau_K(\varphi)$ is progressive, taking $\tau_K(\varphi) = \alpha \cdot \left[1 - \left(\frac{\varphi - \underline{\varphi}}{\overline{\varphi} - \underline{\varphi}}\right)^{\frac{1}{\gamma}}\right]$ with $\alpha \in [0, 1]$ for example, then the active set is $\Phi_A = \{\varphi | \varphi \in [\widehat{\varphi}_1, \widehat{\varphi}_2]\}$, which is illustrated in Figure 6.2. Moreover, I show that the increase of α widens the active set Φ_A and thus lowers the output and increases unemployment.

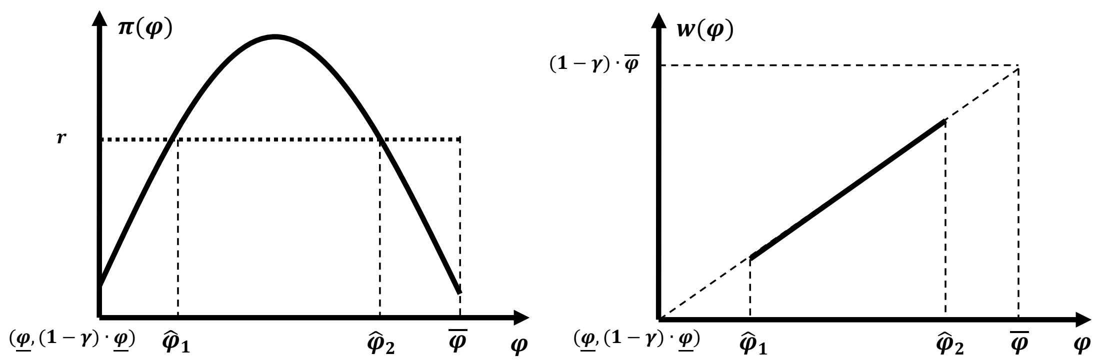


Figure 6.2: **Wage Scheme with a Progressive Tax on Capital Revenue (an example)**

Working Capital Constraint

Hosios condition is satisfied in the baseline with competitive search. Therefore the labor wedge is zero in the baseline. However, the business cycle accounting by Chari, Kehoe and McGrattan (2007) suggests the quantitative importance of labor wedge. This part imposes a working capital constraint upon the baseline to produce a non-trivial labor wedge. As shown in Section 2, total wage payment is $k \cdot \theta(\varphi) \cdot W$ for entrepreneurs with productivity φ and with k units of capital for production. I assume entrepreneurs have to pay part of the wage bill before production such that $k \cdot \theta(\varphi) \cdot W \leq \lambda_w \cdot k$, or equivalently,

$$\theta(\varphi) \leq \frac{\lambda_w}{W}. \tag{6.2}$$

In contrast to the baseline, the equilibrium wage scheme may be distorted in the presence of a constraint on working capital. The following proposition characterizes the equilibrium wedges on productivity, employment, interest rate, and wages, etc in the presence of the working capital constraint.

Corollary 8. (Equilibrium Wedges with Working Capital Constraint) In each period,

1. there exist pairwise cut-off values $(\widehat{\varphi}, \widetilde{\varphi})$ such that

   (a) only entrepreneurs with productivity $\varphi \geq \widehat{\varphi}$ are active in production,

(b) the wage scheme is $w(\varphi) = (1-\gamma) \cdot min\{\varphi, \widetilde{\varphi}\}$,

2. the solution to the pairwise cut-off values $(\widehat{\varphi}, \widetilde{\varphi})$ exists and is unique, and

   (a) $\widehat{\varphi}$ increases with λ and has nothing to do with other variables,

   (b) $\widetilde{\varphi}$ increases with λ, and λ_w,

3. the wedges to aggregate output and employment are

$$
\begin{aligned}
1-\tau_y &\equiv \Lambda(\lambda, \lambda_w) \equiv \frac{\mathbb{E}\left\{max\left(1, \left(\frac{\varphi}{\widetilde{\varphi}}\right)\right) \cdot [min(\varphi, \widetilde{\varphi})]^{\frac{1}{\gamma}} \mid \varphi \in [\widehat{\varphi}, \overline{\varphi}]\right\}}{\left[\mathbb{E}\left\{[min(\varphi, \widetilde{\varphi})]^{\frac{1}{\gamma}} \mid \varphi \in [\widehat{\varphi}, \overline{\varphi}]\right\}\right]^{1-\gamma} \cdot \overline{\varphi}} \in [0,1] \\
1-\tau_n &\equiv \Omega(\lambda, \lambda_w) \equiv \frac{\mathbb{E}\left\{[min(\varphi, \widetilde{\varphi})]^{\frac{1-\gamma}{\gamma}} \mid \varphi \in [\widehat{\varphi}, \overline{\varphi}]\right\}}{\left[\mathbb{E}\left\{[min(\varphi, \widetilde{\varphi})]^{\frac{1}{\gamma}} \mid \varphi \in [\widehat{\varphi}, \overline{\varphi}]\right\}\right]^{1-\gamma}} \in [0,1]
\end{aligned}
$$

   and (τ_y, τ_n) decreases with λ_w.

4. the wedges to ALP and unemployment are

$$
\begin{aligned}
1-\tau_{alp} &= \frac{1-\tau_y}{1-\tau_n} = \frac{\mathbb{E}\left\{max\left(1, \left(\frac{\varphi}{\widetilde{\varphi}}\right)\right) \cdot [min(\varphi, \widetilde{\varphi})]^{\frac{1}{\gamma}} \mid \varphi \in [\widehat{\varphi}, \overline{\varphi}]\right\}}{\mathbb{E}\left\{[min(\varphi, \widetilde{\varphi})]^{\frac{1-\gamma}{\gamma}} \mid \varphi \in [\widehat{\varphi}, \overline{\varphi}]\right\} \cdot \overline{\varphi}} \in [0,1] \\
1+\tau_u &= 1+\tau_n \cdot \left(\frac{1-u^*}{u^*}\right) \in (1, \infty).
\end{aligned}
$$

   and (τ_{alp}, τ_u) decreases with λ_w.

5. the wedges to the interest rate and to the wage are

$$
\begin{aligned}
1-\tau_r &= = \frac{1}{\mathbb{E}\left\{min\left(\left(\frac{\varphi}{\overline{\varphi}}\right)^{\frac{1}{\gamma}}, \left(\frac{\widetilde{\varphi}}{\overline{\varphi}}\right)^{\frac{1}{\gamma}} \cdot \left(\frac{\varphi}{\widetilde{\varphi}}\right)\right) \mid \varphi \in [\widehat{\varphi}, \overline{\varphi}]\right\}} \in [0,1]. \\
1-\tau_w &= \frac{\mathbb{E}\left\{[min(\varphi, \widetilde{\varphi})]^{\frac{1}{\gamma}} \mid \varphi \in [\widehat{\varphi}, \overline{\varphi}]\right\}}{\mathbb{E}\left\{max\left(1, \left(\frac{\varphi}{\widetilde{\varphi}}\right)\right) \cdot [min(\varphi, \widetilde{\varphi})]^{\frac{1}{\gamma}} \mid \varphi \in [\widehat{\varphi}, \overline{\varphi}]\right\}} \in [0,1].
\end{aligned}
$$

   and (τ_r, τ_w) decreases with λ_w.

If λ_w is high enough, then $\widetilde{\varphi} > \overline{\varphi}$ and I am back to the baseline model. Otherwise, as indicated in the above corollary, the optimal wage scheme becomes flattened for $\varphi \in [\widehat{\varphi}, \widetilde{\varphi}]$. The wage scheme with a binding working capital constraint provides a micro foundation for equilibrium wage rigidity, *i.e.*, entrepreneurs choose not to adjust their wage scheme if their productivity $\varphi \in [\widehat{\varphi}, \widetilde{\varphi}]$. I illustrate it in Figure 6.3.

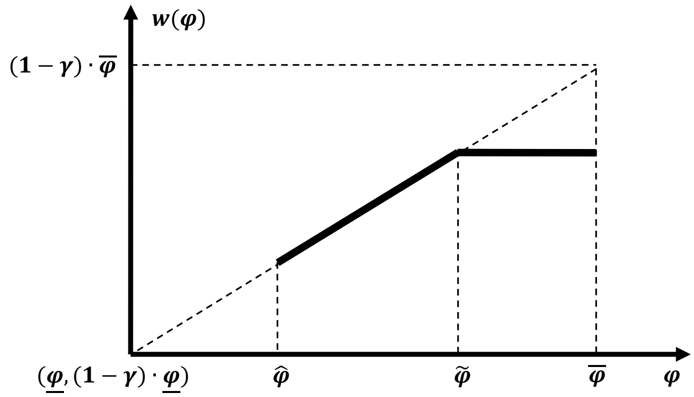


Figure 6.3: **Wage Scheme with Working Capital Constraint**

The possibility of non-trivial labor wedge is the main insight gained from the working capital constraint. The marginal value of being matched with labor increases with entrepreneur's productivity. Thus wage scheme and job-filling rate increase with productivity. However, due to the working capital constraint, high-productivity entrepreneurs would have to cut down the otherwise high wage, which in turn lowers employment and labor expenditure.

Endogenous Recruiting Effort

The recent empirical findings by Davis, Faberman and Haltiwanger (2012) suggest that, in addition to posting vacancies, firm's recruiting effort also includes "increase advertising or search intensity per vacancy, screen applicants more quickly, relax hiring standards, improve working conditions, and offer more attractive compensation to prospective employees".[35] Furthermore, they show that firm's recruiting effort is procyclical. To this end, I follow Pissarides (2000) and Bai, Ríos-Rull and Storesletten (2012) to endogenize firm's search effort.

For simplicity, I assume firm's search effort is made after observing the aggregate state variable, but before the realization of their own productivity level. Entrepreneurs use worker's labor input to increase their own search effort s, which may include advertising and screening effort.[36] More specifically, $\sigma \in (0,1)$ of capital revenue is pledgeable to workers. Matching function is $m\left(v\left(\varphi\right)e\left(\overline{s}\right),l\left(\varphi\right)\right)$, where $\overline{s}$ denotes the average recruiting effort. Each entrepreneur treats $\overline{s}$ as given. In equilibrium, I have $s=\overline{s}$. Denote the modified market tightness as

[35] The recent work by Mukoyama, Patterson and Sahin (2013) complements to Davis, Faberman and Haltiwanger (2012) by focusing on the job search intensity of worker side. Our paper focuses on the endogenous search effort by the firm side.

[36] I also tried an alternative setup to endogenize firm's recruiting effort. It is the entrepreneurs who incur non-pecuniary disutility for recruiting effort. The alternative extension is available upon request.

$\theta(\varphi) \equiv \frac{l(\varphi)}{v(\varphi)e(\overline{s})}$. The job-filling rate and job finding rate are modified as below.

$$\begin{aligned} q(\theta(\varphi), s) &= \frac{m(v(\varphi)e(\overline{s}), l(\varphi))}{v(\varphi)e(\overline{s})} \cdot e(s) = m(1, \theta(\varphi)) \cdot e(s) \\ p(\theta(\varphi), s) &= \frac{m(v(\varphi)e(\overline{s}), l(\varphi))}{l(\varphi)} = m\left(\frac{1}{\theta(\varphi)}, 1\right) = \frac{q(\theta(\varphi), s)}{\theta(\varphi) \cdot e(s)}. \end{aligned}$$

Corollary 9. In equilibrium $s = \overline{s}$. Moreover, given $\overline{s}$, aggregate output and unemployment is adjusted as

$$\begin{aligned} Y &= z \cdot \Lambda(\lambda) \cdot m(e(\overline{s}) \cdot K, L) \\ u &= 1 - \Omega(\lambda) \cdot m\left(\frac{e(\overline{s}) \cdot K}{L}, 1\right), \end{aligned}$$

and the aggregate matching efficiency is $\widehat{\eta} = \Omega(\lambda) \cdot \eta \cdot e(\overline{s})$.

It remains for us to characterize the choice of recruiting effort s. First, given $(s, \overline{s})$, the decision by active entrepreneur-(a, φ) is formulated as below.

$$\pi(\varphi, s) \equiv \max_{s.t.\ p(\theta(\varphi), s) \cdot w(\varphi) = W} \{q(\theta(\varphi), s) \cdot (\varphi - w(\varphi))\}$$

The modified market tightness $\theta(\varphi)$ is pinned down by the FOC $\frac{\partial m(\theta(\varphi),1)}{\partial \theta(\varphi)} = \frac{W}{\varphi}$. Second, given s, the individual decision rule on lending or borrowing depends on $\widehat{\varphi}(s)$, where $\pi(\widehat{\varphi}(s), s) = r$. Therefore s is determined by

$$\max \ \{(1-\sigma) \cdot \{\lambda[1 - F(\widehat{\varphi}(s))] \cdot \mathbb{E}[(\pi(\varphi, s) - r) | \varphi \geq \widehat{\varphi}(s)] + r\} - c(s)\},$$

where $\lambda[1 - F(\widehat{\varphi}(s))] \mathbb{E}[(\pi(\varphi, s) - r) | \varphi \geq \widehat{\varphi}(s)] + r$ denotes the expected capital revenue with search effort s by workers, and σ proportion can be pledgeable to them, and $c(s)$ denotes the effort cost. I assume $e(0) = s_L > 0$. That is, if none of the entrepreneurs exert positive search effort, I am back to the baseline model. Besides, I assume $e'(s) > 0$, $e''(s) < 0$, $c(0) = 0$, $c'(s) > 0$, and $c''(s) \geq 0$. FOC upon the above equation delivers the endogenous choice of recruiting effort. First, the equilibrium search effort $\overline{s}$ increases with (z, η, λ). That is, these shocks will be amplified through the search effort. In particular, since $\widehat{\eta} = \Omega(\lambda) \cdot \eta \cdot e(\overline{s}(z, \lambda, \eta))$, the decrease of either λ or η lowers aggregate matching efficiency in both direct and indirect way. I illustrate the amplification in Figure (6.4).

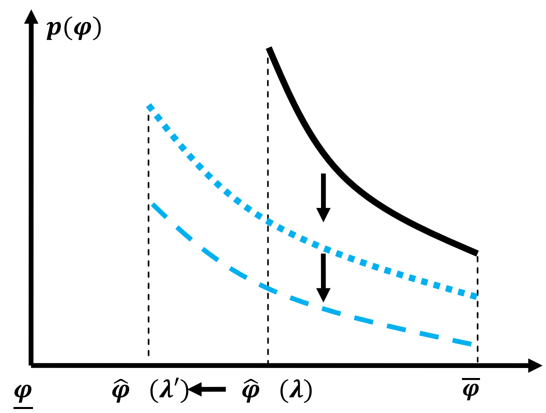


Figure 6.4: **Unemployment Effect of Capital Reallocation with Endogenous Recruiting Effort**

Appendix D - Proofs

Proof on Proposition 1

Proof. Substituting the participation constraint $p(\theta(\varphi))w(\varphi) = W$ into the objective function and using the fact that $p(\theta(\varphi)) = \frac{q(\theta(\varphi))}{\theta(\varphi)}$ yields

$$\pi(\varphi, W) = max \ \{q(\theta(\varphi))\varphi - \theta(\varphi)W\},$$

and thus the FOC is $q'(\theta(\varphi)) = \frac{W}{\varphi}$, which pins down the market tightness $\theta(\varphi)$ in active submarket-$\varphi \in \Phi_A$. Using Implicit Function Theorem and the concavity of $q(\cdot)$ suggests that $\theta(\varphi)$ increases with φ and decreases with W. In turn, I recover the wage scheme as $w(\varphi) = \frac{W}{p(\theta(\varphi))}$. Since $p(\theta(\varphi))$ decreases with $\theta(\varphi)$, we know that $w(\varphi)$ increases with φ. Finally, using Envelope Theorem reveals that $\pi(\varphi, W)$ increases with φ and decreases with W.

□

Proof on Lemma 1

Proof. The net revenue by entrepreneur-(a, φ) is

$$\underset{k\in[0,\lambda\cdot a]}{max}\{\pi(\varphi, W)\cdot k - r\cdot(k-a) + (1-\delta)\cdot a\},$$

where $\pi(\varphi, W)\cdot k$ denotes the capital revenue and $b = k - a$ is the debt if positive and the loan if negative. The above problem can be rewritten as $\underset{k\in[0,\lambda\cdot a]}{max}\ [\pi(\varphi, W) - r]\cdot k + [r + (1-\delta)]\cdot a$. Since the net revenue is linear k, and $k \in [0, \lambda\cdot a]$, only corner solutions, *i.e.*, $k = \lambda a$ or $k = 0$, will be considered. On the one hand, if $\pi(\varphi, W) > r$, the entrepreneurs not only want to engage in production, but also want to borrow as much as they can. On the other hand, if $\pi(\varphi, W) < r$, then the entrepreneurs prefer to lending to others. Since $\pi(\varphi, W)$ increases with φ, if I define the cut-off point $\widehat{\varphi}$ as $\pi(\widehat{\varphi}, W) = r$, then entrepreneurs choose to be active in production with a binding borrowing constraint if and only if $\varphi > \widehat{\varphi}$.

□

Proof on Corollary 1

Proof. First, as shown in Lemma 1, the active set is $\Phi_A \equiv \{\varphi | \varphi \in [\widehat{\varphi}, \overline{\varphi}]\} = \{\varphi_i | \varphi_i \in [\widehat{\varphi}_i, \overline{\varphi}_i]\}$. Therefore, the truncated distribution of productivity by active entrepreneurs is

$$F^A(\varphi) = \int_{\widehat{\varphi}}^{\varphi} f(\varphi | \varphi \geq \widehat{\varphi}) d\varphi = \frac{F(\varphi) - F(\widehat{\varphi})}{1 - F(\widehat{\varphi})}.$$

Second, according to Proposition 1 and Lemma 1, wage scheme $w(\varphi)$ increases with φ and entrepreneurs with higher individual productivity, $q(\varphi)$, is more likely to be matched with workers. Therefore, the productivity distribution of finally matched capital is

$$F^M(\varphi) = \frac{\int_0^\infty \int_{\widehat{\varphi}}^{\varphi} k(\varphi', a) \cdot q(\varphi') \cdot h(\varphi, a) d\varphi da}{\int_{\widehat{\varphi}}^{\overline{\varphi}} k(\varphi', a) \cdot q(\varphi') \cdot dF(\varphi')} = \frac{\int_{\widehat{\varphi}}^{\varphi} q(\varphi') \cdot dF(\varphi')}{\int_{\widehat{\varphi}}^{\overline{\varphi}} q(\varphi') \cdot dF(\varphi')}.$$

Finally, I use the following lemma to prove $F^M(\varphi) < F^A(\varphi) < F(\varphi)$.

Lemma 3. Assume $\epsilon(\varphi) > 0$ for $\varphi \in [\underline{\varphi}, \overline{\varphi}]$. Given any $\widehat{\varphi} \in [\underline{\varphi}, \overline{\varphi}]$, define

$$F_1(\varphi) \equiv \frac{\int_{\widehat{\varphi}}^{\varphi} \epsilon(\varphi') d\varphi'}{\int_{\widehat{\varphi}}^{\overline{\varphi}} \epsilon(\varphi') d\varphi'}, \quad F_2(\varphi) \equiv \frac{\int_{\widehat{\varphi}}^{\varphi} \epsilon(\varphi') \vartheta(\varphi') d\varphi'}{\int_{\widehat{\varphi}}^{\overline{\varphi}} \epsilon(\varphi') \vartheta(\varphi') d\varphi'},$$

where $\vartheta(\varphi)$ increases with φ and is bounded by $[0, 1]$. Then $F_1(\varphi) \leq F_2(\varphi)$.

I leave the proof of this lemma at the end of this part. Now use this lemma to prove $F^M(\varphi) < F^A(\varphi) < F(\varphi)$. First, we can rewrite $F^A(\varphi)$ and $F^M(\varphi)$ as below.

$$F^A(\varphi) = \frac{\int_{\widehat{\varphi}}^{\varphi} f(\varphi') d\varphi'}{\int_{\widehat{\varphi}}^{\overline{\varphi}} f(\varphi') d\varphi'}, \quad F^M(\varphi) = \frac{\int_{\widehat{\varphi}}^{\varphi} f(\varphi') q(\varphi') d\varphi'}{\int_{\widehat{\varphi}}^{\overline{\varphi}} f(\varphi') q(\varphi') d\varphi'}.$$

Therefore, if we treat $\epsilon(\varphi)$ as $f(\varphi)$, and $\vartheta(\varphi)$ as $q(\varphi)$, which has been proved to increase with φ in Proposition 1, then using the above lemma immediately suggests $F^M(\varphi) \leq F^A(\varphi)$, *i.e.*, $F^M(\varphi)$ first-order stochastic dominates (FOSD) $F^A(\varphi)$. Moreover, we can rewrite $F^A(\varphi)$ as below.

$$F^A(\varphi) = \frac{\int_{\underline{\varphi}}^{\varphi} f(\varphi') \cdot \mathbf{1}_{\{\varphi' \geq \widehat{\varphi}\}} d\varphi'}{\int_{\underline{\varphi}}^{\overline{\varphi}} f(\varphi') \cdot \mathbf{1}_{\{\varphi' \geq \widehat{\varphi}\}} d\varphi'}.$$

If we treat $\vartheta(\varphi)$ as $\mathbf{1}_{\{\varphi \geq \widehat{\varphi}\}}$, which increases with φ and bounded by $[0, 1]$ in this scenario, then immediately the above lemma implies $F^A(\varphi) \leq F(\varphi)$. I close this part by proving the aforementioned lemma. Define $F_3(\varphi) \equiv F_2(\varphi) - F_1(\varphi)$. Then we have

$$\begin{aligned} F_3'(\varphi) &= F_2'(\varphi) - F_1'(\varphi) \\ &= \frac{\epsilon(\varphi)\vartheta(\varphi)}{\int_{\widehat{\varphi}}^{\overline{\varphi}} \epsilon(\varphi')\vartheta(\varphi')d\varphi'} - \frac{\epsilon(\varphi)}{\int_{\widehat{\varphi}}^{\overline{\varphi}} \epsilon(\varphi')d\varphi'} \\ &= \epsilon(\varphi)\frac{\left[\vartheta(\varphi)\int_{\widehat{\varphi}}^{\overline{\varphi}} \epsilon(\varphi')d\varphi' - \int_{\widehat{\varphi}}^{\overline{\varphi}} \epsilon(\varphi')\vartheta(\varphi')d\varphi'\right]\cdot\epsilon(\varphi)}{\left[\int_{\widehat{\varphi}}^{\overline{\varphi}} \epsilon(\varphi')\vartheta(\varphi')d\varphi'\right]\cdot\left[\int_{\widehat{\varphi}}^{\overline{\varphi}} \epsilon(\varphi')d\varphi'\right]}. \end{aligned}$$

Now I define $F_4(\varphi) \equiv \vartheta(\varphi)\int_{\widehat{\varphi}}^{\overline{\varphi}} \epsilon(\varphi')d\varphi' - \int_{\widehat{\varphi}}^{\overline{\varphi}} \epsilon(\varphi')\vartheta(\varphi')d\varphi'$. Then we immediately know that, since $\vartheta(\varphi)$ is an increasing function in φ, so is $F_4(\varphi)$. Moreover, notice that $F_4(\widehat{\varphi}) < 0$ and $F_4(\overline{\varphi}) > 0$, and thus there exists a cut-off $\check{\varphi} \in (\widehat{\varphi}, \overline{\varphi})$such that $F_4(\varphi) < 0$ when $\varphi \in (\widehat{\varphi}, \check{\varphi})$ and $F_4(\varphi) > 0$ when $\varphi \in (\check{\varphi}, \overline{\varphi})$. In turn, we know that,

$$F_3'(\varphi) \begin{cases} < 0 & if \;\; \varphi \in (\widehat{\varphi}, \check{\varphi}) \\ > 0 & if \;\; \varphi \in (\check{\varphi}, \overline{\varphi}). \end{cases}$$

Besides, since $F_3(\overline{\varphi}) = F_3(\overline{\varphi}) = 0$, we know that $F_3(\varphi) \equiv F_2(\varphi) - F_1(\varphi) \le 0$ is always satisfied.

□

Proof on Corollary 2

Proof. First, using the result on capital demand mentioned above, the constrained optimization by entrepreneur-(a, φ) can be rewritten as

$$V(a, \varphi; X) = max\left\{log(c) + \beta \cdot \mathbb{E}\left[V(a', \varphi'; X')\,|X\right]\right\},$$

subject to $c + a' = \Psi(\varphi)\cdot a$, where $\Psi(\varphi) = max\{\pi(\varphi) - r, 0\}\cdot\lambda + [r + (1-\delta)]$. Then I substitute the capital demand of Lemma 1 into the budget constraint and thus reach the simplified version of the constrained optimization problem by entrepreneur-(a, φ).

Second, I address the policy function. Guess the value function is linear with own net worth, *i.e.*, $V(a, \varphi) = C(\varphi) + D \cdot log(a)$, then we have

$$V(a, \varphi) = C(\varphi) + D\cdot log(a) = \max_{a' \in (0, \Psi(\varphi)\cdot a)} \left\{log(\Psi(\varphi)\cdot a - a') + \beta\cdot\mathbb{E}\left[C(\varphi') + D\cdot log(a')|\varphi\right]\right\},$$

where $\Psi(\varphi) = max\{\pi(\varphi) - r, 0\}\cdot\lambda + [r + (1-\delta)]$. FOC suggests $a' = \left(\frac{\beta D}{1+\beta D}\right)\cdot\Psi(\varphi)\cdot a$. In turn, the above Bellman equation can be rewritten as

$$C(\varphi) + D\cdot log(a) = log\left[\left(\frac{1}{1+\beta D}\right)\cdot\Psi(\varphi)\cdot a\right] + \beta\cdot\left\{\mathbb{E}\left[C(\varphi')|\varphi\right] + D\cdot log\left[\left(\frac{\beta D}{1+\beta D}\right)\cdot\Psi(\varphi)\cdot a\right]\right\}.$$

Therefore $D = \frac{1}{1-\beta}$, and thus $a' = \beta\cdot\Psi(\varphi)\cdot a$. In turn, $d = \Psi(\varphi)\cdot a - a' = (1-\beta)\cdot\Psi(\varphi)\cdot a$.

□

Proof on Lemma 2

Proof. Denote Φ^* as the efficient set of active capital and labor, *i.e.*, $\Phi^* = \{\varphi|\, l(\varphi) > 0,\, v(\varphi) > 0\}$.

Assume the measure of Φ^* contains at least two types of productivity φ, then for $\varphi_i \in \Phi^*$, the FOC suggests

$$\begin{aligned} \varphi_i \cdot m_v(v(\varphi_i), l(\varphi_i)) &= \mu_K \\ \varphi_i \cdot m_l(v(\varphi_i), l(\varphi_i)) &= \mu_L, \end{aligned}$$

where μ_K and μ_L denotes the Lagrangian multiplier of the constraints on capital and labor respectively. Then we have

$$\frac{m_v(v(\varphi_i), l(\varphi_i))}{m_l(v(\varphi_i), l(\varphi_i))} = \frac{m_v(1, \theta(\varphi_i))}{m_l(1, \theta(\varphi_i))} = \frac{\mu_K}{\mu_L},$$

where the first equation uses the fact that m_v and m_l are homogeneous of degree one. Immediately we know $\theta(\varphi_i)$ is constant for $\varphi_i \in \Phi^*$. Then we know that

$$\varphi_i \cdot m_v(, \theta(\varphi_i)) = \mu_K.$$

Therefore φ_i is unique and is determined by μ_K and μ_L. Thus there is only one element in Φ^*. It then goes without say that $\Phi^* = \{\overline{\varphi}\}$. In turn,

$$Y^* = \int_{\Phi} \varphi m(v^*(\varphi), l^*(\varphi)) d\varphi = \overline{\varphi} \cdot m(K, L).$$

□

Proof on Proposition 2

Proof. First, the clearing condition could be further simplified as $\lambda \cdot [1 - F(\widehat{\varphi})] = 1$. Using Implicit Function Theorem immediately suggests that $\widehat{\varphi}$ increases with λ, and $\lim_{\lambda \to 1} \widehat{\varphi} = \underline{\varphi}$ and $\lim_{\lambda \to \infty} \widehat{\varphi} = \overline{\varphi}$.

Second, The aggregate output is defined as $Y = \int_0^{\infty} \int_{\underline{\varphi}}^{\overline{\varphi}} \varphi v(\varphi, a) q(\varphi) d\varphi da$. Since $v(\varphi, a) = k(\varphi, a) h(\varphi, a) = \lambda a f(\varphi) g(a) \cdot 1_{\varphi \in \Phi^A}$, the output can be rewritten as

$$Y = \left[\int_{\widehat{\varphi}}^{\overline{\varphi}} \varphi \cdot q(\varphi) \cdot dF(\varphi) \right] \cdot \lambda K.$$

When the matching function in sub-labor market φ is $m(l(\varphi), v(\varphi)) = \eta \cdot l(\varphi)^{1-\gamma} v(\varphi)^{\gamma}$, then the matching probability by entrepreneur-φ is $q(\varphi) = \frac{m(l(\varphi), v(\varphi))}{v(\varphi)} = \eta \cdot \theta(\varphi)^{1-\gamma}$. In turn, the FOC is simplified as $q'(\varphi) = \eta \cdot (1-\gamma) \cdot \theta(\varphi)^{-\gamma} = \frac{W}{\varphi}$, and thus $\theta(\varphi, W) = \left[\frac{\eta(1-\gamma)\varphi}{W}\right]^{\frac{1}{\gamma}}$. As a result, we have

$$\begin{aligned} q(\varphi, W) &= \eta \cdot \left[\frac{\eta(1-\gamma)}{W}\right]^{\frac{1-\gamma}{\gamma}} \cdot \varphi^{\frac{1-\gamma}{\gamma}} \\ p(\varphi, W) &= \frac{W}{(1-\gamma) \cdot \varphi} \\ \pi(\varphi, W) &= \eta^{\frac{1}{\gamma}} \gamma (1-\gamma)^{\frac{1}{\gamma}-1} W^{1-\frac{1}{\gamma}} \cdot \varphi^{\frac{1}{\gamma}} \end{aligned}$$

and thus, for $\varphi \in \Phi_A$, the optimal wage scheme is as $w(\varphi, W) = \frac{W}{p(\varphi, W)} = (1-\gamma) \cdot \varphi$. Moreover, the labor resource constraint can be rewritten as

$$\lambda K\int_{\widehat{\varphi}}^{\overline{\varphi}}\theta(\varphi,W)dF(\varphi)=\left[\frac{\eta(1-\gamma)}{W}\right]^{\frac{1}{\gamma}}\cdot K\cdot\left[\lambda\int_{\widehat{\varphi}}^{\overline{\varphi}}\varphi^{\frac{1}{\gamma}}dF(\varphi)\right]=L.$$

Since $\lambda\int_{\widehat{\varphi}}^{\overline{\varphi}}\varphi^{\frac{1}{\gamma}}dF(\varphi)=\lambda\left[1-F(\widehat{\varphi})\right]\cdot\left(\frac{\int_{\widehat{\varphi}}^{\overline{\varphi}}\varphi^{\frac{1}{\gamma}}dF(\varphi)}{1-F(\widehat{\varphi})}\right)=\mathbb{E}_F\left(\varphi^{\frac{1}{\gamma}}\,|\,\varphi\in[\widehat{\varphi},\overline{\varphi}]\right)$, we have

$$\left[\frac{\eta(1-\gamma)}{W}\right]^{\frac{1}{\gamma}}=\frac{L}{K\cdot\mathbb{E}_F\left(\varphi^{\frac{1}{\gamma}}\,|\,\varphi\in[\widehat{\varphi},\overline{\varphi}]\right)}.$$

Therefore the aggregate output can be further rewritten.

$$\begin{aligned}
Y &= \int\int_{\widehat{\varphi}}^{\varphi_{max}} z\cdot\varphi v(\varphi)q(\varphi)d\varphi dG(a).\\
&= z\eta\cdot\left[\frac{\eta(1-\gamma)}{W}\right]^{\frac{1-\gamma}{\gamma}}\cdot K\left[\lambda\int_{\widehat{\varphi}}^{\varphi_{max}}\varphi^{\frac{1}{\gamma}}dF(\varphi)\right]\\
&= \left\{z\eta\left(\mathbb{E}_F\left(\varphi^{\frac{1}{\gamma}}\,|\,\varphi\in[\widehat{\varphi},\overline{\varphi}]\right)\right)^{\gamma}\right\}\cdot K^{\gamma}L^{1-\gamma}\\
&= z\cdot\left(\mathbb{E}_F\left(\varphi^{\frac{1}{\gamma}}\,|\,\varphi\in[\widehat{\varphi},\overline{\varphi}]\right)\right)^{\gamma}\cdot m(K,L).
\end{aligned}$$

which can be immediately verified by using change of variables. Then equilibrium TFP is obtained as below.

$$TFP\equiv\frac{Y}{K^{\gamma}L^{1-\gamma}}=z\cdot\eta\cdot\left(\mathbb{E}\left(\varphi^{\frac{1}{\gamma}}\,|\,\varphi\in[\widehat{\varphi},\overline{\varphi}]\right)\right)^{\gamma}.$$

Now I characterize unemployment. By definition, the total matched labor (and capital) can be formulated as below.

$$N\equiv\int_0^{\infty}\int_{\widehat{\varphi}}^{\overline{\varphi}}v(\varphi,a)q(\varphi)d\varphi da=\left[\int_{\widehat{\varphi}}^{\overline{\varphi}}q(\varphi)\cdot dF(\varphi)\right]\cdot\lambda K=N=\mathbb{E}_F\left[q(\varphi)|\varphi\geq\widehat{\varphi}\right]\cdot K.$$

Moreover, I have

$$\begin{aligned}
N &\equiv \left[\int_{\widehat{\varphi}}^{\overline{\varphi}}q(\varphi)\cdot dF(\varphi)\right]\cdot\lambda K=\left[\eta\int_{\widehat{\varphi}}^{\overline{\varphi}}\left[\frac{\eta(1-\gamma)\varphi}{W}\right]^{\frac{1-\gamma}{\gamma}}\cdot dF(\varphi)\right]\cdot\lambda K\\
&= \eta\left(\int_{\widehat{\varphi}}^{\overline{\varphi}}\varphi^{\frac{1-\gamma}{\gamma}}\cdot dF(\varphi)\right)\left[\frac{L}{K\cdot\mathbb{E}_F\left(\varphi^{\frac{1}{\gamma}}\,|\,\varphi\in[\widehat{\varphi},\overline{\varphi}]\right)}\right]^{1-\gamma}\lambda K=\Omega(\lambda)\cdot m(K,L),
\end{aligned}$$

where $\Omega(\lambda)\equiv\frac{\mathbb{E}_F\left(\varphi^{\frac{1-\gamma}{\gamma}}\,|\,\varphi\in[\widehat{\varphi},\overline{\varphi}]\right)}{\left[\mathbb{E}_F\left(\varphi^{\frac{1}{\gamma}}\,|\,\varphi\in[\widehat{\varphi},\overline{\varphi}]\right)\right]^{1-\gamma}}$. Since $\gamma\in(0,1)$, Jensen's inequality suggests $\mathbb{E}_F\left(\varphi^{\frac{1-\gamma}{\gamma}}\,|\,\varphi\in[\widehat{\varphi},\overline{\varphi}]\right)<$

$\left[\mathbb{E}_F\left(\varphi^{\frac{1}{\gamma}}|\,\varphi\in[\widehat{\varphi},\overline{\varphi}]\right)\right]^{1-\gamma}$ and thus $N<m(K,L)=N^*$. Consequently, unemployment is

$$u\equiv 1-\frac{N}{L}=1-\left\{\frac{\mathbb{E}_F\left(\varphi^{\frac{1-\gamma}{\gamma}}|\,\varphi\in[\widehat{\varphi},\overline{\varphi}]\right)}{\left[\mathbb{E}_F\left(\varphi^{\frac{1}{\gamma}}|\,\varphi\in[\widehat{\varphi},\overline{\varphi}]\right)\right]^{1-\gamma}}.\right\}\cdot\eta\cdot\left(\frac{K}{L}\right)^{\gamma}.$$

Therefore, u decreases with η, but has nothing to do with z.

Now I address the factor price in labor and credit markets in turn. On the one hand, $\left[\frac{\eta(1-\gamma)}{W}\right]^{\frac{1}{\gamma}}=\frac{L}{K\cdot\mathbb{E}_F\left(\varphi^{\frac{1}{\gamma}}|\,\varphi\in[\widehat{\varphi},\overline{\varphi}]\right)}$, we know that

$$W=\eta(1-\gamma)\left[\left(\frac{K}{L}\right)\cdot\mathbb{E}_F\left(\varphi^{\frac{1}{\gamma}}|\,\varphi\in[\widehat{\varphi},\overline{\varphi}]\right)\right]^{\gamma}.$$

On the other hand, I already prove that

$$Y=\left\{\eta\left(\mathbb{E}_F\left(\varphi^{\frac{1}{\gamma}}|\,\varphi\in[\widehat{\varphi},\overline{\varphi}]\right)\right)^{\gamma}\right\}\cdot K^{\gamma}L^{1-\gamma}.$$

Immediately we have $W=\frac{\partial Y}{\partial L}$. Then I need to pin down the interest rate in the credit market. Since $\pi(\widehat{\varphi},W)=r$ and

$$\pi(\varphi,W)=q(\varphi,W)(\varphi-w(\varphi))=\eta\left(\theta(\varphi,W)\right)^{1-\gamma}\gamma\varphi,$$

the market tightness can be rewritten as $\theta(\varphi,W)=\left(\frac{r}{\eta\gamma\widehat{\varphi}}\right)^{\frac{1}{1-\gamma}}\cdot\left(\frac{\varphi}{\widehat{\varphi}}\right)^{\frac{1}{\gamma}}$. In turn, the labor resource constraint can be re-formulated as

$$\lambda K\left(\frac{r}{\eta\gamma\widehat{\varphi}}\right)^{\frac{1}{1-\gamma}}\cdot\int_{\widehat{\varphi}}^{\overline{\varphi}}\left(\frac{\varphi}{\widehat{\varphi}}\right)^{\frac{1}{\gamma}}dF(\varphi)=L.$$

Therefore, the interest rate can be rewritten as

$$\frac{r}{\partial Y/\partial K}=\frac{\widehat{\varphi}^{\frac{1}{\gamma}}}{\mathbb{E}_F\left(\varphi^{\frac{1}{\gamma}}|\,\varphi\in[\widehat{\varphi},\overline{\varphi}]\right)}=\frac{\widehat{\varphi}_i^{\frac{1}{\gamma}}}{\mathbb{E}_{F^i}\left(\varphi_i^{\frac{1}{\gamma}}|\,\varphi_i\in[\widehat{\varphi}_i,\overline{\varphi}_i]\right)}\equiv 1-\tau_K.$$

Since $\mathbb{E}_F\left[\left(\frac{\varphi}{\widehat{\varphi}}\right)^{\frac{1}{\gamma}}|\,\varphi\in[\widehat{\varphi},\overline{\varphi}]\right]$ decreases $\widehat{\varphi}$ and $\widehat{\varphi}$ increases with λ, we know that $\frac{r}{\partial Y/\partial K}=\frac{1}{\mathbb{E}_F\left[\left(\frac{\varphi}{\widehat{\varphi}}\right)^{\frac{1}{\gamma}}|\,\varphi\in[\widehat{\varphi},\overline{\varphi}]\right]}$, which increases with λ. Moreover, we have

$$\lim_{\lambda\to\infty}\frac{r}{\partial Y/\partial K}=\lim_{\alpha\to 0}\frac{r}{\partial Y/\partial K}=1,\ \lim_{\overline{\varphi}_i\to\infty}\frac{r}{\partial Y/\partial K}=1-\frac{\alpha}{\gamma}.$$

Finally, I show why $\Omega(\lambda)$ increases with λ. It is immediately done by using Assumption 1. Furthermore, I make further characterization on $\Omega(\lambda)$. Notice that we can rewrite $\Omega(\lambda)$ as $\Omega(\lambda)=$

$\frac{\lambda^\gamma \cdot \int_{\widehat{\varphi}}^{\overline{\varphi}} \varphi^{\frac{1-\gamma}{\gamma}} dF(\varphi)}{\left[\int_{\widehat{\varphi}}^{\overline{\varphi}} \varphi^{\frac{1}{\gamma}} dF(\varphi)\right]^{1-\gamma}}$. Denote $A(\lambda) = \int_{\widehat{\varphi}}^{\overline{\varphi}} \varphi^{\frac{1-\gamma}{\gamma}} dF(\varphi)$ and $B(\lambda) = \int_{\widehat{\varphi}}^{\overline{\varphi}} \varphi^{\frac{1}{\gamma}} dF(\varphi)$, then after some algebraic manipulation, we have

$$\Omega'(\lambda) = \frac{\lambda^\gamma}{B(\lambda)^{1-\gamma}} \left\{ \left(\frac{\gamma}{\lambda}\right) A(\lambda) + A'(\lambda) \left[1 - (1-\gamma)\left(\frac{A(\lambda)}{B(\lambda)}\right)\widehat{\varphi}(\lambda)\right]\right\},$$

where

$$A'(\lambda) = -\left(\widehat{\varphi}(\lambda)\right)^{\frac{1-\gamma}{\gamma}} \cdot f(\widehat{\varphi}(\lambda)) \cdot \widehat{\varphi}'(\lambda) = -\left(\widehat{\varphi}(\lambda)\right)^{\frac{1-\gamma}{\gamma}} \cdot f(\widehat{\varphi}(\lambda)) \cdot \left[\frac{1}{\lambda^2} \cdot \frac{1}{f(\widehat{\varphi})}\right] = -\left(\widehat{\varphi}(\lambda)\right)^{\frac{1-\gamma}{\gamma}} \cdot \left(\frac{1}{\lambda^2}\right).$$

Therefore, $\Omega'(\lambda) > 0$ if and only if

$$\left(\frac{\gamma}{\lambda}\right) A(\lambda) + A'(\lambda)\left[1-(1-\gamma)\left(\frac{A(\lambda)}{B(\lambda)}\right)\widehat{\varphi}(\lambda)\right] = \left(\frac{1}{\lambda}\right)\left\{\gamma\int_{\widehat{\varphi}}^{\overline{\varphi}} \varphi^{\frac{1-\gamma}{\gamma}} dF(\varphi) - \left(\frac{1}{\lambda}\right)\left(\widehat{\varphi}(\lambda)\right)^{\frac{1-\gamma}{\gamma}}\left[1-(1-\gamma)\left(\frac{\int_{\widehat{\varphi}}^{\overline{\varphi}} \varphi^{\frac{1-\gamma}{\gamma}} dF(\varphi)}{\int_{\widehat{\varphi}}^{\overline{\varphi}} \varphi^{\frac{1}{\gamma}} dF(\varphi)}\right)\widehat{\varphi}(\lambda)\right]\right\} > 0,$$

or, equivalently, $\Omega'(\lambda) > 0$ if and only if

$$\gamma \quad > \quad \left(\frac{1}{\lambda}\right) \cdot \left[\frac{\left(\widehat{\varphi}(\lambda)\right)^{\frac{1-\gamma}{\gamma}}}{\int_{\widehat{\varphi}}^{\overline{\varphi}} \varphi^{\frac{1-\gamma}{\gamma}} dF(\varphi)}\right] \cdot \left[1-(1-\gamma)\left(\frac{\int_{\widehat{\varphi}}^{\overline{\varphi}} \varphi^{\frac{1-\gamma}{\gamma}} dF(\varphi)}{\int_{\widehat{\varphi}}^{\overline{\varphi}} \varphi^{\frac{1}{\gamma}} dF(\varphi)}\right)\widehat{\varphi}(\lambda)\right].$$

Notice that

$$\begin{aligned}
\left(\frac{1}{\lambda}\right) \cdot \left[\frac{\left(\widehat{\varphi}(\lambda)\right)^{\frac{1-\gamma}{\gamma}}}{\int_{\widehat{\varphi}}^{\overline{\varphi}} \varphi^{\frac{1-\gamma}{\gamma}} dF(\varphi)}\right] \cdot \left[1-(1-\gamma)\left(\frac{\int_{\widehat{\varphi}}^{\overline{\varphi}} \varphi^{\frac{1-\gamma}{\gamma}} dF(\varphi)}{\int_{\widehat{\varphi}}^{\overline{\varphi}} \varphi^{\frac{1}{\gamma}} dF(\varphi)}\right)\widehat{\varphi}(\lambda)\right] \quad &= \quad \left(\frac{1}{\lambda}\right) \cdot \left\{\frac{\left(\widehat{\varphi}(\lambda)\right)^{\frac{1-\gamma}{\gamma}}}{\int_{\widehat{\varphi}}^{\overline{\varphi}} \varphi^{\frac{1-\gamma}{\gamma}} dF(\varphi)} - (1-\gamma) \cdot \frac{\left(\widehat{\varphi}(\lambda)\right)^{\frac{1}{\gamma}}}{\int_{\widehat{\varphi}}^{\overline{\varphi}} \varphi^{\frac{1}{\gamma}} dF(\varphi)}\right\} \\
&= \quad \left(\frac{1}{\lambda}\right) \cdot \left\{\frac{1}{\int_{\widehat{\varphi}}^{\overline{\varphi}} \left(\frac{\varphi}{\widehat{\varphi}(\lambda)}\right)^{\frac{1-\gamma}{\gamma}} dF(\varphi)} - \frac{1-\gamma}{\int_{\widehat{\varphi}}^{\overline{\varphi}} \left(\frac{\varphi}{\widehat{\varphi}(\lambda)}\right)^{\frac{1}{\gamma}} dF(\varphi)}\right\}.
\end{aligned}$$

Therefore $\Omega'(\lambda) > 0$, or Assumption 1 is held, if and only if

$$\mathbb{E}_F\left[\left(\frac{\varphi}{\widetilde{\varphi}}\right)^{\frac{1}{\gamma}} | \varphi \in (\widetilde{\varphi}, \overline{\varphi})\right] \cdot \left\{1 - \left(\frac{1}{\gamma}\right) \cdot \left[\frac{1 - F(\widetilde{\varphi})}{\widetilde{\varphi} \cdot f(\widetilde{\varphi})}\right]\right\} \leq 1.$$

□

Proof on Corollary 3

Proof. There are at least two ways to prove this result. On the the one hand, we can verify this claim by the following reasoning. Since I have proved that the optimal wage scheme in active sub-labor markets is $w(\varphi) = (1-\gamma)\varphi$, we know that all active entrepreneurs gets γ proportion of realized output. Then by definition, the aggregate accumulated wealth by entrepreneurs, $\left\{\int \Psi(\varphi) dF(\varphi)\right\} \cdot K$, should equal to the capital stock after depreciation, $(1-\delta) \cdot K$, plus γ proportion of the aggregate output, γY. On the other hand, we can prove the result by straightforward calculation as below. As defined in the context, $\Psi(\varphi)=\lambda\cdot max\left\{\pi(\varphi)-r,0\right\}+r+(1-\delta)$. Then we have

$$\begin{aligned}\int\Psi(\varphi)dF(\varphi) &= \int\left[\lambda\cdot max\left\{\pi(\varphi)-r,0\right\}+r+(1-\delta)\right]dF(\varphi)\\ &= \lambda\cdot\int max\left\{\pi(\varphi)-r,0\right\}dF(\varphi)+r+(1-\delta)\\ &= \left\{\lambda\cdot\int_{\widehat{\varphi}}^{\overline{\varphi}}\left[\left(\frac{\varphi}{\widehat{\varphi}}\right)^{\frac{1}{\gamma}}-1\right]+1\right\}\cdot r+(1-\delta).\end{aligned}$$

Then using the clearing condition in credit market, *i.e.*, $\lambda=1/\left[1-F(\widehat{\varphi})\right]$, then we have

$$\begin{aligned}\left\{\int\Psi(\varphi)dF(\varphi)\right\}\cdot K &= \left\{\lambda\cdot\int_{\widehat{\varphi}}^{\overline{\varphi}}\left[\left(\frac{\varphi}{\widehat{\varphi}}\right)^{\frac{1}{\gamma}}-1\right]+1\right\}\cdot rK+(1-\delta)K\\ &= \left\{\mathbb{E}_F\left[\left(\frac{\varphi}{\widehat{\varphi}}\right)^{\frac{1}{\gamma}}-1|\varphi\in[\widehat{\varphi},\overline{\varphi}]\right]+1\right\}\cdot(1-\tau)\cdot\left(\frac{\partial Y}{\partial K}\cdot K\right)+(1-\delta)K\\ &= \gamma Y+(1-\delta)K.\end{aligned}$$

Therefore the aggregate transition dynamics is obtained as below.

$$K_{t+1}=\beta\cdot\left[\int_{\Phi}\Psi_t(\varphi)dF_t(\varphi)\right]\cdot K_t=\beta\cdot\left[\gamma Y_t+(1-\delta)K_t\right].$$

□

Proof on Proposition 3

Proof. Given w, the active entrepreneur's decision is

$$\pi(\varphi)\cdot k=\max_{l}\left\{\eta\varphi k^{\gamma}l^{1-\gamma}-wl\right\}.$$

FOC suggests $l=(\eta\varphi)^{\frac{1}{\gamma}}\left(\frac{1-\alpha}{w}\right)^{\frac{1}{\gamma}}k$, which in turn implies

$$\pi(\varphi) = \gamma\left(\eta\varphi\right)^{\frac{1}{\gamma}}\left(\frac{1-\alpha}{w}\right)^{\frac{1-\gamma}{\gamma}}.$$

Since $\pi(\varphi)$ increases with φ, the cut-off point $\widehat{\varphi}$ is determined by $\pi(\widehat{\varphi},w)=r$. The capital and labor demand then is obtained as below.

$$\begin{aligned}k(\varphi,a) &= \begin{cases}\lambda\cdot a & if\ \varphi\geq\widehat{\varphi}\\ 0 & if\ \varphi<\widehat{\varphi}\end{cases}\\ l(\varphi,a) &= (\eta\varphi)^{\frac{1}{\gamma}}\left(\frac{1-\alpha}{w}\right)^{\frac{1}{\gamma}}k(\varphi,a).\end{aligned}$$

In turn, the clearing conditions in credit market, $\int_0^{\infty}\int_{\widehat{\varphi}}^{\overline{\varphi}}k(\varphi,a)h(\varphi,a)d\varphi da=K$, can be simplified as $\lambda\cdot[1-F(\widehat{\varphi})]=1$. Meanwhile, the resource constraint in the labor market, $\int_0^{\infty}\int_{\widehat{\varphi}}^{\overline{\varphi}}l(\varphi,a)h(\varphi,a)d\varphi da=L$, can be rewritten as below.

$$\left[\frac{\eta(1-\alpha)}{w}\right]^{\frac{1}{\gamma}}\left(\lambda K\int_{\widehat{\varphi}}^{\overline{\varphi}}\varphi^{\frac{1}{\gamma}}dF(\varphi)\right)=L.$$

Since $\lambda \cdot [1 - F(\widehat{\varphi})] = 1$, we have $\lambda \int_{\widehat{\varphi}}^{\overline{\varphi}} \varphi^{\frac{1}{\gamma}} dF(\varphi) = \mathbb{E}_F \left(\varphi^{\frac{1}{\gamma}} | \, \varphi \in [\widehat{\varphi}, \overline{\varphi}] \right)$, and therefore $\left[\frac{\eta(1-\alpha)}{w} \right]^{\frac{1}{\gamma}} = \frac{L}{K \cdot \mathbb{E}_F \left(\varphi^{\frac{1}{\gamma}} | \, \varphi \in [\widehat{\varphi}, \overline{\varphi}] \right)}$.

By definition, aggregate output is

$$
\begin{aligned}
Y &= \int_0^\infty \int_{\widehat{\varphi}}^{\overline{\varphi}} y(\varphi, a) h(\varphi, a) d\varphi da \\
&= \int_0^\infty \int_{\widehat{\varphi}}^{\overline{\varphi}} \left(\frac{\pi(\varphi, w) k(\varphi, a)}{\gamma} \right) h(\varphi, a) d\varphi da \\
&= \left[\frac{\eta(1-\alpha)}{w} \right]^{\frac{1-\gamma}{\gamma}} \left(\lambda K \int_{\widehat{\varphi}}^{\overline{\varphi}} \varphi^{\frac{1}{\gamma}} dF(\varphi) \right) \\
&= \left\{ \eta \left(\mathbb{E}_F \left(\varphi^{\frac{1}{\gamma}} | \, \varphi \in [\widehat{\varphi}, \overline{\varphi}] \right) \right)^\gamma \right\} \cdot K^\gamma L^{1-\gamma},
\end{aligned}
$$

and thus $TFP \equiv \frac{Y}{K^\gamma L^{1-\gamma}} = \eta z \left(\mathbb{E}_F \left(\varphi^{\frac{1}{\gamma}} | \, \varphi \in [\widehat{\varphi}, \overline{\varphi}] \right) \right)^\gamma$.

□

Proof on Corollary 4

Proof. By definition, we have

$$
\begin{aligned}
l_t - l_{t+1} &= k_t(\varphi_t, a_t) q_t(\varphi_t) - \mathbb{E} \left[k_{t+1}(\varphi_{t+1}, a_{t+1}) \cdot q_{t+1}(\varphi_{t+1}) \right] \\
&= \lambda_t a_t \cdot \mathbf{1}_{\{\varphi_t \geq \widehat{\varphi}_t\}} q_t(\varphi_t) - \mathbb{E}_{\varphi^{t+1}} \left[\lambda_{t+1} a_{t+1} \cdot \mathbf{1}_{\{\varphi_{t+1} \geq \widehat{\varphi}_{t+1}\}} q_{t+1}(\varphi_{t+1}) \right] \\
&= \lambda_t a_t \cdot \mathbf{1}_{\{\varphi_t \geq \widehat{\varphi}_t\}} q_t(\varphi_t) - \lambda_{t+1} a_{t+1} \left[\rho \cdot \mathbf{1}_{\{\varphi_t \geq \widehat{\varphi}_{t+1}\}} q_{t+1}(\varphi_t) + (1-\rho) \cdot \int_\Phi \mathbf{1}_{\{\varphi \geq \widehat{\varphi}_{t+1}\}} q_{t+1}(\varphi) \cdot dF(\varphi) \right] \\
&= a_t \cdot \left\{ \lambda_t \cdot \mathbf{1}_{\{\varphi_t \geq \widehat{\varphi}_t\}} q_t(\varphi_t) - \lambda_{t+1} \beta \Psi(\varphi_t) \cdot \left[\rho \cdot \mathbf{1}_{\{\varphi_t \geq \widehat{\varphi}_t\}} q_{t+1}(\varphi_t) + (1-\rho) \cdot \int_\Phi \mathbf{1}_{\{\varphi \geq \widehat{\varphi}_{t+1}\}} q_{t+1}(\varphi) \cdot dF(\varphi) \right] \right\} \\
&\equiv a_t \cdot \Delta_{t,t+1}(\varphi^t).
\end{aligned}
$$

□

Proof on Corollary 5

Proof. The first part is obvious. When it comes to growth rate, by definition and using the results of the first part, we have

$$
\begin{aligned}
\mathbb{E} \left[\frac{k_{t+1}}{k_t} | (k_t, \varphi_t; X_t) \right] &= \beta \cdot \left(\frac{\Psi_t(\varphi_t)}{\lambda_t} \right) \cdot \mathbb{E} \left[\mathbf{1}_{\{\varphi' \geq \widehat{\varphi}(X')\}} \cdot \lambda_{t+1} | (\varphi_t, X_t) \right] \\
&= \beta \cdot \left(\frac{\Psi_t(\varphi_t)}{\lambda_t} \right) \cdot \mathbb{E} \left\{ \left[\rho \cdot \mathbf{1}_{\{\varphi_t \geq \widehat{\varphi}_{t+1}\}} + (1-\rho) \cdot (1 - F(\widehat{\varphi}_{t+1})) \right] \cdot \lambda_{t+1} | (\varphi_t, X_t) \right\}.
\end{aligned}
$$

Meanwhile,

$$
\begin{aligned}
\mathbb{E}\left[\frac{n_{t+1}}{n_t}|(n_t,\varphi_t;X_t)\right] &= \frac{\mathbb{E}\left[k_t(\varphi'_{t+1},a_{t+1})q_{t+1}(\varphi'_{t+1})\right]}{k_t(\varphi_t,a_t)q_t(\varphi_t)} \\
&= \beta\cdot\Psi_t(\varphi_t)\left(\frac{\lambda_{t+1}}{\lambda_t}\right)\left[\rho\cdot\left(\frac{q_{t+1}(\varphi_t)}{q_t(\varphi_t)}\right)+(1-\rho)\cdot\left(\frac{\int_{\widehat{\varphi}_{t+1}}^{\overline{\varphi}} q(\varphi_{t+1})dF(\varphi_{t+1})}{q_t(\varphi_t)}\right)\right].
\end{aligned}
$$

□