Financial Constraints, Endogenous Markups, and Self-fulfilling Equilibria

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Abstract

We show that self-fulfilling equilibria and indeterminacy can easily arise in a simple financial accelerator model with reasonable parameter calibrations and without increasing returns in production. A key feature for generating indeterminacy in our model is the countercyclical markup due to the procyclical loan to output ratio. We illustrate, via simulations, that our financial accelerator model can generate rich business cycle dynamics, including hump-shaped output in response to demand shocks as well as serial autocorrelation in output growth rates.

Keywords: Financial Constraints, Endogenous Markups, Self-fulfilling Equilibria, Indeterminacy.

JEL codes: E2, E44

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1 Introduction

Since the seminal work of Bernanke and Gertler (1990)[1] the financial accelerator model with collateral constraints has been widely used to explain the amplification of shocks to business cycles. Since firms and businesses face borrowing costs to finance their working capital which depends on the collateral value of their assets and output, any downturn or negative shock that depresses the value of their collateral will curtail their ability to finance investments and increase their operating costs. This, in turn, will amplify the downturn. Conversely, any positive shock that appreciates the value of a firm's collateral will decrease the cost of external finance, increase profitability, and amplify the effect of the initial shock. This mechanism, however, suggests the possibility of self-fulfilling multiple equilibria: Optimistic expectations of higher output may well lead to increased lending to financially constrained firms. Even though our model has no increasing returns in production, the relaxation of the borrowing constraint implies that unit marginal costs can increase with output as firms compete for more labor and capital. In such a case markups can become countercyclical and factor returns can increase sufficiently so that the expectation of higher output can become self-fulfilling. The purpose of this paper is to show that multiple equilibria and indeterminacy can easily arise in a simple financial accelerator model with realistic parameter calibrations, and that the model can reasonably match some of the quantitative features of economic data.

Our paper is related to several other papers on financial constraints and business cycle fluctuations that ascribe a significant part of such fluctuations to financial shocks. Examples include the work of Jermann and Quadrini (2011), Liu, Wang and Zha (2011), Gertler and Kiyotaki (2011), among many others. But where do financial shocks come from? A growing literature links financial constraints to asset bubbles as a source of financial shocks. For example, Farhi and Tirole (2011), Miao and Wang (2011), Wang and Wen (2012), Miao, Wang and Xu (2012), and Ventura and Martin (2012) study asset bubbles in economies with borrowing constraints. They show that the growth and burst of asset bubbles can generate endogenous fluctuations in borrowing limits which result in booms and busts in the real economy. A shortcoming of such asset bubble models is that the bubbleless steady state is a sink, and cannot explain the recurrent fluctuations in the borrowing limits unless bubbles arrive exogenously.

Our paper is also related to the recent paper of Liu and Wang (2010). They show that financial constraints can generate indeterminacy through an endogenous TFP channel as a

[1] See also Bernanke, Gertler and Gilchrist (1995, 1996, 1999), and Kiyotaki and Moore (1997).

result of resource reallocation across firms. In their model, firms differ in productivity and in the absence of credit constraints, only the most productive firms survive while the unproductive firms with high costs perish. Some unproductive firms however continue to produce as the more productive firms are financially constrained by the value of their assets. An expected increase in aggregate output increases the value of the assets of all firms, and relaxes their borrowing constraints. This relaxation in the borrowing constraints allows more productive firms to expand production. This in turn pushes up the factor prices and increases the cost of production for the unproductive firms. As some of the unproductive firms stop producing the resource reallocation towards more productive firms generates endogenous increasing return to scale. Liu and Wang (2010) show that their model is isomorphic to the Benhabib-Farmer (1994) model after aggregation.

This paper provides an alternative and complementary mechanism for indeterminacy to Liu and Wang (2010). While they emphasize reallocation effects of financial constraints, we focus on an endogenous markup channel. For this purpose, we introduce borrowing constraints into an otherwise standard Dixit-Stiglitz monopoly competition model. Firms rent capital and hire labor in the competitive markets to produce differentiated intermediate goods. The firms however may default on their promise or contract to repay their debt. We assume therefore that firms face borrowing constraints when financing their working capital, determined by the fraction of firm revenues and assets that the creditors can recover, minus some fixed collection costs. This constrains the output as well as the unit marginal costs of firms. Given the fixed collection costs however, if households expect a higher equilibrium output, they will be willing to increase their lending to firms, even if the marginal costs of firms rise and their markups decline as they compete for additional labor and capital.[2] At the new equilibrium both output and factor returns will be higher. Despite the income effects on labor supply, the increase in wages associated with lower markups will allow employment and output to increase, so the optimistic expectations of higher output will be fulfilled.

We describe the baseline model in the next section. Section 2.6 provides the main results characterizing the parameter ranges where equilibrium is indeterminate, as well as examples, graphical illustrations and a discussion of our parameter calibrations. Section 3 offers extensions that relaxes the fixed cost component of the borrowing constraint, eliminates capacity utilization, and provides examples of indeterminacy under these extensions. The section 4 introduces sunspots into the discrete-time version of the model, calibrates it to match the mo-

[2] The countercyclical markup is consistent with data (see Rotemberg and Woodford (1999)).

ments of US data, and generates impulse responses to technology and sunspot shocks. Section 5 concludes.

2 A baseline model

2.1 Firms

To illustrate the driving features of our model, we start with a simple benchmark model of monopolistic competition and borrowing constraints for intermediate goods producers. The production side is a standard Dixit and Stiglitz model of monopolistic competition. There is a competitive final goods producer that combines a continuum of intermediate goods $Y_t(i)$ to produce final goods Y_t according to the technology

$$Y_t = \left[\int Y_t^{\frac{\sigma-1}{\sigma}}(i)di\right]^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

where $\sigma \geq 1$. The final goods producer solves

$$\max_{y_t(i)} \left[\int Y_t^{\frac{\sigma-1}{\sigma}}(i)di\right]^{\frac{\sigma}{\sigma-1}} - \int P_t(i)Y_t(i)di. \tag{2}$$

where $P_t(i)$ the price of the i-th type of intermediate goods. The first-order conditions lead to the following inverse demand functions for intermediate goods:

$$P_t(i) = Y_t^{-\frac{1}{\sigma}}(i)Y_t^{\frac{1}{\sigma}}, \tag{3}$$

where the aggregate price index is

$$1 = \left[\int P_t^{1-\sigma}(i)di\right]^{\frac{1}{1-\sigma}}. \tag{4}$$

Intermediate goods producers. The technology for producing intermediate goods is given by

$$Y_t(i) = AK_t^{\alpha}(i)N_t^{1-\alpha}(i), \tag{5}$$

where $A > 0$, $0 < \alpha < 1$. We assume symmetry: the technology for producing intermediate goods is the same for all i. The profit for i'th intermediate good producer is

$$\Pi_t(i) = P_t(i)Y_t(i) - w_tN_t(i) - r_tK_t(i). \tag{6}$$

Denote by $\phi_t = \frac{1}{A}(\frac{r_t}{\alpha})^{\alpha}(\frac{w_t}{1-\alpha})^{1-\alpha}$ the unit cost for the intermediate goods firms. Then their profit is:

$$\Pi_t(i) = P_t(i)Y_t(i) - \phi_tY_t(i). \tag{7}$$

The Financial Constraint. Unlike the final goods producer, we assume that producers of intermediate goods face financial constraints due to limited enforcement. In our simple benchmark model we assume that in the beginning of each period, the i'th intermediate goods firm decides to rent capital $K_t(i)$ from the households and hire labor $N_t(i)$. To cover its short term operating costs the firm borrows from the households and promises to pay $w_tN_t(i) + r_tK_t(i) \equiv b_t(i)$at he end of the period. However the firm may default on its contract or promise. We assume that if the firm does not pay its debt $b_t(i)$, the households can recover a fraction $\xi < 1$ of the firm's revenue $P_t(i)Y_t(i)$ by incurring a liquidation cost f. [3] One possibility is that the firm must pay the labor wages as production takes place , and that creditors can always redeem the physical capital, but that the interest on borrowing may not be fully recoverable. So if the household can recover $\xi P_t(i)Y_t(i) - f$, they will lend to the firm only if $\xi P_t(i)Y_t(i) - f$ can at least cover the wage bill plus principal and interest. Knowing that the household cannot recover more than $\xi P_t(i)Y_t(i) - f$, the firm will have no incentive to repay more than $\xi P_t(i)Y_t(i) - f$. The incentive-compatiblity constraint for the firm then is:

$$P_t(i)Y_t(i) - [w_tN_t(i) + r_tK_t(i)] \geq P_t(i)Y_t(i) - [\xi P_t(i)Y_t(i) - f], \tag{8}$$

or

$$w_tN_t(i) + r_tK_t(i) \leq \xi P_t(i)Y_t(i) - f. \tag{9}$$

After substituting $P_t(i)$ from equation (3) into equation (8), the profit maximization for the $i'th$ firm becomes

$$\max_{Y_t(i)} Y_t^{1-\frac{1}{\sigma}}(i)Y_t^{\frac{1}{\sigma}} - \phi_t Y_t(i), \tag{10}$$

subject to

$$\phi_t Y_t(i) + f \leq \xi Y_t^{1-\frac{1}{\sigma}}(i)Y_t^{\frac{1}{\sigma}}. \tag{11}$$

Given w_t, r_t, final output Y_t, and the borrowing constraint (11), the feasible choices of $Y_t(i)$ are represented by the shaded area in Figure 1.[4]

[3] To calibrate $\xi < 1$ we note that outstanding credit market debt for domestic non-financial business corporate and non-corporate sectors in the US in 2012 stood at 12 trillion, or about 77% of GDP.See the Federal Reserve "Flow of Funds report", June 07, 2012, table D.3 in particular, at http://www.federalreserve.gov/releases/z1/Current/z1.pdf

[4] In an alternative formulation the firm also borrows enough to directly purchase its capital stock in addition to its needs for operating costs: $w_tN_t(i) + (1 + r_t) K_t(i)$. If for simplicity we assume that lenders can always recover the capital stock in case of default but only a fraction of the output, the constraint becomes $w_tN_t(i) + (1 + r_t) K_t(i) \leq \xi P_t(i)Y_t(i) + K_t(i) - f$. After cancelling $K_t(i)$ from both sides, we again have the constraint (9). In this case however debt would exceed GDP, since it would include the borrowed capital stock. Of course some part of the capital stock may represent business equity that also yielding a competitive return of r_t,rather than debt. In such a case however equity returns and principal may be subordinated to debt, but for simplicity we may abstract from these considerations.

If we denote by $\mu_t(i)$ the Lagrangian multiplier of constraint (11), the first-order conditions for the profit maximization are

$$r_t K_t(i) = \alpha \phi_t Y_t(i), \tag{12}$$

$$w_t N_t = (1-\alpha)\phi_t Y_t(i), \tag{13}$$

and

$$(1-\frac{1}{\sigma})P_t(i) - \phi_t + \mu_t(i)[\xi(1-\frac{1}{\sigma})P_t(i) - \phi_t] = 0, \tag{14}$$

with the slackness condition

$$\left[\xi Y_t^{1-\frac{1}{\sigma}}(i) Y_t^{\frac{1}{\sigma}} - \phi_t Y_t(i) - f\right]\mu_t(i) = 0. \tag{15}$$

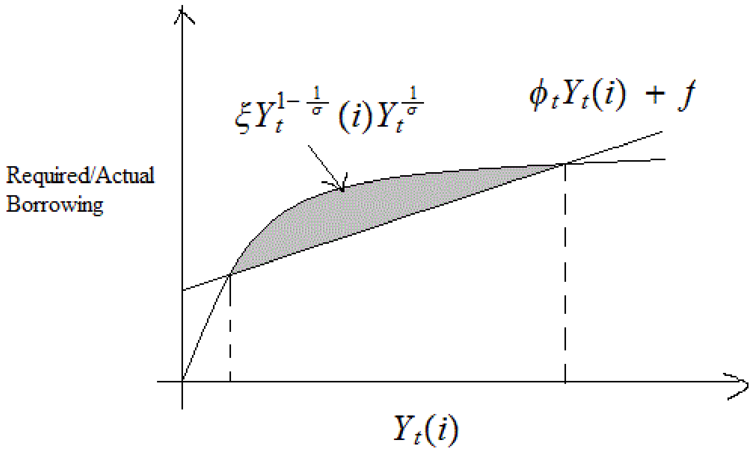


Figure 1. The Credit Constraints and Feasible Ouptut Choice.

2.2 Households

We now turn to the intertemporal optimization problem faced by a representative consumer. To facilitate the stability analysis of a steady state, we set our model in continuous time. The instantaneous utility of the representative consumer is given by

$$\log C_t - \psi \frac{N_t^{1+\chi}}{1+\chi}, \tag{16}$$

where C is consumption, N is labor supply, and $\chi \geq 0$. Taking the market interest rate r_t and wage w_t as given, the representative consumer maximizes

$$\int_0^\infty [\log C_t - \psi \frac{N_t^{1+\chi}}{1+\chi}] e^{-\rho t} dt, \tag{17}$$

subject to

$$\dot{K}_t = r_t e_t K_t - \delta(e_t) K_t + w_t N_t - C_t + \Pi_t, \tag{18}$$

where K_t is the capital stock and K_0 is given. We model endogenous capacity utilization along the lines of Greenwood, Hercowitz and Huffman (1988). For simplicity we assume that the households choose the capacity utilization rate e_t. A higher e_t implies that the capital is more intensively utilized, at the cost of faster depreciation, so that $\delta(e_t)$ is a convex increasing function. The parameter ρ represents the discount rate, and Π_t is total profit of all firms.

We note at this point that the indeterminacy results that follow will hold even in the absence of variable capacity utilization, but we include it in our model to improve calibration results in section .

The first-order conditions for the consumer's optimization problem are given by

$$\frac{\dot{C}_t}{C_t} = r_t e_t - \rho - \delta(e_t), \tag{19}$$

$$r_t = \delta'(e_t), \tag{20}$$

and

$$\psi N_t^{\chi} = \frac{1}{C_t} w_t. \tag{21}$$

2.3 Equilibrium

The equilibrium in the economy is a collection of price processes $\{w_t, r_t, P_t(i)\}$ and quantities $\{K_t(i), N_t(i), Y_t(i), Y_t, K_t, N_t, e_t, \Pi_t\}$, such that a) given the prices and the aggregate Π_t, the households choose K_t and N_t to maximize their utility; b) given $P_t(i)$, the final good firm chooses $\{Y_t(i)\}$ to maximize its profits defined in (2); c) given w_t, r_t, and the financial constraint (11), the intermediate goods producers maximizes its profit by choosing $K_t(i)$ and $N_t(i)$; and all markets clear. Since firms are symmetric, we have $K_t(i) = K_t$, $N_t(i) = N_t$, $P_t(i) = 1$, $Y_t(i) = Y_t$ and $\Pi_t(i) = \Pi_t = Y_t - w_t N_t - r_t K_t$. The budget constraint becomes

$$\dot{K}_t = Y_t - C_t - \delta K_t. \tag{22}$$

The wage w_t and the interest rate r_t are

$$w_t = (1-\alpha)\phi_t \frac{Y_t}{N_t}, \tag{23}$$

and

$$r_t = \alpha \phi_t \frac{Y_t}{K_t}. \tag{24}$$

Equation (14) becomes

$$(1 - \frac{1}{\sigma}) - \phi_t + \mu_t(\xi(1 - \frac{1}{\sigma}) - \phi_t) = 0. \tag{25}$$

We then have the following lemma regarding the financial constraint (11).

Lemma 1 *If $\xi(1 - \frac{1}{\sigma}) < \phi_t < 1 - \frac{1}{\sigma}$, then the financial constraint binds; that is*

$$\phi_t Y_t(i) + f \leq \xi Y_t^{1-\frac{1}{\sigma}}(i) Y_t^{\frac{1}{\sigma}}. \tag{26}$$

Based on fact that $Y_t(i) = Y_t$, the constraint implies

$$\phi_t = \xi - \frac{f}{Y_t} \tag{27}$$

The intuition for Lemma 1 is as follows. The firms' profit function is $\Pi_t(i) = Y_t^{1-\frac{1}{\sigma}}(i)Y_t^{\frac{1}{\sigma}} - \phi_t Y_t(i)$ and if the profit for the marginal unit evaluated at equilibrium is $(1 - \frac{1}{\sigma}) - \phi_t > 0$, the firms would have the incentive to increase their output. This profit on the marginal unit exceeds the revenue that households can recover in case of default if $\phi_t \leq \xi(1-\frac{1}{\sigma})$. Therefore the original output level cannot be optimal because firms would be able to borrow and produce more to increase their production and their profits. If $\xi(1 - \frac{1}{\sigma}) < \phi_t < 1 - \frac{1}{\sigma}$ however, firms would not be able to increase their production since the borrowing constraint binds: an additional unit of output would allow the firms to borrow only an additional $\xi(1 - \frac{1}{\sigma})$, which is not enough to cover the marginal unit production cost ϕ_t.[5]

[5] Formally,

$$\begin{aligned}
(1 - \frac{1}{\sigma}) - \phi_t &= -\mu_t(\xi(1 - \frac{1}{\sigma}) - \phi_t) \\
0 &> \frac{1}{-\mu_t} = \frac{(\xi(1 - \frac{1}{\sigma}) - \phi_t)}{(1 - \frac{1}{\sigma}) - \phi_t} \\
sign\left((1 - \frac{1}{\sigma}) - \phi_t\right) &= -sign(\xi(1 - \frac{1}{\sigma}) - \phi_t) \\
\xi(1 - \frac{1}{\sigma}) - \phi_t &< (1 - \frac{1}{\sigma}) - \phi_t \text{ if } \xi < 1 \\
\xi(1 - \frac{1}{\sigma}) &< \phi_t < (1 - \frac{1}{\sigma})
\end{aligned}$$

We will focus on the parameters that make financial constraint (11) always binding in equilibrium. To summarize, the following system of equations fully characterize the equilibrium

$$\begin{aligned}
\frac{\dot{C}_t}{C_t} &= \phi_t \frac{\alpha Y_t}{K_t} - \rho - \delta(e_t), && (28)\\
\dot{K}_t &= Y_t - \delta(e_t)K_t - C_t, && (29)\\
\psi N_t^{\chi} &= \frac{1}{C_t}\phi_t \frac{(1-\alpha)\,Y_t}{N_t}, && (30)\\
Y_t &= A(e_t K_t)^{\alpha} N_t^{1-\alpha}, && (31)\\
\phi_t \frac{\alpha Y_t}{e_t K_t} &= \delta'(e_t) && (32)\\
\phi_t &= \xi - \frac{f}{Y_t}, && (33)
\end{aligned}$$

subject to the constraint $\xi(1-\frac{1}{\sigma}) < \phi_t < 1-\frac{1}{\sigma}$. Following Greenwood, Hercowitz and Huffman (1988) let the depreciation function be given by

$$\delta(e_t) = \delta_0 \frac{e_t^{1+\nu}}{1+\nu} \tag{34}$$

We then have

$$\phi \frac{\alpha Y}{eK} = \delta'(e) = \delta_0 e^{\nu} \tag{35}$$

2.4 Steady state

We first solve for the deterministic steady state. Denote by X the steady state value of X_t. Unfortunately, the model does not have a full analytical solution for the steady state with the fixed financial cost. In the following, we describe the major steps for solving for steady state $\{Y, K, N, e, c, \phi, r, w\}$. We first express the other variables as a function of the steady state ϕ recursively.

1. Using the first-order condition $\phi \frac{\alpha Y}{K} = \delta_0 e^{\nu+1}$, we have

$$\delta(e) = \frac{1}{1+\nu}\phi \frac{\alpha Y}{K}$$

.

2. Equation (28) then implies $\frac{\nu}{1+\nu}\phi\frac{\alpha Y}{K} = \rho$ so we have

$$K = \frac{\nu}{1+\nu}\frac{\phi\alpha}{\rho}Y$$

3. Combining the above, we have $\delta(e) = \delta_0 \frac{e^{1+\nu}}{1+\nu} = \frac{1}{1+\nu}\phi\frac{\alpha Y}{K} = \frac{\rho}{\nu}$, or

$$e = \left[\frac{(1+\nu)\rho}{\nu\delta_0}\right]^{\frac{1}{1+\nu}} \tag{36}$$

We normalize δ_0 such that $e = 1$.

4. To solve N we use

$$\begin{aligned} \frac{C}{Y} &= 1 - \delta(e)\frac{K}{Y} \\ &= 1 - \frac{\rho}{\nu}\frac{\nu}{1+\nu}\frac{\phi\alpha}{\rho} \\ &= 1 - \frac{\phi\alpha}{1+\nu} \end{aligned}$$

so that

$$N = \left[\phi\frac{(1-\alpha)}{\frac{C}{Y}}\frac{1}{\psi}\right]^{\frac{1}{1+\chi}}$$

5. We then obtain the output

$$Y = K^\alpha N^{1-\alpha} = \left(\frac{\nu}{1+\nu}\frac{\phi\alpha}{\rho}Y\right)^\alpha \left[\phi\frac{(1-\alpha)}{1-\frac{\phi\alpha}{1+\nu}}\frac{1}{\psi}\right]^{\frac{1-\alpha}{1+\chi}}$$

or

$$Y = \left(\frac{\nu}{1+\nu}\frac{\phi\alpha}{\rho}\right)^{\frac{\alpha}{1-\alpha}} \left[\phi\frac{(1-\alpha)}{1-\frac{\phi\alpha}{1+\nu}}\frac{1}{\psi}\right]^{\frac{1}{1+\chi}} \equiv Y(\phi)$$

6. Finally from the definition of $\phi = \xi - \frac{f}{Y}$, we have

$$f = (\xi - \phi)Y(\phi) \equiv \Psi(\phi), \tag{37}$$

which determines the steady-state value of ϕ. In what follows we can treat the steady state value of marginal cost ϕ as a parameter and allow f to adjust.

7. For the existence of a steady state however we will need to assume $\xi(1-\frac{1}{\sigma}) < \phi < 1 - \frac{1}{\sigma}$. Define $\Psi(\phi) = (\xi - \phi)Y(\phi)$ and $\bar{\Psi} = \max_{0\leq\phi\leq\xi}\Psi(\phi)$. Notice also that

$$\Psi(\xi) = \Psi(0) = 0. \tag{38}$$

Lemma 2 *If $0 < f < \bar{\Psi}$, then equation (37) has at least two solutions such that*

$$\Psi(\phi) - f = 0 \tag{39}$$

Lemma 3 *For $0 < f < \Psi(\xi(1-\frac{1}{\sigma}))$, there is a steady state ϕ such that $\xi(1-\frac{1}{\sigma}) < \phi < \xi$.*

Proof: Since $\Psi(\xi) - f < 0$ and $\Psi(\xi(1-\frac{1}{\sigma})) - f > 0$, by the intermediate value theorem there is a steady state ϕ that lies between $\xi(1-\frac{1}{\sigma})$ and ξ such that $\Psi(\phi) - f = 0$. ■

2.5 Log-linearization

After obtaining the steady state (Y, K, N, C, ϕ), we log-linearize the system of equations around the steady state value. We denote by $\hat{X}_t$ the percentage deviation of variable X_t from its steady state value X_t, that is, $\hat{X}_t = \log X_t - \log X$. Let $\dot{c}_t = \frac{d(\log C_t - \log C)}{dt}$ and $\dot{k}_t = \frac{d(\log K_t - \log K)}{dt}$. Then the log-linearized system of equations is

$$\begin{aligned}
\dot{c}_t &= \rho[\hat{Y}_t - \hat{K}_t + \hat{\phi}_t] && (40)\\
\dot{k}_t &= \frac{(1+\nu)\delta}{\alpha\phi}(\hat{Y}_t - \hat{K}_t) - \left(\frac{(1+\nu)\delta}{\alpha\phi} - \delta\right)(\hat{C}_t - \hat{K}_t) && (41)\\
&\quad -\delta(\hat{Y}_t + \hat{\phi}_t - \hat{K}_t) && \\
\chi\hat{N}_t &= \hat{\phi}_t + \hat{Y}_t - \hat{N}_t - \hat{C}_t && (42)\\
\hat{Y}_t &= \alpha(\hat{K}_t + \hat{e}_t) + (1-\alpha)\hat{N}_t && (43)\\
\hat{e}_t &= \frac{1}{1+\nu}(\hat{\phi}_t + \hat{Y}_t - \hat{K}_t) && (44)\\
\hat{\phi}_t &= \frac{f/Y}{\xi - f/Y}\hat{Y}_t \equiv \gamma\hat{Y}_t && (45)
\end{aligned}$$

where $\delta = \frac{\rho}{\nu}$. Note that γ can also be defined by the steady state value of ϕ as $\gamma = \frac{\xi-\phi}{\phi}$. Eliminating $\hat{N}_t$ from equation (42), $\hat{\phi}_t$ from equation (45), and $\hat{e}_t$ from equation (44) allows us to obtain the expression for $\hat{Y}_t$ in terms of capital and consumption. We first substitute $\hat{e}_t$ out of the production function which gives

$$\begin{aligned}
\hat{Y}_t &= \frac{1}{1+\nu-(1+\gamma)\alpha}[(1+\nu)(1-\alpha)\hat{N}_t + \alpha\nu\hat{K}_t] && (46)\\
&\equiv \omega_1\hat{N}_t + \omega_2\hat{K}_t
\end{aligned}$$

where $\omega_1 = \frac{(1+\nu)(1-\alpha)}{1+\nu-(1+\gamma)\alpha}$ and $\omega_2 = \frac{\alpha\nu}{1+\nu-(1+\gamma)\alpha}$. It is easy to check that $\omega_1 + \omega_2 > 1$ if $\gamma > 0$. Finally we need to substitute out $\hat{N}_t$. Combining the labor demand and labor supply curves we have

$$\hat{Y}_t = \lambda_1\hat{K}_t + \lambda_2\hat{C}_t, \qquad (47)$$

where $\lambda_1 = \frac{\omega_2(1+\chi)}{\chi+1-(1+\gamma)\omega_1}$, $\lambda_2 = \frac{-\omega_1}{\chi+1-(1+\gamma)\omega_1}$.

Using the factor $\hat{\phi}_t = \gamma \hat{Y}_t$ from (45), the log-linearized Euler condition becomes:

$$\dot{c}_t = \rho[(1+\gamma)\left(\lambda_1 \hat{K}_t + \lambda_2 \hat{C}_t\right) - \hat{K}_t]. \tag{48}$$

Then equation (41) yields

$$\begin{aligned} \dot{k}_t &= \left\{\frac{(1+\nu)\delta}{\alpha\phi}\lambda_1 - \delta(1+\gamma)\lambda_1\right\}\hat{K}_t \\ &\quad + \left\{\frac{(1+\nu)\delta}{\alpha\phi}(\lambda_2 - 1) + \delta - \delta(1+\gamma)\lambda_2\right\}\hat{C}_t \end{aligned} \tag{49}$$

In a matrix form

$$\begin{bmatrix} \dot{k}_t \\ \dot{c}_t \end{bmatrix} = J \begin{bmatrix} \hat{K}_t \\ \hat{C}_t \end{bmatrix} \tag{50}$$

where

$$J = \begin{bmatrix} \frac{(1+\nu)\delta}{\alpha\phi}\lambda_1 - \delta(1+\gamma)\lambda_1] & \frac{(1+\nu)\delta}{\alpha\phi}(\lambda_2 - 1) + \delta - \delta(1+\gamma)\lambda_2 \\ \rho\left[(1+\gamma)\lambda_1 - 1\right] & \rho(1+\gamma)\lambda_2 \end{bmatrix} \tag{51}$$

Finally using the factor $\rho = \delta\nu$, we have

$$J = \delta \begin{bmatrix} \frac{(1+\nu)}{\alpha\phi}\lambda_1 - (1+\gamma)\lambda_1 & \frac{(1+\nu)}{\alpha\phi}(\lambda_2 - 1) + 1 - (1+\gamma)\lambda_2 \\ \nu\left[(1+\gamma)\lambda_1 - 1\right] & \nu(1+\gamma)\lambda_2 \end{bmatrix}$$

2.6 Dynamics around the steady state

The local dynamics around the steady state is determined by the roots of J. The trace of the J is

$$\text{Trace}\,(J) = \delta\left[\frac{(1+\nu)}{\alpha\phi}\lambda_1 - (1+\gamma)\lambda_1 + \nu(1+\gamma)\lambda_2\right] \tag{52}$$

and the determinant of J is

$$\det(J) = \left\{[(1+\gamma)\lambda_1 - 1 + \lambda_2]\left(\frac{1+\nu}{\alpha\phi} - 1\right) - \gamma\lambda_2\right\}\delta^2\nu \tag{53}$$

The roots of J, x_1 and x_2 satisfy the following constraints

$$x_1 + x_2 = \text{Trace}\,(J)\,, \tag{54}$$

and

$$x_1 x_2 = \det(J). \tag{55}$$

If $\det(J) > 0$ and $\text{Trace}(J) < 0$, then the roots x_1 and x_2 will both be negative, and the model will have local indeterminacy around the steady state. Since given other parameters the trace and determinant are functions of γ and ϕ, we will first examine the possibility of indeterminacy in the parameter space of γ and ϕ. We will then use the mapping between (γ, ϕ) and (f, ξ) to establish the possibility of indeterminacy supported by the deep parameters of the model.

Proposition 1 *Let γ and ϕ satisfy the following two constraints*

$$(1+\gamma) > \frac{(1+\nu)(1+\chi)}{\alpha(1+\chi)+(1+\nu)(1-\alpha)} \tag{56}$$

and

$$1+\gamma \quad < \quad \min(\frac{1+\nu}{\alpha}, \frac{\left(\frac{1+\nu}{\phi}\right)(1+\chi)}{\alpha(1+\chi)+(1+\nu)(1-\alpha)}, \\ \frac{(1-\alpha)(1+\chi)}{(1+\nu)(1-\alpha)\frac{1}{1+\nu-\alpha\phi}+(1+\chi)\alpha}+1) \tag{57}$$

Then

$$Trace\,(J) < 0, \det(J) > 0 \tag{58}$$

Proof: See Appendix A1. ■

To gain intuition for self-fulfilling expectations of higher output and higher factor rewards, we first focus on labor demand and supply curves incorporating the equilibrium effects of the borrowing constraint on marginal costs and markups. The labor demand curve is given by

$$\hat{w}_t = (1+\gamma)\hat{Y}_t - \hat{N}_t = \frac{(1+\gamma)(1+\nu)(1-\alpha)}{1+\nu-(1+\gamma)\alpha} - 1 \tag{59}$$

and the labor supply curve in the economy is

$$\hat{w}_t = \hat{C}_t + \chi\hat{N}_t. \tag{60}$$

The slope of the labor market demand curve is positive and steeper than that of the labor supply curve under the condition $(1+\gamma) > \frac{(1+\nu)(1+\chi)}{\alpha(1+\chi)+(1+\nu)(1-\alpha)}$ of the Proposition above. The indeterminacy result then parallels the results in Benhabib and Farmer (1994) and Wen (1998). However unlike their works, our model has no increasing returns in the production technology. Instead indeterminacy arises from the borrowing constraints and their indirect effects on marginal costs through wages and the rental rate on capital. If households expect a higher equilibrium output, they will be willing to increase their lending to firms. Given positive fixed collection costs f, an expected increase in output levels relaxes the borrowing constraint so that the unit marginal costs of firms, $\phi_t = \xi - \frac{f}{Y_t}$, can rise and markups can fall. This implies that as firms compete for inputs, factor rewards will also increases with Y_t. The labor demand curve incorporating these general equilibrium effects on marginal costs will then be positively sloped and steeper than the labor supply curve. Normally, higher output levels increase the demand for leisure, so barring inferiority in preferences, the higher demand for labor will be contained by the income effect on labor supply. However if the labor demand slopes up more steeply than labor supply, employment will increase robustly as the labor supply curve shifts to the left with income effects. The rise in labor hours as well as the accumulation of capital will raise output, so that the optimistic output expectations of households will be self-fulfilling.

Before turning to calibrations, we formally state the indeterminacy result of the paper. We define the set

$$\Omega = \{(\gamma, \phi) | constraint(56), constraint(57) are\ satisfied\}$$

.

Proposition 2 *For* $(\gamma, \phi) \in \Omega$, $\phi < 1 - \frac{1}{\sigma}$ *and* $\gamma = \frac{\xi - \phi}{\phi} < \frac{1}{\sigma - 1}$ *(that is* $\xi \left(1 - \frac{1}{\sigma}\right) < \phi$*), construct the set* $\Theta = \{(\xi, f) : |$ *such that* $\gamma = \frac{\xi - \phi}{\phi}$, $f = (\xi - \phi) Y(\phi)$ *and* $(\gamma, \phi) \in \Omega\}$*. For* $(\xi, f) \in \Theta$*, the model is indeterminate.*

Proof: For $(\xi, f) \in \Theta$, by construction we can find a solution for ϕ and γ such that

$$\gamma = \frac{\xi - \phi}{\phi};\ f = (\xi - \phi) Y(\phi) \tag{61}$$

and since $(\gamma, \phi) \in \Omega$, by Proposition 1, we have $Trace < 0, \det(J) > 0$. So the model is indeterminate around the steady state. ■

2.7 Calibrations

To calibrate our model we use some standard parameter values in the literature. We set the quarterly discount rate $\rho = 0.01$, so the quarterly discount factor is $\beta = \frac{1}{1+\rho} = 0.99$. We make the labor supply fully elastic, so $\chi = 0$. The factor share of capital is, $\alpha = \frac{1}{3}$.The steady state depreciation rate is given by equation (34), where we normalized the steady state utilization rate $e = 1$ in equation (36). This gives $\delta_0 = 0.04333$. If we assume that the lifetime of new equipment averages 30 quarters, or that $\delta(e) = 0.0333$, then we can solve for $\nu = 0.3$. The markup in our model is the inverse of the steady state marginal cost $\frac{1}{\phi} - 1$. We set the steady state markup to 12%, which implies a steady state marginal cost $\phi = \xi - \frac{f}{Y(\phi)} = 0.88$. This also represents the steady state ratio of non-financial private business debt to GDP. As noted earlier, this ratio in the data is a little lower at 0.77, and excludes all private debt issued by financial institutions, mostly to consumers, as well as state and federal debt. (See footnote 3.) Note that choosing the steady state markup and therefore ϕ constrains the parameters ξ

and f through equation (37): $\phi = \xi - \frac{f}{Y(\phi)}$. Furthermore the steady state marginal cost ϕ together parameters ξ and σ must satisfy the inequality constraints of Proposition 2, $\xi\left(1-\frac{1}{\sigma}\right) < \phi < 1-\frac{1}{\sigma}$. In models with constant markups, the steady state Dixit-Stiglitz elasticity σ is usually calibrated to immediately obtain a markup of $10-15\%$, and often σ is set to 10 (see for example Dotsey and King (2005) or Sbordone (2008), p. 20). We set $\sigma = 10$ as well, even though σ does not exactly determine ϕ in our model but constrains it. Finally we can calibrate the financial constraint parameters to assure that the markup is indeed 12%. If we set $\xi = 0.9768$ then equation (37) implies a steady state value of the fixed liquidation cost $f = 0.1908$ in case of a default. Below in Figure 2 we illustrate the regions of indeterminacy in the $\xi - f$ plane. Finally for the above calibration we can easily check that the constraints $\xi\left(1-\frac{1}{\sigma}\right) < \phi < 1-\frac{1}{\sigma}$ are satisfied, so by Proposition 2 the steady state is indeed indeterminate, with the value of output at this steady state equal to 1.9711.

Note that for our calibrated parameters, the implied liquidation costs amount to 12% of the firm's total sale revenue[6]. For 88 firms that reorganized during 1982-1993, Alderson and Betker (1995) find that the liquidation costs measured as the percent of loss in going-concern value is large. For example, only 25% of firms have liquidation costs less than 12.8%. The mean and median liquidation costs among all the firms are 36.5% and 34.7%, respectively. Using a very comprehensive sample of corporate bankruptcies, Bris, Welch and Zhu (2006) find the direct expenses alone accounts for 8.1% of pre-bankruptcy assets for firms who filed for Bankruptcy in the United States under Chapter 7 of the U.S. Bankruptcy Code, and 16.9% for firms who filed for Bankruptcy Under Chapter 11[7] (see Bris, Welch and Zhu (2006), table X, p. 1281). Similarly for international data Thorburn (2000) reports that direct expenses on average account for 19.1% (with medium 13.2%) of the market value of assets in Sweden. So the implied liquidation costs in our model are in line with the data. In addition, both Thorburn (2000) and Bris, Welch and Zhu (2006) find that the expenses to assets ratio declines significantly with firm's scale, suggesting an important fixed cost component in the direct liquidation expenses.

Figure 2 illustrates the combinations of f and ξ that yield indeterminacy with the other parameters set to $\nu = 0.3$, $\alpha = \frac{1}{3}$, $\rho = 0.01$, $\chi = 0$. The feasible parameter values for f and ξ are graphed in these two shaded areas. Consider the borrowing constraint $f = (\xi - \phi)Y(\phi)$. For a given ξ there exist a minimum f and a maximum f consistent with the steady state

[6] The liquidation costs include the fixed cost f and the loss in output $(1-\xi)Y$. So in the steady state they account for $f/Y + 1 - \xi$ fraction of total output (sale revenue).

[7] Chapter 7 expenses mainly include expenses on debtor's attorney, accountant, and trustee. Chapter 11 expenses mainly include debtor expenses and unsecured creditors' committee expenses.

equilibrium such that $\frac{\sigma-1}{\sigma}\xi < \phi < \frac{\sigma-1}{\sigma}$. Notice that if $f = 0$ and $\phi = \xi$, as long as $\xi < \frac{\sigma-1}{\sigma}$, the condition $\frac{\sigma-1}{\sigma}\xi < \phi < \frac{\sigma-1}{\sigma}$ is automatically satisfied. This implies that for $\xi < \frac{\sigma-1}{\sigma}$ the minimum f is zero. But if $\xi \geq \frac{\sigma-1}{\sigma}$ then $f = 0$ (hence $\phi = \xi$) is no longer consistent with the equilibrium. If f is too small, then ϕ will be larger than $\frac{\sigma-1}{\sigma}$. Since $f = (\xi - \phi)Y(\phi)$ is decreasing in ϕ, the lower bound for f is $f_{\min}(\xi) = (\xi - \frac{\sigma-1}{\sigma})Y(\frac{\sigma-1}{\sigma})$ for $\xi \geq \frac{\sigma-1}{\sigma}$. We can write it as $f_{\min}(\xi) = \max(\left(\xi - \frac{\sigma-1}{\sigma}\right) Y(\frac{\sigma-1}{\sigma}), 0)$ for $0 < \xi \leq 1$. On the other hand if f is too large, then the marginal cost will fall below $\frac{\sigma-1}{\sigma}\xi$. Now maximizing f over $\frac{\sigma-1}{\sigma}\xi < \phi < \frac{\sigma-1}{\sigma}$ the upper bound for f for a given ξ is $f_{\max}(\xi) = \frac{1}{\sigma}\xi Y(\frac{\sigma-1}{\sigma}\xi)$. For these feasible parameters, if f is greater than some cut-off level, then the implied γ will be bigger than $\gamma_{\min}$. It turns out that the condition $\gamma < \gamma_{\max}$ is automatically satisfied. The cut-off f can be determined by

$$f_{cut}(\xi) = \max\{\frac{\gamma_{\min}}{1+\gamma_{\min}}\xi Y(\frac{\xi}{1+\gamma_{\min}}), f_{\min}(\xi)\}.$$

For any f such that $f_{cut}(\xi) \leq f < f_{\max}(\xi)$, we have $\gamma > \gamma_{\min}$, so the model is locally indeterminate around the steady state. In Figure 2, the indeterminacy region is shown in red.

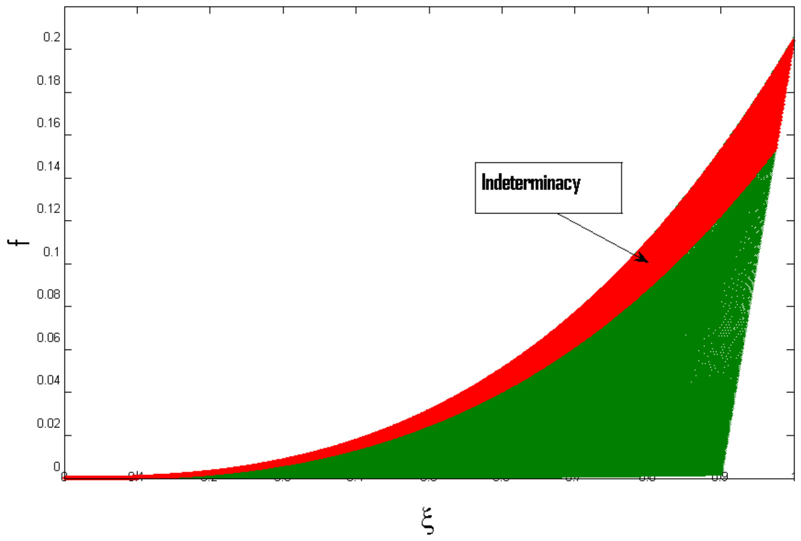


Figure 2. Parameter Spaces for Indeterminacy.

The shaded areas (the red areas together with the green areas) are the feasible ξ and f. The upper shaded areas with red color yields indeterminacy around the steady state.

3 Discussion and extensions of the model

3.1 The role of fixed costs

In this section we argue that it is not the fixed liquidation costs per se that generates indeterminacy. It is the procyclical leverage generated by fixed liquidity costs that is the source of indeterminacy. Note that with fixed costs the debt to GDP ratio

$$\frac{b_t}{Y_t} = \xi - \frac{f}{Y_t} \tag{62}$$

is procyclical. In what follows, we construct an example in which the firm's borrowing constraint is

$$\phi_t Y_t(i) \leq \xi(\frac{Y_t}{Y}) Y_t^{1-\frac{1}{\sigma}}(i) Y_t^{\frac{1}{\sigma}}, \tag{63}$$

where $\xi_t = \xi(\frac{Y_t}{Y}) < \frac{\sigma-1}{\sigma}$ is an increasing function of $\frac{Y_t}{Y}$ with $\xi(1) = \phi < \frac{\sigma}{\sigma-1}$ and $\xi'(1) = \gamma\phi$. In this case, the marginal cost is $\phi_t = \xi_t$. The condition $\xi_t \frac{\sigma-1}{\sigma} < \phi_t < \frac{\sigma-1}{\sigma}$ is automatically satisfied, so the borrowing constraint is binding. The equilibrium can be characterized by a system of nonlinear equations similar to equations (28) to (33), except that equation (33) is now replaced by $\phi_t = \xi_t = \xi(\frac{Y_t}{Y})$. The log-linearized system of equations, however, is exactly the same as equations (40) to (45). So we can directly invoke Proposition 2 if

$$\gamma_{\min}(\phi) < \gamma < \gamma_{\max}(\phi) \tag{64}$$

for $0 < \phi < \frac{\sigma-1}{\sigma}$. The more general function $\xi(\frac{Y_t}{Y})$ which replaces the fixed liquidation cost eliminates the constraint relating γ and ϕ in the benchmark model, and provides more flexibility in generating a range of parameters such that indeterminacy holds.

3.2 The role of capacity utilization

With the more general function $\xi(\frac{Y_t}{Y})$, we now show that endogenous capacity utilization is not essential for indeterminacy even though it makes indeterminacy possible for a wider range of parameters, as demonstrated by Wen (1998). The equilibrium is characterized by:

$$\frac{\dot{C}_t}{C_t} = \phi_t \frac{\alpha Y_t}{K_t} - \rho - \delta, \tag{65}$$

$$\dot{K}_t = Y_t - \delta(e_t) K_t - C_t, \tag{66}$$

$$\psi N_t^{\chi} = \frac{1}{C_t} \phi_t \frac{(1-\alpha) Y_t}{N_t}, \tag{67}$$

$$Y_t = A K_t{}^{\alpha} N_t^{1-\alpha}, \tag{68}$$

$$\phi_t = \xi_t = \xi(\frac{Y_t}{Y}). \tag{69}$$

The local dynamics around the steady state is:

$$\left[\begin{array}{c} \dot{k}_t \\ \dot{c}_t \end{array}\right] = \left[\begin{array}{cc} \frac{\rho+\delta}{\alpha\phi}\lambda_1 - \delta & \frac{\rho+\delta}{\alpha\phi}\lambda_2 - \frac{\rho+\delta}{\alpha\phi} + \delta \\ (\rho+\delta)\left[(1+\gamma)\lambda_1 - 1\right] & (\rho+\delta)(1+\gamma)\lambda_2 \end{array}\right] \left[\begin{array}{c} \hat{K}_t \\ \hat{C}_t \end{array}\right] \tag{70}$$

where $\lambda_1 = \frac{\alpha(1+\chi)}{\chi+1-(1+\gamma)(1-\alpha)}$ and $\lambda_2 = \frac{-(1-\alpha)}{\chi+1-(1+\gamma)(1-\alpha)}$.[8]

Proposition 3 *The model is indeterminate if*

$$\frac{(1+\chi)}{1-\alpha} - 1 < \gamma < \frac{1}{1-\alpha}\frac{\frac{1}{\phi} - \frac{\delta}{\rho+\delta}}{1 - \frac{\delta}{\rho+\delta}}(1+\chi) - 1, \tag{71}$$

and

$$\gamma < \frac{(1-\alpha)(1+\chi)}{(1+\alpha\chi) + (1-\alpha)\frac{\delta}{\frac{\rho+\delta}{\alpha\phi} - \delta}}. \tag{72}$$

Proof: See Appendix A2. ■

Example 1 *Suppose* $\chi = 0$, $\sigma = 10, \rho = 0.01$, $\alpha = \frac{1}{3}$, $\sigma = 10$, *and* $\delta = 0.0333$. *We assume* $\xi(1) = 0.88$ *and* $\xi'(1) = 0.88\gamma$. *This implies* $\phi = 0.88$ *in the steady state. The model is indeterminate if* $0.5 < \gamma < 0.5582$.

4 Simulation exercises

In this section, we write the model in discrete time and solve it by log-linearizing the equations that characterize the equilibrium around the steady state. We adopt a standard parameterization: $\beta = \frac{1}{1+\rho} = 0.99$, $\alpha = 1/3$, $\delta = 0.033, \sigma = 10$ and $\nu = 0.3$. We set $\xi = 0.9768$, $f = 0.1908$ and fix the productivity level to $A = 1$. These parameter values imply steady state values $\phi = 0.88$ and $\gamma = \frac{f/Y}{\xi - f/Y} = 0.11$. We begin without fundamental shocks. In the case of indeterminacy, the model's solution takes the form

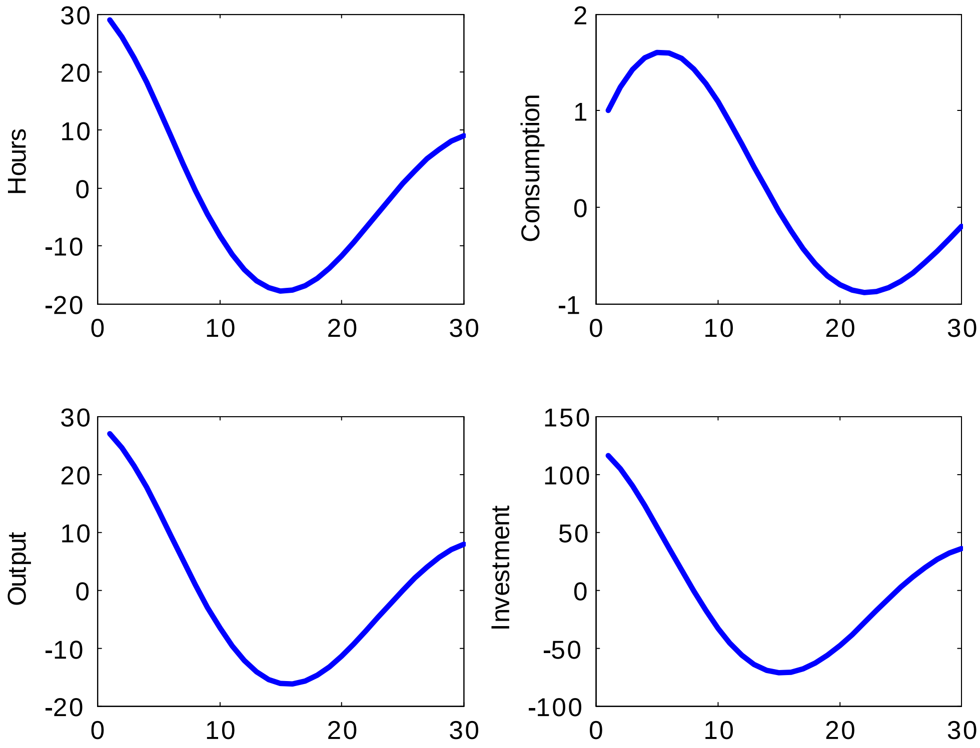
$$\left(\begin{array}{c} \hat{K}_{t+1} \\ \hat{C}_{t+1} \end{array}\right) = M \left(\begin{array}{c} \hat{K}_t \\ \hat{C}_t \end{array}\right) + \left(\begin{array}{c} 0 \\ \varepsilon_{t+1} \end{array}\right) \tag{73}$$

where M is a two-by-two matrix and $\varepsilon_{t+1} = \hat{C}_{t+1} - E_t\hat{C}_{t+1}$ is the sunspot shock. The remaining variables can be written as functions of $\hat{K}_t$ and $\hat{C}_t$:

$$\left(\begin{array}{c} \hat{Y}_t \\ \hat{I}_t \\ \hat{N}_t \\ \hat{e}_t \end{array}\right) = H \left(\begin{array}{c} \hat{K}_t \\ \hat{C}_t \end{array}\right) \tag{74}$$

[8] With minor modifications and reinterpretations, it is possible to transform our model so that its local dynamics around the steady states associated with (70) are isomorphic to the dynamics in Wen (1998).

where H is a four-by-two matrix. Figure 3 shows the impulse responses of output, investment, consumption and hours to an unexpected one percentage increase in the initial consumption level induced by the agent's optimistic expectations about future income.



Figre 3. Impulse Responses to a consumption (sunspot) Shock.

The impulse response functions resemble those obtained in the models with increasing returns to scale. From Figure 3 we see that output, investment, consumption and hours commove. The impulse responses also demonstrate that labor is slightly more volatile than output, an important feature of the data that the standard RBC model has difficulty explaining with a TFP shock. The impulse responses also show cycles in output, investment, consumption and hours, so the model has the potential to explain the boom-bust patterns often observed in data. However, as in the models with increasing returns to scale, the extremely large impact of autonomous consumption on output and investment seems empirically unjustified. In the impact period, one percentage increase in consumption leads to a 27 percent increase in output and a 116 percent increase in investment.

These volatile responses of output and investment can be understood by studying the effect of consumption on labor. Equating the labor demand (59) and labor supply (60) we have

$$\hat{N}_t = \frac{1}{\frac{(1+\gamma)(1+\nu)(1-\alpha)}{1+\nu-(1+\gamma)\alpha} - 1 - \chi} \hat{C}_t. \tag{75}$$

where $\frac{(1+\gamma)(1+\nu)(1-\alpha)}{1+\nu-(1+\gamma)\alpha} - 1$ is the slope of the labor demand curve and χ is the slope of the labor supply curve. When these two slopes are close, a one percentage increase in autonomous consumption increase can lead to huge increases in labor and hence output. Denote s as the ratio of steady state investment to income. Then from the resource constraint,

$$s\hat{I}_t + (1-s)\hat{C}_t = \hat{Y}_t, \tag{76}$$

so it is clear that the combination of smooth consumption and volatile income will make investment even more volatile as $s << 1$. In the current calibration $s = 0.23$. So the response of investment upon impact will be about 4.4 times that of output.

Table 1 reports some basic moments of the linearized model assuming that sunspots are the only driving force. All moments for the model are calculated analytically. The table shows that all variables are positively correlated with output. The correlations between them are also highly persistent. By our construction, the sunspots are i.i.d., so the persistence of the variables is not due to the persistence of exogenous shocks, but comes from the internal propagation mechanism of the model. Table 1 confirms that labor is slightly more volatile than output. The relative volatility of labor is 1.10 in the data and 1.08 in the model while the relative volatility of labor is 0.53 in the real business cycle model (see table 2).

Table 1: Sample and Model Moments

	US Sample			Model		
var	σ_X/σ_Y	$corr(X,Y)$	$corr(X_t, X_{t-1})$	σ_X/σ_Y	$corr(X,Y)$	$corr(X_t, X_{t-1})$
Y	1.00	1.00	0.87	1.00	1.00	0.91
N	1.01	0.88	0.92	1.08	0.99	0.91
C	0.52	0.83	0.90	0.07	0.48	0.97
I	3.33	0.92	0.92	4.31	0.99	0.91
ϕ	0.32	0.16	0.70	0.11	1.00	0.91

Note: Variables (Y, N, C, I, ϕ) denote output, labor (in hours), consumption, investment and marginal cost respectively. The marginal cost in the data can be computed via $\phi = \frac{labor\ share}{1-\alpha}$. σ_X/σ_Y is the standard deviation of variable X relative to output, $corr(X,Y)$ computes the correlation between X and output, and $corr(X_t, X_{t-1})$ computes the first-order autocorrelation of X_t.

To better match the relative volatilities of consumption and output we now introduce a TFP shock into the model. We assume that the technology level in the economy follows an AR(1) process

$$\hat{A}_{t+1} = \rho_a \hat{A}_t + \sigma_a \varepsilon_{at+1}. \tag{77}$$

Following Benhabib and Wen (2004), we assume the sunspots shocks and technology shocks are correlated. Following King and Rebelo (1999) , we assume $\rho_a = 0.98$. The technology shock ε_{at} and sunspot shocks ε_t are assumed to be perfectly correlated and the relative volatility of sunspot and technology shocks is set to $\sigma_s/\sigma_\varepsilon = 1.5$. These bring the relative volatility of consumption closer to data. The moments with correlated TFP shocks and sunspots shocks are in Table 2.

Table 2: Moments with correlated TFP and Sunspot Shocks

	Model with Correlated Shocks			The RBC Model		
var	σ_X/σ_Y	$corr(X,Y)$	$corr(X_t, X_{t-1})$	σ_X/σ_Y	$corr(X,Y)$	$corr(X_t, X_{t-1})$
Y	1.00	1.00	0.98	1.00	1.00	0.95
N	0.96	0.95	0.99	0.53	0.73	0.90
C	0.37	0.55	0.99	0.62	0.86	0.99
I	3.38	0.96	0.98	2.65	0.88	0.91
ϕ	0.11	1.00	0.98	0	N.A	N.A

The RBC model refers to $f = 0$, so $\gamma = 0$, and $\phi = \xi$ is a constant. We select the parameter values such that the two models have the same steady state. For the RBC model, we use TFP shocks with $\rho_a = 0.98$ as the only driving force.

Hump-Shaped output Dynamics The above simulation exercises show that our model with indeterminacy has a similar ability to that of the RBC model to match some key moments in the data. In the simulation exercise that follows, we illustrate how our indeterminacy model can also predict some aspects of actual fluctuations that standard RBC models cannot explain, such as the hump-shaped, trend-reverting impulse response of output to transitory demand shocks, and the substantial serial correlation in output growth rates in the data (see Cogley and Nason(1995)). Since there is significant empirical evidence favoring demand shocks as a main source of business cycles, (e.g., see Blanchard and Quah(1989), Watson (1993), Cogley and Nason (1995) and Benhabib and Wen (2004)), it is important to examine whether demand shocks can generate persistent business cycles. We consider two types of demand shocks as in Benhabib and Wen (2004): government spending shocks and preference shocks. With preference shocks the period-by-period utility function is now given by $U = \exp(\Delta_t) \log C_t - \psi \frac{N_t^{1+\chi}}{1+\chi}$. We assume that the preference shocks Δ_t follow an AR(1) process, namely $\Delta_t = \rho_\Delta \Delta_{t-1} + \varepsilon_{\Delta t}$. With government spending in period t, G_t, the resource constraint changes to $\dot{K}_t = Y_t - \delta(e_t)K_t - C_t - G_t$. We assume that $\log(G_t) = \rho_g \log(G_{t-1}) + \varepsilon_{gt}$. We choose $\rho_g = \rho_\Delta = 0.90$ as in Benhabib and Wen (2004). To highlight the effect of indeterminacy on the propagation mechanism of RBC models, we graph the impulse responses to a persistent government spending shock with and without indeterminacy in Figure 4. Figure 5 graphs the impulse response of the model to a persistent preference shock. For the model without indeterminacy we set $f = 0$ and reset $\xi = 0.88$ so that the models with and without indeterminacy have the same steady state. We set the steady state government spending to GDP ratio to 0.2 as in Benhabib and Wen (2004).

Several features of Figure 4 deserve particular mention. First, in the case of $f = 0$, the marginal cost $\phi_t = \xi$ is a constant. Hence the impulse responses of our model with financial constraints resemble those of a standard RBC model. Figure 4 and Figure 5 show that the standard RBC model has difficulty in generating business cycle fluctuations. Figure 4 shows that consumption and investment move against each other after a positive government spending shock. An increase in government spending generates a negative wealth effect, which reduces both consumption and leisure. The decrease in leisure leads to an increase in output, and an increase in output together with a decrease in consumption imply that investment has to increase. Second, even though the model generates comovement without indeterminacy under persistent preference shocks, the responses of output to such demand shocks are monotonic. Neither government spending shocks nor preference shocks can generate the hump-shaped output dynamics observed in the data. And these monotonic and persistent output responses to demand shocks mostly come from the persistence of shocks, not from an inner propagation mechanism of the model. If the persistence of the shocks is reduced, the persistence of output responses will be reduced accordingly. Third, when the model is indeterminate, the responses of output to both the government spending shocks and the preference shocks are dramatically changed. Figure 4 and Figure 5 clearly shows persistent and hump-shaped responses of output to both shocks. In addition, these persistent responses of output are not due to the persistence in shocks. As Figure 3 has already demonstrated, the model with indeterminacy can generate persistent fluctuations even under i.i.d shocks. Figure 4 and Figure 5 again highlight the similarity of our indeterminacy model with those based on increasing returns to scale, so it has the ability to explain other puzzles. For example, Benhabib and Wen (2004) demonstrate that their indeterminacy model based on increasing returns to scale can explain the forcecastable movement puzzle pointed out by Rotemberg and Woodford (1996). It is easy to show that our indeterminacy model can also replicate the highly forecastable comovements observed in changes in output, hours, investment and consumption highlighted by Rotemberg and Woodford (1996). To avoid repetition, we skip such a simulation exercise and refer readers to Benhabib and Wen (2004). In brief, these simulation exercises illustrate the ability of our indeterminacy model to replicate rich business cycle dynamics observed in the data.

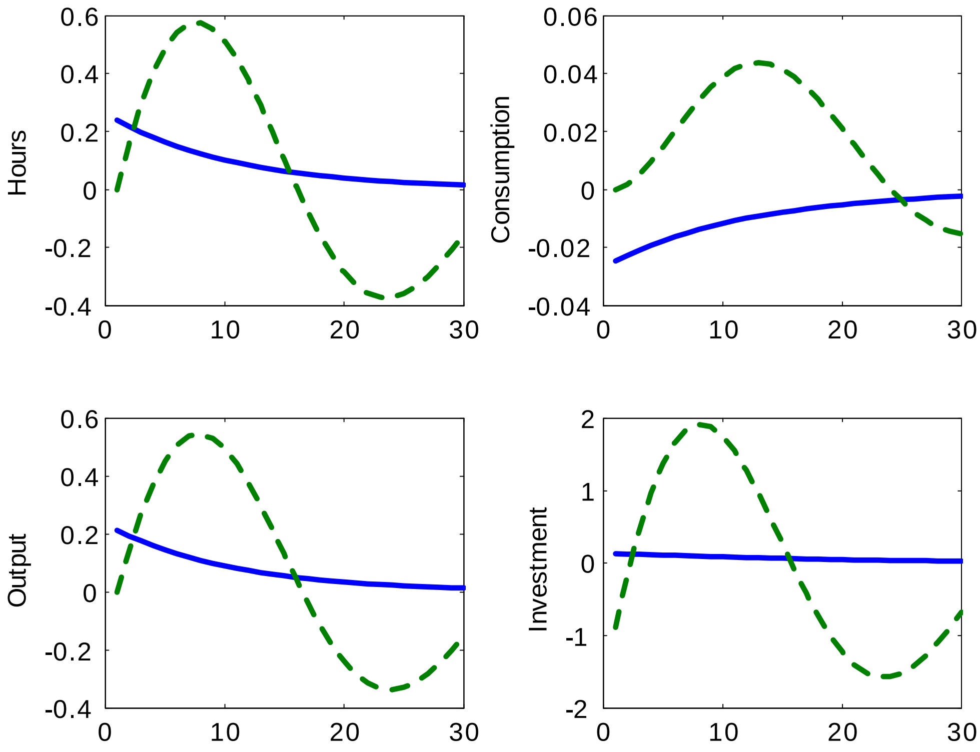


Figure 4. Impulse responses to a government spending shock.

Solid lines are responses under determinacy ($f = 0$) and dashed lines are responses under indeterminacy.

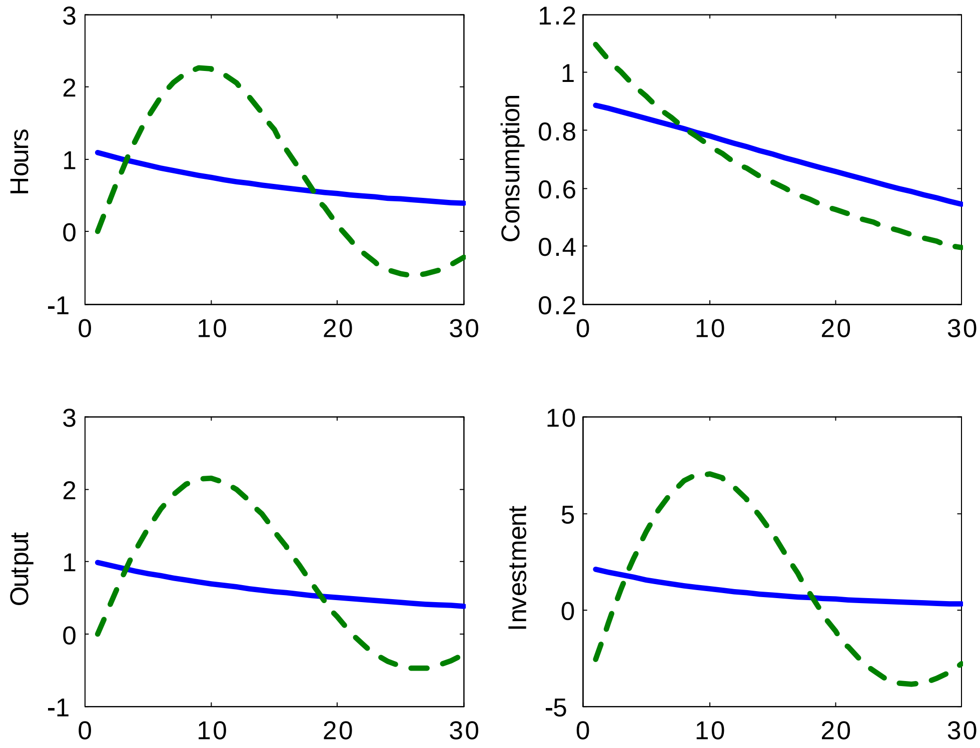


Figure 5. Impulse responses to a preferenc shock.

Solid lines are responses under determinacy ($f = 0$) and dashed lines are responses under indeterminacy.

5 Conclusion

We conclude that borrowing or collateral constraints can be a source of self-fulfilling fluctuations in economies that have no increasing returns to scale in production. Expectations of higher output can relax borrowing constraints, and firms can expand their output by bidding up factor prices and eliciting a labor supply response that allows the initial expectations to be fulfilled. The parameter ranges and markups that allow self-fulfilling expectations to occur are within realistic ranges and compatible with US macroeconomic data. Simulating our data we obtain moments and impulse responses that match the US macroeconomic data reasonably well.

Appendix

A Proofs

A. 1 The Proofs of Proposition 1

First we substitute for $\lambda_1 = \frac{\omega_2(1+\chi)}{\chi+1-(1+\gamma)\omega_1}$ and $\lambda_2 = \frac{-\omega_1}{\chi+1-(1+\gamma)\omega_1}$, and we obtain

$$\frac{\text{Trace}\,(J)}{\delta} = \frac{1}{\chi+1-(1+\gamma)\omega_1}\left[\left(\frac{1+\nu}{\alpha\bar{\phi}} - 1 - \gamma\right)\omega_2(1+\chi) - \nu(1+\gamma)\omega_1\right]. \tag{A.1}$$

The determinant of J is

$$\frac{\det(J)}{\delta^2\nu} = \left(\frac{1+\nu}{\alpha\phi} - 1\right)\frac{(1+\chi)[\omega_2(1+\gamma)-1]+\gamma\omega_1}{\chi+1-(1+\gamma)\omega_1} + \frac{\gamma\omega_1}{\chi+1-(1+\gamma)\omega_1}. \tag{A.2}$$

If $(1+\gamma) > \frac{(1+\nu)(1+\chi)}{\alpha(1+\chi)+(1+\nu)(1-\alpha)}$, we have

$$\begin{aligned}
(1+\gamma)\omega_1 - 1 &= \frac{(1+\gamma)(1+\nu)(1-\alpha)}{1+\nu-(1+\gamma)\alpha} - 1, \\
&> \frac{\frac{(1+\nu)(1+\chi)}{\alpha(1+\chi)+(1+\nu)(1-\alpha)}(1+\nu)(1-\alpha)}{1+\nu - \frac{(1+\nu)(1+\chi)}{\alpha(1+\chi)+(1+\nu)(1-\alpha)}\alpha} - 1, \\
&= \chi + 1 - 1 > \chi.
\end{aligned}$$

It follows that

$$\frac{1}{\chi+1-(1+\gamma)\omega_1} < 0, \tag{A.3}$$

and

$$\begin{aligned}
&\left(\frac{1+\nu}{\alpha\phi} - 1 - \gamma\right)\omega_2(1+\chi) - \nu(1+\gamma)\omega_1 \\
= \;& \frac{(1+\nu)-\alpha\phi(1+\gamma)}{\alpha\phi}\frac{\alpha\nu(1+\chi)}{1+\nu-(1+\gamma)\alpha} - \nu(1+\gamma)\frac{(1+\nu)(1-\alpha)}{1+\nu-(1+\gamma)\alpha} \\
= \;& \frac{\nu}{1+\nu-(1+\gamma)\alpha}\{\frac{1+\nu}{\phi}(1+\chi) - (1+\gamma)[\alpha(1+\chi)+(1+\nu)(1-\alpha)]\} \\
> \;& 0.
\end{aligned} \tag{A.4}$$

So the trace is negative. Finally we have

$$\begin{aligned}
&\left(\frac{1+\nu}{\alpha\phi} - 1\right)\frac{(1+\chi)[\omega_2(1+\gamma)-1]+\gamma\omega_1}{\chi+1-(1+\gamma)\omega_1} + \frac{\gamma\omega_1}{\chi+1-(1+\gamma)\omega_1}, \qquad \text{(A.5)}\\
= \;& \frac{1+\nu}{(1+\gamma)\omega_1 - \chi - 1}\left\{(1+\chi)(1-\alpha) - [(1-\alpha)(1+\nu)/(1+\nu-\alpha\phi) + (1+\chi)\alpha]\,\gamma\right\} \\
> \;& 0
\end{aligned}$$

so the determinant is positive. Q.E.D.

A. 2 The Proofs of Proposition 3

In the case without capacity utilization, the trace is

$$\begin{aligned} Trace &= \frac{\rho+\delta}{\alpha\phi}\lambda_1 + (\rho+\delta)(1+\gamma)\lambda_2 - \delta, \\ &= \frac{\rho+\delta}{\chi+1-(1+\gamma)(1-\alpha)}(\frac{\alpha(1+\chi)}{\alpha\phi} - (1+\gamma)(1-\alpha)) - \delta. \end{aligned}$$

The second line uses $\lambda_1 = \frac{\alpha(1+\chi)}{\chi+1-(1+\gamma)(1-\alpha)}$ and $\lambda_2 = \frac{-(1-\alpha)}{\chi+1-(1+\gamma)(1-\alpha)}$. The determinant is

$$\begin{aligned} \frac{\det(J)}{(\rho+\delta)} &= \frac{\rho+\delta}{\alpha\phi}[(1+\gamma)\lambda_1 + \lambda_2 - 1] + \delta[1-(1+\gamma)(\lambda_1+\lambda_2)] \qquad (A.6) \\ &= [\frac{\rho+\delta}{\alpha\phi} - \delta][(1+\gamma)\lambda_1 + \lambda_2 - 1] - \gamma\lambda_2\delta \end{aligned}$$

The necessary and sufficient condition for indeterminacy is that Trace $(J) < 0$ and $\frac{\det(J)}{(\rho+\delta)} > 0$. Under the condition $\chi + 1 < (1+\gamma)(1-\alpha)$ the trace is negative if

$$\frac{1}{\chi+1-(1+\gamma)(1-\alpha)}(\frac{1+\chi}{\phi} - (1+\gamma)(1-\alpha)) < \frac{\delta}{\rho+\delta}. \qquad (A.7)$$

or

$$\frac{(1+\chi)}{\phi} - (1+\gamma)(1-\alpha) > [\chi+1-(1+\gamma)(1-\alpha)]\,\frac{\delta}{\rho+\delta}. \qquad (A.8)$$

If we rearrange terms we have

$$\left(\frac{1}{\phi} - \frac{\delta}{\rho+\delta}\right)(1+\chi) > (1+\gamma)(1-\alpha)\frac{\rho}{\rho+\delta}. \qquad (A.9)$$

So the necessary and sufficient condition for the trace to be negative is:

$$(1+\chi) < (1+\gamma)(1-\alpha) < \frac{\frac{1}{\phi} - \frac{\delta}{\rho+\delta}}{1-\frac{\delta}{\rho+\delta}}(1+\chi). \qquad (A.10)$$

We now check the sign of the determinant. Substituting $\lambda_1 = \frac{\alpha(1+\chi)}{\chi+1-(1+\gamma)(1-\alpha)}, \lambda_2 = \frac{-(1-\alpha)}{\chi+1-(1+\gamma)(1-\alpha)}$
, we obtain

$$\begin{aligned} \frac{\det(J)}{(\rho+\delta)} &= [\frac{\rho+\delta}{\alpha\phi} - \delta]\frac{(1-\alpha)(1+\chi) - \gamma(1+\alpha\chi)}{(1+\gamma)(1-\alpha)-1-\chi} \\ &\quad - \frac{\gamma\delta(1-\alpha)}{(1+\gamma)(1-\alpha)-1-\chi} \qquad (A.11) \end{aligned}$$

Under the condition $\chi + 1 - (1+\gamma)(1-\alpha) < 0$, $\det(J) > 0$ is equivalent to

$$(1-\alpha)(1+\chi) > \gamma(1+\alpha\chi) + \gamma(1-\alpha)\frac{\delta}{\frac{\rho+\delta}{\alpha\phi} - \delta} \qquad (A.12)$$

which implies that γ can not be too big. Q.E.D.

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