Stock Market Bubbles and Unemployment*

Jianjun Miao[†], Pengfei Wang[‡], and Lifang Xu[§]

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Abstract

This paper introduces endogenous credit constraints in a search model of unemployment. These constraints generate multiple equilibria supported by self-fulfilling beliefs. A stock market bubble exists through a positive feedback loop mechanism. The collapse of the bubble tightens the credit constraints, causing firms to reduce investment and hirings. Unemployed workers are hard to find jobs generating high and persistent unemployment.

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[†]Department of Economics, Boston University, 270 Bay State Road, Boston, MA 02215. Email: miaoj@bu.edu. Tel.: 617-353-6675.

[‡]Department of Economics, Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong. Tel: (+852) 2358 7612. Email: pfwang@ust.hk

[§]Department of Economics, Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong. Email: tsuilf@ust.hk

1 Introduction

This paper provides a theoretical study that links unemployment to the stock market bubbles and crashes. Our theory is based on three observations from the U.S. labor, credit, and stock markets. First, the U.S. stock market has experienced booms and busts and these large swings may not be explained entirely by fundamentals. Shiller (2005) documents extensive evidence on the U.S. stock market behavior and argues that many episodes of stock market booms are attributed to speculative bubbles. Second, the stock market booms and busts are often accompanied by the credit market booms and busts. A boom is often driven by a rapid expansion of credit to the private sector accompanied by rising asset prices. Following the boom phase, asset prices collapse and a credit crunch arises. This leads to a large fall in investment and consumption and an economic recession may follow.[1] Third, the stock market and unemployment are highly correlated.[2] Figure 1. plots the post-war U.S. monthly data of the real Standard and Poor's Composite Stock Price Index constructed by Robert Shiller and the unemployment rate downloaded from the Bureau of Labor Statistics (BLS).[3] This figure shows that, during recessions, the stock price fell and the unemployment rate rose. In particular, during the recent Great Recession, the unemployment rate rose from 5.0 percent at the onset of the recession to a peak of 10.1 percent in October 2009, while the stock market fell by more than 50 percent from October 2007 to March 2009.

Motivated by the preceding observations, we build a search model with credit constraints, based on Blanchard and Gali (2010). The Blanchard and Gali model is isomorphic to the Diamond-Mortensen-Pissarides (DMP) search and matching model of unemployment (Diamond (1982), Mortensen (1982), and Pissarides (1985)). Our key contribution is to introduce credit constraints in a way similar to Miao and Wang (2011a,b,c, 2012a,b).[4] The presence of this type of credit constraints can generate a stock market bubble through a positive feedback loop

[1] See, e.g., Collyns and Senhadji (2002), Goyal and Yamada (2004), Gan (2007), and Chaney, Sraer, and Thesmar (2009) for empirical evidence.

[2] See Farmer (2012b) for a regression analysis.

[3] The stock price index data is downloaded from Robert Shiller's website: http://www.econ.yale.edu/~shiller/data.htm

[4] The modeling of credit constraints is closely related to Kiyotaki and Moore (1997), Alvarez and Jermann (2000), Albuquerque and Hopenhayn (2004), and Jermann and Quadrini (2012).

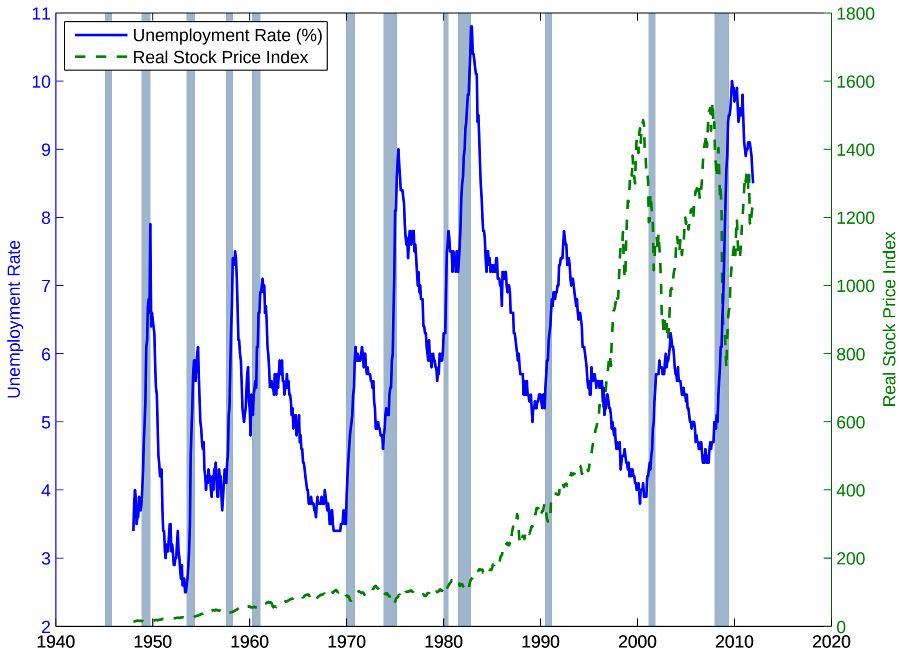

Figure 1: The unemployment rate and the stock market value. Shaded areas indicate NBER recessions.

mechanism. The intuition is the following: When investors have optimistic beliefs about the stock market value of a firm's assets, the firm wants to borrow more using its assets as collateral. Lenders are willing to lend more in the hope that they can recover more if the firm defaults. Then the firm can finance more investment and hiring spending. This generates higher firm value and justifies investors' initial optimistic beliefs. Thus, a high stock market value of the firm can sustain in equilibrium.

There is another equilibrium in which no one believes that firm assets have a high value. In this case, the firm cannot borrow more to finance investment and hiring spending. This makes firm value indeed low, justifying initial pessimistic beliefs. We refer to the first type of equilibrium as the bubbly equilibrium and to the second type as the bubbleless equilibrium. Both types can coexist due to self-fulfilling beliefs. In the bubbly equilibrium, firms can hire more workers and hence the market tightness is higher, compared to the bubbleless equilibrium. In addition, in the bubbly equilibrium, an unemployed worker can find a job more easily (i.e., the job-finding rate is higher) and hence the unemployment rate is lower.

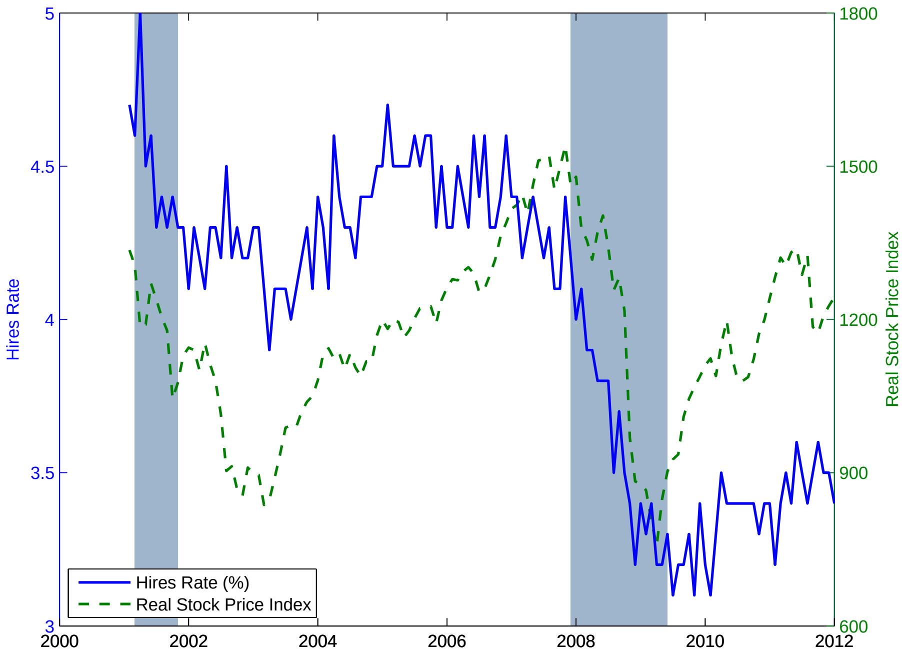

Figure 2: The hires rate and the stock market value. Shaded areas indicate NBER recessions.

After analyzing these two types of equilibria, we follow Weil (1987), Kocherlakota (2009) and Miao and Wang (2011a,b,c, 2012a,b) and introduce a third type of equilibrium with stochastic bubbles. Agents believe that there is a small probability that the stock market bubble may burst. After the burst of the bubble, it cannot re-emerge by rational expectations. We show that this shift of beliefs can also be self-fulfilling. After the burst of the bubble, the economy enters a recession with a persistent high unemployment rate. The intuition is the following. After the burst of the bubble, the credit constraints tighten, causing firms to reduce investment and hiring. An unemployed worker is then harder to find a job, generating high unemployment.

Our model can help explain the high unemployment during the Great Recession. Figures 2 and 3 plot the hires rate and the job-finding rate using the Job Openings and Labor Turnover Survey (JOLTS) data set.[5] These figures reveal that both the job-finding rate and the hires

[5] To be consistent with our model and the Blanchard and Gali (2010) model, we define the job-finding rate as the ratio of hires to unemployment. We first use the hires rate in the private sector and total employment in the private sector from JOLTS to calculate the number of hires, then use the unemployment rate and civilian employment from BLS to calculate the unemployed labor force, and finally derive the job-finding rate by dividing hires by unemployment. Our construction is different from that in Shimer (2005) for the DMP model.

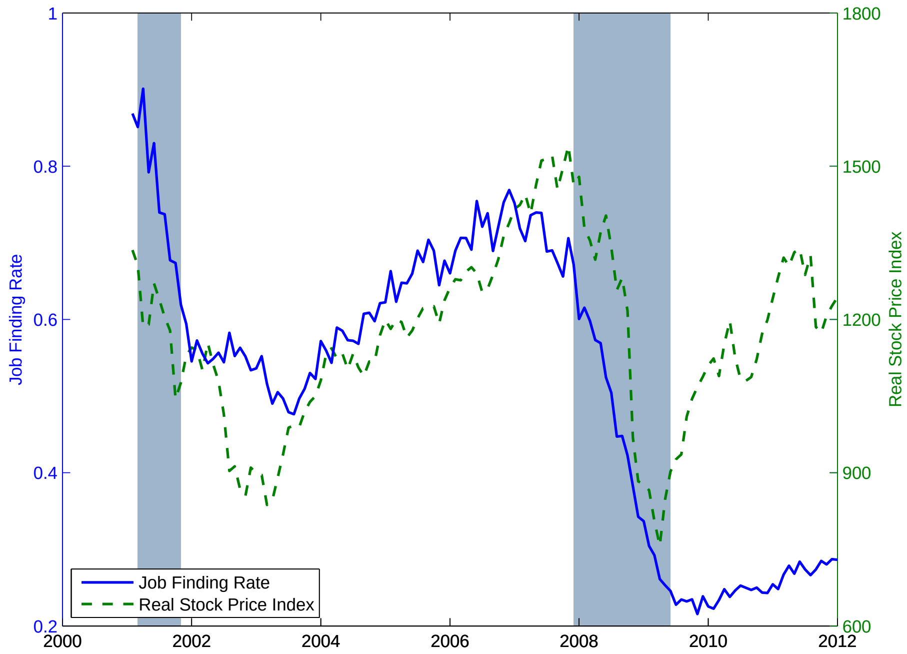

Figure 3: The job finding rate and the stock market value. Shaded areas indicate NBER recessions.

rate fell sharply following the stock market crash during the Great Recession. In particular, the hires rate and the job-finding rate fell from 4.4 percent and 0.7, respectively, at the onset of the recession to about 3.1 percent and 0.25, respectively, in the end of the recession.

While it is intuitive that unemployment is related to the stock market bubbles and crashes, it is difficult to build a theoretical model that features both unemployment and the stock market bubbles in a search framework.[6] To the best of our knowledge, we are aware of two approaches in the literature. The first approach is advocated by Farmer (2010a,b,c,d, 2012a,b). The idea of this approach is to replace the wage bargaining equation by the assumption that employment is demand determined.[7] In particular, Farmer assumes that the stock market value is determined by an exogenously specified belief function, rather than the present value of future

[6] As shown by Santos and Woodford (1997), rational bubbles can typically be ruled out in infinite-horizon models by transversality conditions. Bubbles can be generated in overlapping-generations models (Tirole (1985)) or in infinite-horizon models with borrowing constraints (Kocherlakota (1992,2009) and Wang and Wen (2011)). See Brunnermeier (2009) for a short survery of the literature on bubbles.

[7] One motivation of replacing the wage bargaining equation follows from Shimer's (2005) finding that Nash bargained wages make unemployment too smooth. Hall (2005) argues that any wage in the bargaining set can be supported as an equilibrium.

dividends. For any given beliefs, there is an equilibrium which makes the beliefs self-fulfilling. A shift in beliefs that lower stock prices reduces aggregate demand and raises unemployment. This approach of modeling stock prices seems ad hoc since anything can happen. The second approach is proposed by Kocherlakota (2011). He combines the overlapping generations model of Samuelson (1958) with the DMP model. The overlapping generations model can generate bubbles in an intrinsically useless asset. As in Farmer's approach, Kocherlakota also assumes that output is demand determined by removing the job creation equation in the DMP model. He then separates labor markets from asset markets. The two are connected only through the exchange of the goods owned by asset market participants (or households) and the different goods produced by workers. He assumes that both households and workers have finite lives, but firms are owned by infinitely lived people not explicitly modeled in the paper.

Our approach is different from the previous two approaches in three respects. First, we introduce endogenous credit constraints into an infinite-horizon search model.[8] The presence of credit constraints generates multiple equilibria through self-fulfilling beliefs. Unlike the Kocherlakota (2011) model, we focus on bubbles in the stock market value of the firm, but not in an intrinsically useless assets. A distinctive feature of stocks is that dividends are endogenous. Unlike Farmer's approach, our approach implies that the stock price is endogenously determined by both fundamentals and beliefs. In addition, in our model the crash of bubbles makes the stock price return to the fundamental value often modeled in the standard model. Second, unlike Kocherlakota (2011), we study both steady state and transitional dynamics. We also introduce stochastic bubbles and show that the collapse of bubbles raises unemployment. Kocherlakota (2011) does not model stochastic bubbles. But he shows that the unemployment rate is the same in a bubbly equilibrium as it is in a bubbleless equilibrium, as long as the interest rate is sufficiently low in the latter. He then deduces that labor market outcomes are unaffected by a bubble collapse, as long as monetary policy is sufficiently accommodative.

Third, our model has some policy implications different from Farmer's and Kocherlakota's models. In our model, the root of the existence of a bubble is the presence of credit constraints.

[8] Gu and Wright (2011), He, Wright, and Zhu (2011), Rocheteau and Wright (2012) also introduce credit constraints into search models and show that bubbles may appear. But they do not study the relation between stock market bubbles and unemployment.

Improving credit markets can prevent the emergence of a bubble so that the economy cannot enter the bad equilibrium with high and persistent unemployment driven by self-fulfilling beliefs. Our model also implies that raising unemployment insurance benefits during a recession may exacerbate the recession because an unemployed worker is reluctant to search for a job. This result is consistent with the prediction in the DMP model. However, it is different from Kocherlakota's result that an increase in unemployment insurance benefits funded by the young lowers the unemployment rate. We also show that the policy of hiring subsidies after the stock market crash can help the economy recovers from the recession faster. However, this policy cannot solve the inefficiency caused by credit constraints and hence the economy will enter a steady state with unemployment rate higher than that in the steady state with stock market bubbles.

The remainder of the paper proceeds as follows. Section 2 presents the model. Section 3 presents the equilibrium system and analyzes a benchmark model with a perfect credit market. Sections 4 and 5 study the bubbleless and bubbly equilibria, respectively. Section 6 introduces stochastic bubbles and show how the collapse of bubbles generates a recession and persistent and high unemployment. Section 7 discusses some policy implications focusing on the unemployment benefit and hiring subsidies. Section 8 concludes. Appendix A contains technical proofs. In Appendix B, we show that the Blanchard-Gali setup is isomorphic to the DMP setup, even with credit constraints. The key difference is that in the Blanchard-Gali setup vacancies are immediately filled by paying hiring costs, while in the DMP setup it takes time to fill a vacancy and employment is generated by a matching function of vacancy and unemployment. Since vacancy is not the focus of our study, we adopt the Blanchard-Gali framework.

2 The Model

Consider a continuous-time setup without aggregate uncertainty, based on the Blanchard and Gali (2010) model in discrete time. We follow Miao and Wang (2011a) and introduce credit constraints into this setup. To facilitate exposition, we sometimes consider a discrete-time approximation in which time is denoted by $t = 0, dt, 2dt, ...$ The continuous-time model is the limit when dt goes to zero.

2.1 Households

There is a continuum of identical households of measure unity. Each household consists of a continuum of members of measure unity. The representative household derives utility according to the following utility function:

$$\int_0^\infty e^{-rt} C_t dt, \tag{1}$$

where $r \in (0,1)$ represents the subjective discount rate, C_t represents consumption. As in Merz (1995) and Andolfatto (1996), we assume full risk sharing within a large family. For simplicity, we do not consider disutility from work, as is standard in the search literature (e.g., Pissarides (2000)).[9]

The representative household receives wages from work and unemployment benefits from the government and chooses consumption and share holdings so as to maximize the utility function in (1) subject to the budget constraint:

$$\dot{X}_t = rX_t - C_t + w_t N_t + c\,(1 - N_t) - T_t,\ X_0 \text{ given}, \tag{2}$$

where X_t represents wealth, N_t represents employment, w_t represents the wage rate, $c > 0$ represents the constant unemployment compensation, and T_t represents lump-sum taxes. Suppose that the unemployment compensation is financed by lump sum taxes T_t. Define the unemployment rate by

$$U_t = 1 - N_t. \tag{3}$$

Since we have assumed that there is no aggregate uncertainty and that each household has linear utility in consumption, the return on any asset is equal to the subjective discount rate r.

2.2 Firms

There is a continuum of firms of measure unity, owned by households. Each firm $j \in [0,1]$ hires N_t^j workers and purchases K_t^j machines to produce output Y_t^j according to a Leontief technology $Y_t^j = A \min\{K_t^j, N_t^j\}$, which means that each worker requires one machine to produce.[10] We further assume that one unit of capital costs κ. Each firm j meets an opportunity to hire

[9] One can introduce disutility from work by adopting the utility function in Blanchard and Gali (2010).
[10] We introduce physical capital in the model so that it can be used as collateral.

H_t^j new workers in a frictional labor market with Poisson probability πdt in a small time interval $[t, t + dt]$. The Poisson shock is independent across firms. Employment in firm j evolves according to

$$N_{t+dt}^j = \begin{cases} (1 - sdt)N_t^j + H_t^j & \text{with probability } \pi dt \\ (1 - sdt)N_t^j & \text{with probability } 1 - \pi dt \end{cases}, \tag{4}$$

where $s > 0$ represents the exogenous separation rate. Define aggregate employment as $N_t = \int N_t^j dj$ and total hires as

$$H_t \equiv \int H_t^j dj = \pi \int_{\mathcal{J}} H_t^j dj,$$

where $\mathcal{J} \subset [0, 1]$ represents the set of firms having hiring opportunities. The second equality in the preceding equation follows from a law of large numbers. We can then write the aggregate employment dynamics as

$$N_{t+dt} = (1 - sdt) N_t + H_t dt. \tag{5}$$

In the continuous-time limit, this equation becomes

$$\dot{N}_t = -sN_t + H_t. \tag{6}$$

Following Blanchard and Gali (2010), define an index of market tightness as the ratio of aggregate hires to unemployment:

$$\theta_t = \frac{H_t}{U_t}. \tag{7}$$

It also represents the job-finding rate. Assume that the total hiring costs for firm j are given by $G_t H_t^j$, where G_t is an increasing function of market tightness θ_t :

$$G_t = \psi \theta_t^\alpha, \tag{8}$$

where $\psi > 0$ and $\alpha > 0$ are parameters. Intuitively, if total hires in the market are large relative to unemployment, then workers will be relatively scarce and a firm's hiring will be relatively costly.

Let $V_t(N_t^j)$ denote the market value of firm j before observing the arrival of an hiring opportunity. It satisfies the following Bellman equation in the discrete-time approximation:[11]

$$\begin{aligned} V_t(N_t^j) &= \max_{H_t^j} \quad (A - w_t) N_t^j dt - \left(\kappa H_t^j + G_t H_t^j\right) \pi dt \qquad (9) \\ &+ e^{-rdt} V_{t+dt}\left((1 - sdt)N_t^j + H_t^j\right) \pi dt + e^{-rdt} V_{t+dt}\left((1 - sdt)N_t^j\right)(1 - \pi dt), \end{aligned}$$

[11] The continuous-time Bellman equation is given by (A.1) in the appendix.

where κ represents the price of capital. Note that the discount rate is r since firms are owned by the risk-neutral households with the subjective discount rate r.

Assume that hiring and investment are financed by internal funds and external debt:

$$(\kappa + G_t) H_t^j \leq (A - w_t) N_t^j + L_t^j, \tag{10}$$

where L_t^j represents debt. We abstract from external equity financing. Our key insights still go through as long as external equity financing is limited. Following Carlstrom and Fuerst (1997), Jermann and Quadrini (2012), and Miao and Wang (2011a,b,c, 2012a,b), we consider intra-period debt without interest payments for simplicity. As in Miao and Wang (2011a,b,c, 2012a,b), we assume that the firm face the following credit constraint:[12]

$$L_t^j \leq e^{-rdt} V_{t+dt}(\xi N_t^j), \tag{11}$$

where $\xi \in (0, 1]$ is a parameter representing the degree of financial frictions. This constraint can be justified as an incentive constraint in an optimal contracting problem with limited commitment. Because of the enforcement problem, lenders require the firm to pledge its assets as collateral. In our model, the firm has assets (or capital) N_t^j due to the Leontief technology. It pledges assets N_t^j as collateral. If the firm defaults on debt, lenders can capture ξN_t assets of the firm. The remaining fraction $1 - \xi$ accounts for default costs. Lenders and the firm renegotiate the debt and lenders keep the firm running in the next period. Thus lenders can get the threat value $e^{-rt} V_{t+dt}(\xi N_t^j)$. Suppose that the firm has all the bargaining power as in Jermann and Quadrini (2012). Then the credit constraint in (11) represents an incentive constraint so that the firm will never default in an optimal lending contract. In the continuous-time limit as $dt \to 0$, (11) becomes

$$L_t^j \leq V_t(\xi N_t^j), \tag{12}$$

It follows from (10) and (11) that we can write down the combined constraint:

$$(\kappa + G_t) H_t^j \leq (A - w_t) N_t^j + V_t(\xi N_t^j). \tag{13}$$

[12] One may introduce intertemporal debt with interest payments as in Miao and Wang (2011a). This modeling introduces an additional state variable (i.e., debt) and complicates the analysis without changing our key insights.

Note that our modeling of credit constraints is different from that in Kiyotaki and Moore (1997). In their model, when the firm defaults lenders immediately liquidate firm assets. The collateral value is equal to the liquidation value. In our model, when the firm defaults, lenders reorganize the firm and renegotiate the debt. Thus, the collateral value is equal to the going concern value of the firm.

2.3 Nash Bargaining

Suppose that the wage rate can be negotiated continually and is determined by Nash bargaining at each point of time as in the DMP model. Because a firm employs multiple workers in our model, we consider the Nash bargaining problem between a household member and a firm with existing workers N_t. We need to derive the marginal values to the household and to the firm when an additional household member is employed.

We can show that the marginal value of an employed worker V_t^N satisfies the following asset-pricing equation:

$$rV_t^N = w_t + s\left(V_t^U - V_t^N\right) + \dot{V}_t^N. \tag{14}$$

The marginal value of an unemployed V_t^U satisfies the following asset-pricing equation:

$$rV_t^U = c + \theta_t\left(V_t^N - V_t^U\right) + \dot{V}_t^U. \tag{15}$$

The marginal household surplus is given by

$$S_t^H = V_t^N - V_t^U. \tag{16}$$

It follows (14) and (15) that

$$rS_t^H = w_t - c - (s + \theta_t)\, S_t^H + \dot{S}_t^H. \tag{17}$$

The marginal firm surplus is given by

$$S_t^F = \frac{\partial V_t\left(N_t^j\right)}{\partial N_t^j}. \tag{18}$$

The Nash bargained wage solves the following problem:

$$\max_{w_t} \; \left(S_t^H\right)^\eta \left(S_t^F\right)^{1-\eta}, \tag{19}$$

subject to $S_t^H \geq 0$ and $S_t^F \geq 0$, where $\eta \in (0,1)$ denotes the relative bargaining power of the worker. The two inequality constraints state that there are gains from trade between the worker and the firm.

2.4 Equilibrium

Let $N_t = \int_0^1 N_t^j dj$, $H_t = \int_0^1 H_t^j dj$, and $Y_t = \int_0^1 Y_t^j dj$ denote aggregate employment, total hires, and aggregate output. A search equilibrium consists of trajectories of $(Y_t, N_t, C_t, U_t, \theta_t, H_t, w_t)_{t\geq 0}$ and value functions V_t^N, V_t^U, and V_t such that (i) firms solve problem (9), (ii) V_t^N and V_t^U satisfy the Bellman equations (14) and (15), (iii) the wage rate solves problem (19), and (iv) markets clear in that equations (3), (6), and (7) hold and

$$C_t + (\kappa + G_t) H_t = Y_t = AN_t. \tag{20}$$

3 Equilibrium System

In this section, we first study a single firm's hiring decision problem. We then analyze how wages are determined by Nash bargaining. Finally, we derive the equilibrium system by differential equations.

3.1 Hiring Decision

Consider firm j's dynamic programming problem. Conjecture that firm value takes the following form:

$$V_t(N_t^j) = Q_t N_t^j + B_t, \tag{21}$$

where Q_t and B_t are variables to be determined. Because the firm's dynamic programming problem does not give a contraction mapping, two types of solutions are possible. In the first type, $B_t = 0$ for all t. In the second type, $B_t \neq 0$ for some t. In this case, we will impose conditions later such that $B_t > 0$ for all t and interpret it as a bubble. The following proposition characterizes these solutions:

Proposition 1 *Suppose*

$$\mu_t = \frac{Q_t}{\kappa + G_t} - 1 > 0. \tag{22}$$

Then firm value takes the form in (21), where (B_t, Q_t) satisfies the following differential equations:

$$\dot{B}_t = rB_t - \pi\mu_t B_t, \tag{23}$$

$$\dot{Q}_t = (r + s - \xi\pi\mu_t)Q_t - (A - w_t)(1 + \pi\mu_t), \tag{24}$$

and the transversality condition

$$\lim_{T\to\infty} e^{-rT} B_T = \lim_{T\to\infty} e^{-rT} Q_T = 0. \tag{25}$$

The optimal hiring is given by

$$H_t^j = \frac{(A - w_t + Q_t\xi_t) N_t^j + B_t}{\kappa + G_t}. \tag{26}$$

We use $\pi\mu_t$ to denote the Lagrange multiplier associated with the credit constraint (13). The first-order condition for problem (9) with respect to H_t^j gives

$$(1 + \mu_t)(\kappa + G_t) = Q_t. \tag{27}$$

Thus, we may interpret μ_t as the external finance premium. If $\mu_t = 0$, then the borrowing constraint does not bind and the model reduces to the case with perfect capital markets. Condition (22) ensures that the credit constraint binds so that we can derive the optimal hiring in equation (26). Equation (23) is an asset-pricing equation for the bubble B_t. It says that the rate of return on the bubble, r, is equal to the sum of capital gains, $\dot{B}_t/B_t$, and dividend yields, $\pi\mu_t$. The intuition for the presence of dividend yields is similar to that in Miao and Wang (2011): One dollar bubble raises collateral value by one dollar, which allows the firm to borrow one more dollar to finance hiring and investment costs. As a result, the firm can hire more workers and firm value rises by $\pi\mu_t$.

We may interpret Q_t as the shadow value of capital or labor (recall the Leontief production function). Equation (22) shows that optimal hiring must be such that the marginal benefit Q_t is equal to the marginal cost $(1 + \mu_t)(\kappa + G_t)$. The marginal cost exceeds the actual costs $\kappa + G_t$ during credit constraints. We thus may also interpret μ_t as an external financing premium. Equation (24) is an asset pricing equation. It says that the return on capital rQ_t is equal to "dividends" $(A - w_t) + \pi\mu_t (A - w_t + \xi Q_t)$, minus the loss of value due to separation sQ_t, plus capital gains $\dot{Q}_t$. Note that dividends consist of profits $A - w_t$ and the shadow value of funds $\pi\mu_t (A - w_t + \xi Q_t)$.

3.2 Nash Bargained Wage

Next, we derive the equilibrium wage rate, which solves problem (19). To analyze this problem, we consider a discrete-time approximation. In this case, the values of an employed and an unemployed satisfy the following equations:

$$V_t^N = w_t dt + e^{-rdt}[sdtV_{t+dt}^U + (1 - sdt) V_{t+dt}^N],$$

$$V_t^U = cdt + e^{-rdt} \left[\theta_t dt V_{t+dt}^N + (1 - \theta_t dt) V_{t+dt}^U\right].$$

Thus, the household surplus is given by

$$\begin{aligned} S_t^H &= V_t^N - V_t^U \\ &= (w_t - c) dt + e^{-rdt} (1 - sdt - \theta_t dt) \left(V_{t+dt}^N - V_{t+dt}^U\right) \\ &= (w_t - c) dt + e^{-rdt} (1 - sdt - \theta_t dt) S_{t+dt}^H. \end{aligned} \tag{28}$$

Turn to the firm surplus. Let $\mu_t \pi$ be the Lagrange multiplier associated with constraint (10). If $\mu_t > 0$, then both this constraint and constraint (11) bind. Apply the envelop theorem to problem (9) to derive

$$\begin{aligned} S_t^F &= \frac{\partial V_t \left(N_t^j\right)}{\partial N_t^j} = (1 + \pi\mu_t) (A - w_t) dt \\ &\quad + e^{-rdt} (\xi\pi\mu_t dt + 1 - sdt) \frac{\partial V_{t+dt} \left(N_{t+dt}^j\right)}{\partial N_{t+dt}^j} \\ &= (1 + \pi\mu_t) (A - w_t) dt + e^{-rdt} (\xi\pi\mu_t dt + 1 - sdt) S_{t+dt}^F. \end{aligned} \tag{29}$$

Note that the continuous-time limit of this equation is (24) since $S_t^F = Q_t$ by (21).

Using equations (28) and (29), we can rewrite problem (19) as

$$\begin{aligned} &\max_{w_t} \left[(w_t - c) dt + e^{-rdt} (1 - sdt - \theta_t dt) S_{t+dt}^H\right]^\eta \\ \times &\left[(1 + \pi\mu_t) (A - w_t) dt + e^{-rdt} (\xi\pi\mu_t dt + 1 - sdt) S_{t+dt}^F\right]^{1-\eta}. \end{aligned}$$

The first-order condition implies that

$$\eta S_t^F = (1-\eta)(1+\pi\mu_t) S_t^H. \tag{30}$$

Note that this sharing rule is different from the standard Nash bargaining solution in the DMP model. The presence of credit constraints induces a wedge given by $\mu_t > 0$. An additional dollar increase in the wage rate raises the value of the employed by one dollar. But it decreases firm value by $(1+\pi\mu_t)$ dollars, which is more than one dollar. The additional cost $\pi\mu_t$ to the firm reflects costly external finance caused by the credit constraints.

Since we have assumed that wage is negotiated continually, equation (30) also holds in rates of change as in Pissarides (2000, p. 28). We thus obtain

$$\eta \dot{S}_t^F = (1-\eta)(1+\pi\mu_t) \dot{S}_t^H + (1-\eta)\pi\dot{\mu}_t S_t^H. \tag{31}$$

Substituting equations (17), (24), and $S_t^F = Q_t$ into the above equation yields

$$\begin{aligned} & \eta \left[(r+s-\xi\pi\mu_t)Q_t - (A-w_t)(1+\pi\mu_t)\right] \\ = \; & (1-\eta)(1+\pi\mu_t)\left[(r+\theta_t+s) S_t^H - w_t + c\right] + (1-\eta)\pi\dot{\mu}_t S_t^H \end{aligned}$$

Using equation (30) and $S_t^F = Q_t$, we can solve the above equation for the wage rate:

$$w_t = \eta \left[A + \frac{\xi\pi\mu_t + \theta_t}{1+\pi\mu_t} Q_t + \frac{\pi\dot{\mu}_t Q_t}{(1+\pi\mu_t)^2}\right] + (1-\eta)c. \tag{32}$$

This equation shows that the Nash bargained wage is equal to a weighted average of the unemployment benefit and a term consisting of three components. The weight is equal to the relative bargaining power. The first component is productivity A. The second component is related to the value from external financing and the threat value of the worker, $(\xi\pi\mu_t + \theta_t) Q_t / (1+\pi\mu_t)$. Workers are rewarded for the saving of external funds to finance hiring costs. Holding everything else constant, a higher external finance premium leads to a higher wage rate. The market tightness or the job-finding rate, θ_t, affects a household's threat value. Holding everything else constant, a higher value of market tightness, θ_t, implies that a searcher can more easily find a job and hence he demands a higher wage. The third component reflects the change in the cost of external financing, the term related to $\dot{\mu}_t$ because wages are continually negotiated. The last two components are also positively related to Q_t, holding everything else constant. The intuition is that workers get higher wages when the marginal Q of the firm is higher.

3.3 Equilibrium

Finally, we conduct aggregation and impose market-clearing conditions. We then obtain the equilibrium system.

Proposition 2 *Suppose $\mu_t > 0$, where μ_t satisfies (22). Then the equilibrium dynamics for $(B_t, Q_t, N_t, U_t, \theta_t, H_t, w_t)$ satisfy the system of equations (23), (24), (6), (3), (7), (32), and*

$$H_t = \pi \frac{(A - w_t) N_t + Q_t \xi N_t + B_t}{\kappa + G_t}, \tag{33}$$

where G_t is given by (8). The transversality condition in (25) also holds.

It follows from this proposition that there are two types of equilibrium. In the first type, $B_t = 0$ for all t. In the second type, $B_t \neq 0$ for some t. Because firm value cannot be negative, we restrict attention to the case with $B_t > 0$ for all t. We call the first type of equilibrium the bubbleless equilibrium and the second type the bubbly equilibrium. Intuitively, if $N_t^j = 0$, the firm has no worker or capital, one may expect its intrinsic value should be zero. Thus, the positive term $B_t > 0$ represents a bubble in firm value.

3.4 A Benchmark with Perfect Credit Markets

Before analyzing the model with credit constraints, we first consider a benchmark without credit constraints. In this case, the Lagrange multiplier associated with the credit constraint is zero, i.e., $\mu_t = 0$. Since Appendix B shows that this model is isomorphic to a standard DMP model as in Chapter 1 of Pissarides (2000), we will follow a similar analysis.

We still conjecture that firm value takes the form given in (21). Following a similar analysis for Proposition 1, we can show that

$$Q_t = \kappa + G_t = \kappa + \psi \theta_t^{\alpha}, \tag{34}$$

$$rQ_t = A - w_t - sQ_t + \dot{Q}_t, \tag{35}$$

$$rB_t = \dot{B}_t.$$

By the transversality condition, we deduce that $B_t = 0$. It follows that a bubble cannot exist for the model with perfect credit markets.

The wage rate is determined by Nash bargaining as in Section 3.2. We can show that the Nash bargained wage satisfies

$$w_t = \eta \left(A + \theta_t Q_t\right) + (1 - \eta)\, c. \tag{36}$$

Using (34) and (36), we can rewrite (35) as

$$\dot{Q}_t = (r + s)\, Q_t - A + \eta \left(A + \theta_t Q_t\right) + (1 - \eta)\, c. \tag{37}$$

Using (3), (6), and (7), we obtain

$$\dot{N}_t = -sN_t + \theta_t (1 - N_t), \tag{38}$$

An equilibrium can be characterized by a system of differential equations (37) and (38) for (Q_t, N_t), where we use (34) to substitute for θ_t.

Now, we analyze the steady state for the above equilibrium system. Equations (34) and (35) give the steady state relation between w and θ :

$$A - w = (r + s) \left(\psi \theta^{\alpha} + \kappa\right). \tag{39}$$

We plot this relation in Figure 4 and call it the job creation curve, following the literature on search models, e.g., Pissarides (2000). In the (θ, w) space it slopes down: Higher wage rate makes job creation less profitable and so leads to a lower equilibrium ratio of new hires to unemployed workers. It replaces the demand curve of Walrasian economics.

Equations (34) and (36) give another steady state relation between w and θ :

$$w = \eta \left(A + \theta \left(\psi \theta^{\alpha} + \kappa\right)\right) + (1 - \eta)\, c. \tag{40}$$

We plot this relation in Figure 4 and call it the wage curve, as in Pissarides (2000). This curve slopes up: At higher market tightness the relative bargaining strength of market participants shifts in favor of workers. It replaces the supply curve of Walrasian economics.

The steady state equilibrium $\left(\bar{\theta}, \bar{w}\right)$ is at the intersection of the two curves. Clearly, when θ goes to infinity the wage curve approaches positive infinity and the job creation curve approaches negative infinity. When θ approaches zero, we impose the assumption

$$(1 - \eta)\,(A - c) > (r + s)\, \kappa, \tag{41}$$

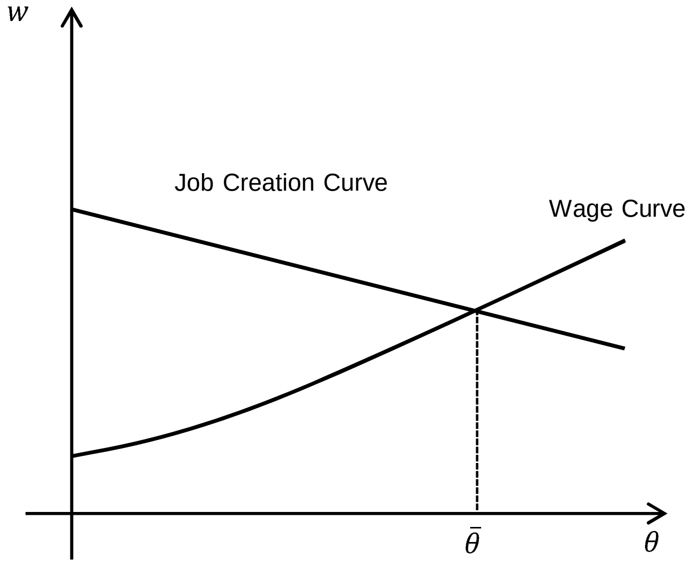

Figure 4: The job creation and wage curves for the steady state equilibrium with perfect credit markets.

so that the job creation curve is above the wage curve at $\theta = 0$. The preceding properties of the two curves ensure the existence and uniqueness of the steady state equilibrium $\left(\bar{\theta}, \bar{w}\right)$.

Once we obtain $\left(\bar{\theta}, \bar{w}\right)$, the other steady state equilibrium variables can be easily derived. For example, we can determine $\left(\bar{H}, \bar{U}\right)$ using equations $H = s\left(1 - U\right)$ and $H = \theta U$. The first equation is analogous to the Beveridge curve and is downward sloping as illustrated in Figure 5.

Turn to local dynamics. We linearize the equilibrium system (37) and (38) where θ_t is replaced by a function of Q_t using (34). We then obtain the linearized system:

$$\begin{bmatrix} \dot{Q}_t \\ \dot{N}_t \end{bmatrix} = \begin{bmatrix} + & 0 \\ + & - \end{bmatrix} \begin{bmatrix} Q_t - \bar{Q} \\ N_t - \bar{N} \end{bmatrix}.$$

Given the sign pattern of the matrix, the determinant is negative. Thus, the steady state is a saddle point. Note that N_t is predetermined and Q_t is non-predetermined. Since the differential equation for Q_t does not depend on N_t, Q_t must be constant along the transition path. This implies that θ_t must also be constant along the transition path.

If N_0 or U_0 is out of the steady state, say $U_0 > \bar{U}$, then the market tightness is relatively

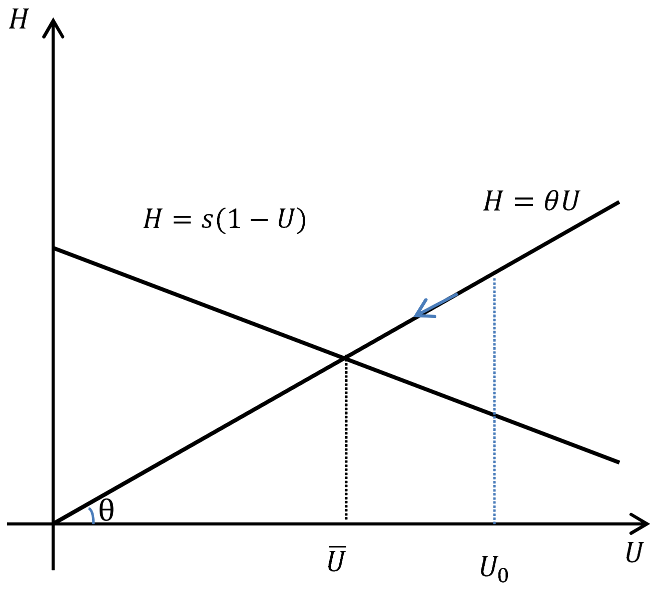

Figure 5: Determination of hiring and unemployment for the benchmark model with perfect credit markets.

low. An unemployed worker is harder to find a job and hence he bargains a lower wage. This causes firm value to rise initially, inducing firms to hire more workers immediately. As a result, unemployment falls. During the transition path, firms adjust hiring to maintain the ratio of hires and unemployment constant, until reaching the steady state.

4 Bubbleless Equilibrium

From then on, we focus on the model with credit constraints. In this case, multiple equilibria may emerge. In this section, we analyze the bubbleless equilibrium in which $B_t = 0$ for all t. We first characterize the steady state and then study transition dynamics.

4.1 Steady State

We use Proposition 2 to show that the bubbleless steady-state equilibrium (Q, N, U, θ, H, w) satisfies the following system of six algebraic equations:

$$0 = (r + s)Q - (A - w) - \pi\mu\left(A - w + \xi Q\right), \tag{42}$$

$$H = \pi \frac{(A - w) N + Q\xi N}{\kappa + G}, \tag{43}$$

$$0 = -sN + H, \tag{44}$$

$$U = 1 - N, \tag{45}$$

$$\theta = U/H, \tag{46}$$

$$w = \eta \left[A + \frac{\xi\pi\mu + \theta}{1 + \pi\mu} Q \right] + (1 - \eta) c, \tag{47}$$

where

$$\mu = \frac{Q}{\kappa + G} - 1, \tag{48}$$

$$G = \psi\theta^{\alpha}. \tag{49}$$

Solving the above system yields:

Proposition 3 *If*

$$0 < \xi < s\frac{1 - \pi}{\pi} - r, \tag{50}$$

$$A - c > \frac{\kappa s}{\pi (1 - \eta)} \left[\frac{r + \pi\xi}{\xi + r} + \eta \frac{\xi\pi\mu^*}{1 + \pi\mu^*} \frac{1 - \pi}{\xi + r} \right], \tag{51}$$

where

$$\mu^* = \frac{1 - \pi}{\pi} \frac{s}{\xi + r} - 1, \tag{52}$$

then there exists a unique bubbleless steady-state equilibrium $(Q^*, N^*, U^*, \theta^*, H^*, w^*)$ *satisfying*

$$Q^* = \frac{1 - \pi}{\pi} \frac{s}{\xi + r} (\kappa + \psi\theta^{*\alpha}), \tag{53}$$

$$N^* = \frac{\theta^*}{s + \theta^*}, \tag{54}$$

where θ^* *is the unique solution to the equation for* θ *:*

$$\frac{(1 - \eta)(A - c)}{\kappa + \psi\theta^{\alpha}} = \frac{s}{\pi} \left[\frac{r + \pi\xi}{\xi + r} + \eta \frac{\xi\pi\mu^* + \theta}{1 + \pi\mu^*} \frac{1 - \pi}{\xi + r} \right]. \tag{55}$$

Condition (50) ensures that $\mu^* > 0$ so that we can apply Proposition 2 in a neighborhood of the steady state. The steady state can be derived using the job creation and wage curves analogous to those discussed in Section 3.4. We first substitute H in (43) into (44) to derive

$$s = \pi \frac{(A - w) + Q\xi}{\kappa + G}. \tag{56}$$

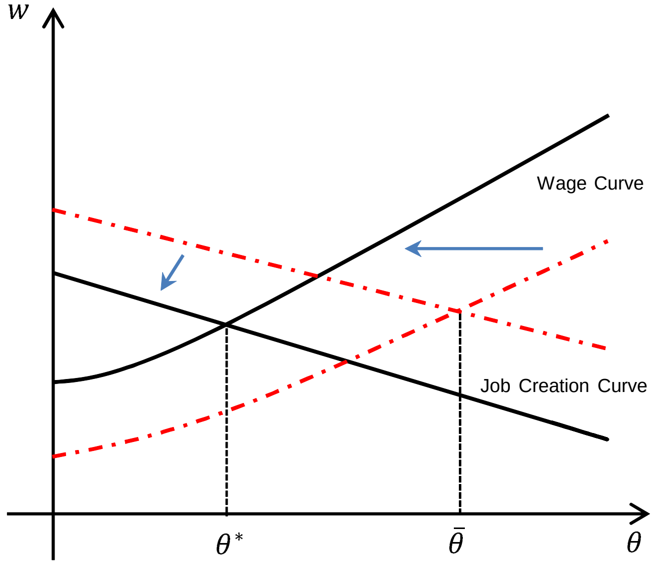

Figure 6: The job creation and wage curves for the bubbleless steady state equilibrium.

Eliminating $A - w$ in this equation and in equation (42), we can solve for Q :

$$Q = \frac{1 + \pi\mu}{r + s + \xi} \frac{s}{\pi} (\kappa + G). \tag{57}$$

Combining the above equation with (48), we obtain the solution for μ in (52). Plugging this solution into (57) and substituting the resulting expression for Q in (56), we obtain

$$A - w = \frac{s(r + \pi\xi)}{\pi(\xi + r)} (\kappa + \psi\theta^\alpha). \tag{58}$$

This equation defines w as a function of θ and gives the job creation curve. It is downward sloping as illustrated in Figure 6.

Next, substituting (57), (49), and (52) into equation (47), we can express w as a function of θ.

$$w = \eta \left[A + \frac{\xi\pi\mu^* + \theta}{r + s + \xi} \frac{s}{\pi} (\kappa + \psi\theta^\alpha) \right] + (1 - \eta) c. \tag{59}$$

This equation gives the upward sloping wage curve. The equilibrium (θ^*, w^*) is determined by the intersection of the job creation and wage curves as illustrated in Figure 6. As in Section 3.4, the equilibrium (H^*, U^*) is determined the Figure 5.

What is the impact of credit constraints? Figure 6 also plots the job creation and wage curves for the benchmark model with perfect credit markets. It is straightforward to show that, in the presence of credit constraints, both the job creation and wage curves shift to the left. As a result, credit constraints lower the steady state market tightness. The impact on wage is ambiguous. We can then use Figure 5 to show that credit constraints reduce hiring and raise unemployment.

Proposition 4 *Suppose that conditions (41), (50), and (51) are satisfied. Then $\theta^* < \bar{\theta}$, $H^* < \bar{H}$, and $U^* > \bar{U}$. Namely, the labor market tightness and hiring are lower, but unemployment is higher, in the bubbleless steady state with credit constraints than in the steady state with perfect credit markets.*

4.2 Transition Dynamics

Turn to transition dynamics. The predetermined state variable for the equilibrium system is N_t and the nonpredetermined variables are $(Q_t, U_t, \theta_t, H_t, w_t)$. Simplifying this system, we can represent it by a system of three differential equations for three unknowns (Q_t, N_t, μ_t): (24), (32), and (38). In this simplified system, we have to represent θ_t and w_t in terms of (Q_t, N_t, μ_t). To this end, we use (22) and (8) to solve for θ :

$$\theta_t = \left[\frac{Q_t/(1+\mu_t) - \kappa}{\psi}\right]^{\frac{1}{\alpha}}. \tag{60}$$

We then use (7), (33), and (8) to solve for w_t :

$$w_t = A - \frac{\theta_t\left(\kappa + \psi\theta_t^{\alpha}\right)(1 - N_t)}{\pi N_t} + \xi Q_t. \tag{61}$$

To study local dynamics around the bubbleless steady state, one may linearize the preceding simplified equilibrium system and compute eigenvalues. Since this system is highly nonlinear, we are unable to derive an analytical result. We thus use a numerical example to illustrate transition dynamics.[13] We set $s = 0.1$, $\pi = 0.02$, $\kappa = 0.1$, $\eta = 0.5$, $\psi = 0.04$, $\alpha = 1$, $c = 0.4$, $A = 1$, and $\xi = 0.9$. For these parameter values, conditions (50) and (51) are satisfied.

[13] We use the reverse shooting method to numerically solve the system of differential equations (see, e.g., Judd (1998)).

We compute the steady state $(Q, N, \mu) = (0.707, 0.887, 4.385)$. We find that all three eigenvalues associated with the linearized system around the steady state are real. Two of them are positive and one is negative. The negative eigenvalue corresponds to the predetermined variable N_t. Thus, the steady state is a saddle point and the system is saddle path stable.

Figure 7 plots the transition paths. Suppose that the unemployment rate is initially low relative to the steady state. Then the market tightness is relatively high. Thus, an unemployed worker is easier to find a job and hence bargains a higher wage. This in turns lowers firm value and marginal Q, causing a firm to reduce hiring initially. In addition, because the initial unemployment rate is low, the initial output is high. The firm then gradually increases hiring. However, the increase is slower than the exogenous separation rate, causing the unemployment rate to rise gradually. Unlike the case of perfect credit markets analyzed in Section 3.4, the market tightness θ_t is not constant during adjustment. In fact, it falls gradually. As a result, the job-finding rate falls gradually, leading the wage rate to fall too. Output also falls over time, but firm value rises. The increase in firm value is due to the increase in marginal Q. The gradual rise in marginal Q is due to two effects. First, because hires rise over time, the firm uses more external financing and hence the external finance premium μ_t rises over time. Second, since wage falls over time, the profits rise over time.

5 Bubbly Equilibrium

We now turn to the bubbly equilibrium in which $B_t > 0$ for all t. We first study steady state and then examine transition dynamics.

5.1 Steady State

We use Proposition 2 to show that the bubbleless steady state equilibrium $(B, Q, N, U, \theta, H, w)$ satisfies the following system of seven equations: (42), (44), (45), (46), (47) and

$$0 = rB - \pi\mu B, \tag{62}$$

$$H = \pi \frac{(A - w) N + (Q\xi N + B)}{\kappa + G}, \tag{63}$$

where μ and G satisfy (48) and (49).

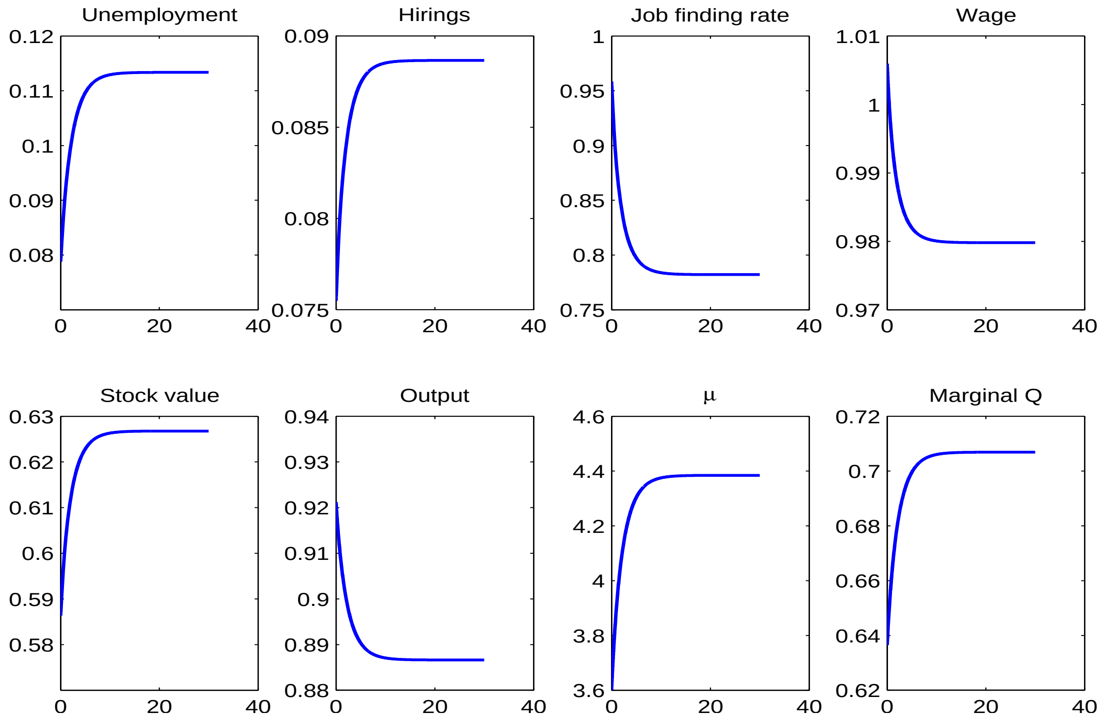

Figure 7: Transitional dynamics for the bubbleless equilibrium. We set parameter values as $s = 0.1$, $\pi = 0.02$, $\kappa = 0.1$, $\eta = 0.5$, $\psi = 0.04$, $\alpha = 1$, $c = 0.4$, $A = 1$, and $\xi = 0.9$.

Solving the above system yields:

Proposition 5 *If*

$$0 < \xi < \frac{s(1-\pi)}{r+\pi} - r, \tag{64}$$

$$A - c > \frac{r+\pi}{\pi(1-\eta)}\left[\eta\frac{\xi\pi\mu_b}{1+\pi\mu_b} - \xi + \frac{r+s+\xi}{1+r}\right], \tag{65}$$

where $\mu_b = r/\pi$, *then there exists a bubbly steady-state equilibrium* $(B, Q_b, N_b, U_b, w_b, \theta_b)$ *satisfying*

$$\frac{B}{N_b} = (\kappa + \psi\theta_b^\alpha)\left[\frac{s}{\pi} - \frac{r+\pi}{\pi}\frac{r+s+\xi}{1+r}\right] > 0, \tag{66}$$

$$Q_b = \frac{r+\pi}{\pi}(\kappa + \psi\theta_b^\alpha), \tag{67}$$

$$N_b = \frac{\theta_b}{s+\theta_b}, \tag{68}$$

where θ_b *is the unique solution to the equation for* θ :

$$\frac{(1-\eta)(A-c)}{\kappa+\psi\theta^\alpha} = \frac{r+\pi}{\pi}\left[\eta\frac{\xi\pi\mu_b+\theta}{1+\pi\mu_b} - \xi + \frac{r+s+\xi}{1+r}\right]. \tag{69}$$

Condition (64) ensures that $B > 0$. In addition, it also guarantees that condition (50) holds so that a bubbleless steady-state equilibrium also exists. To see how the steady-state θ_b is determined, we derive the job creation and wage curves as in the case of bubbleless equilibrium. First, we plug equation (63) into (44) to derive

$$s = \pi\frac{A - w + Q\xi + B/N}{\kappa + \psi\theta^\alpha}. \tag{70}$$

Then use equation (62) to derive $\mu_b = r/\pi$. Using (48) yields

$$Q = \frac{r+\pi}{\pi}(\kappa + G). \tag{71}$$

Plugging this equation and (49) into (42) yields

$$A - w = \frac{(r+\pi)(r+s-r\xi)}{\pi(1+r)}(\kappa + \psi\theta^\alpha). \tag{72}$$

Substituting the expression for $A - w$ in the above equation into (70), we can derive the expression for B/N in (66). The above equation also gives w as a function of θ. In Figure 8, we plot this function and call the resulting curve the job creation curve. As in the case for the bubbleless equilibrium, this curve is downward sloping.

Next, substituting $\mu = \mu_b = r/\pi$, (71), and (49) into (47), we can express wage w as a function of θ :

$$w = \eta \left[A + \frac{\xi r + \theta}{1 + r} \frac{r + \pi}{\pi} (\kappa + \psi \theta^{\alpha}) \right] + (1 - \eta) c, \tag{73}$$

This gives the upward sloping wage curve as illustrated in Figure 8. The equilibrium (θ_b, w_b) is at the intersection of the two curves. As in the case of the bubbleless steady state, condition (65) ensures the existence of an intersection point. Equation (69) expresses the solution for θ_b in a single nonlinear equation.

How does the stock market bubble affect steady-state output and unemployment? To answer this question, we compare the bubbleless and the bubbly steady states. In the appendix, we show that both the job creation curve and the wage curve shift to the right in the presence of bubbles as illustrated in Figure 8. The intuition is the following: In the presence of a stock market bubble, the collateral value rises and the credit constraint is relaxed. Thus, a firm can finance more hires and create more jobs for a given wage rate. This explains why the job creation curve shifts to the right. Turn to the wage curve. For a given level of market tightness, the presence of a bubble puts the firm in a more favorable bargaining position because more jobs are available. This allows the firm to negotiate a lower wage rate.

The above analysis shows that the market tightness is higher in the bubbly steady state than in the bubbleless steady state. This in turn implies that hires and output are higher and unemployment is lower in the bubbly steady state than in the bubbleless steady state by Figure 5. Note that the comparison of the wage rate is ambiguous depending on the magnitude of the shifts in the two curves. If the job creation curve shifts more than the wage curve, then the wage rate should rise in the bubbly steady state. Otherwise, the wage rate should fall in the bubbly steady state.

We summarize the above result in the following:

Proposition 6 *Suppose that conditions (41), (51), (64), and (65) hold. Then in the steady state,* $\bar{\theta} > \theta_b > \theta^*$, $\bar{H} > H_b > H^*$, *and* $\bar{U} < U_b < U^*$.

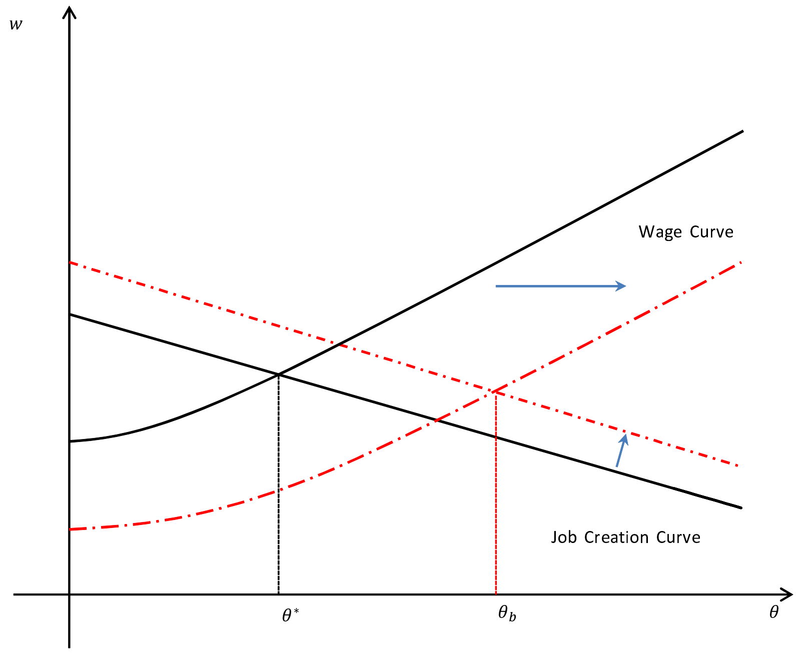

Figure 8: The job creation and wage curves for the bubbly steady state equilibrium.

How is the bubbly steady-state equilibrium with credit constraints compared to the steady-state equilibrium with perfect credit markets analyzed in Section 3.4? We can easily check that the presence of bubbles in the model with credit constraints shifts the job creation curve in Figure 5 to the right, but it shifts the wage curve to the left in Figure 4.[14] It seems that the impact on the market tightness is ambiguous. In the appendix, we show that the effect of the wage curve shift dominates so that $\bar{\theta} > \theta_b$. As a result, $H_b < \bar{H}$, and $U_b > \bar{U}$. The intuition is that even though the presence of bubbles can relax credit constraints and allows the firm to hire more workers, wages absorb the rise in firm value and reduce the firm's incentive to hire.

[14] We only need to check that

$$\frac{(r+\pi)(r+s-r\xi)}{1+r} < r+s,$$

and

$$\frac{\xi r+\theta}{1+r}\frac{r+\pi}{\pi} > \theta \text{ for all } \theta > 0.$$

5.2 Transition Dynamics

Turn to transition dynamics. As in the bubbleless equilibrium, the predetermined state variable for the equilibrium system is still N_t. But we have one more nonpredetermined variable, which is the stock price bubble B_t. Following a similar analysis for the bubbleless equilibrium in Section 4.2, we can simplify the equilibrium system and represent it by a system of four differential equations for four unknowns (Q_t, N_t, μ_t, B_t). We are unable to derive an analytical result for stability of the bubbly steady state. We thus use a numerical example to illustrate local dynamics. We still use the same parameter values given in Section 4.2. We note that the conditions in Proposition 5 are satisfied. Thus, both bubbleless and bubbly equilibria exist. In addition, one can check that these conditions are also satisfied for $\xi = 1$, implying that multiple equilibria can exist, even though there is no efficiency loss at default.

We find the steady state $(Q, N, \mu, B) = (0.271, 0.953, 0.500, 0.603)$. We then linearize around this steady state and compute eigenvalues. We find that three of the eigenvalues are positive and real and only of them is negative and real and corresponds to the predetermined variable N_t. Thus, the steady state is a saddle point and the system is saddle path stable.

Figure 9 plots the transition dynamics. Suppose the unemployment rate is initially low relative to the steady state. For a similar intuition analyzed before, the initial hiring rate must be lower than the steady state level and then gradually rises to the steady state. Other equilibrium variables follow similar patterns to those in Figure 7 during adjustment, except for bubbles. Bubbles rise gradually to the steady state value. By (23), the growth rate of bubbles is equal to the interest rate minus the shadow value of funds, $r - \pi\mu_t$. As the shadow value of external funds rises over time, the growth rate of bubbles decreases and until it reaches zero.

6 Stochastic Bubbles

So far, we have studied deterministic bubbles. In this section, we follow Blanchard and Watson (1982), Weil (1987), and Miao and Wang (2011a) and introduce stochastic bubbles. Suppose that initially the economy has a stock market bubble. But the bubble may burst in the future. The bursting event follows a Poisson process and the arrival rate is given by $\delta > 0$. When

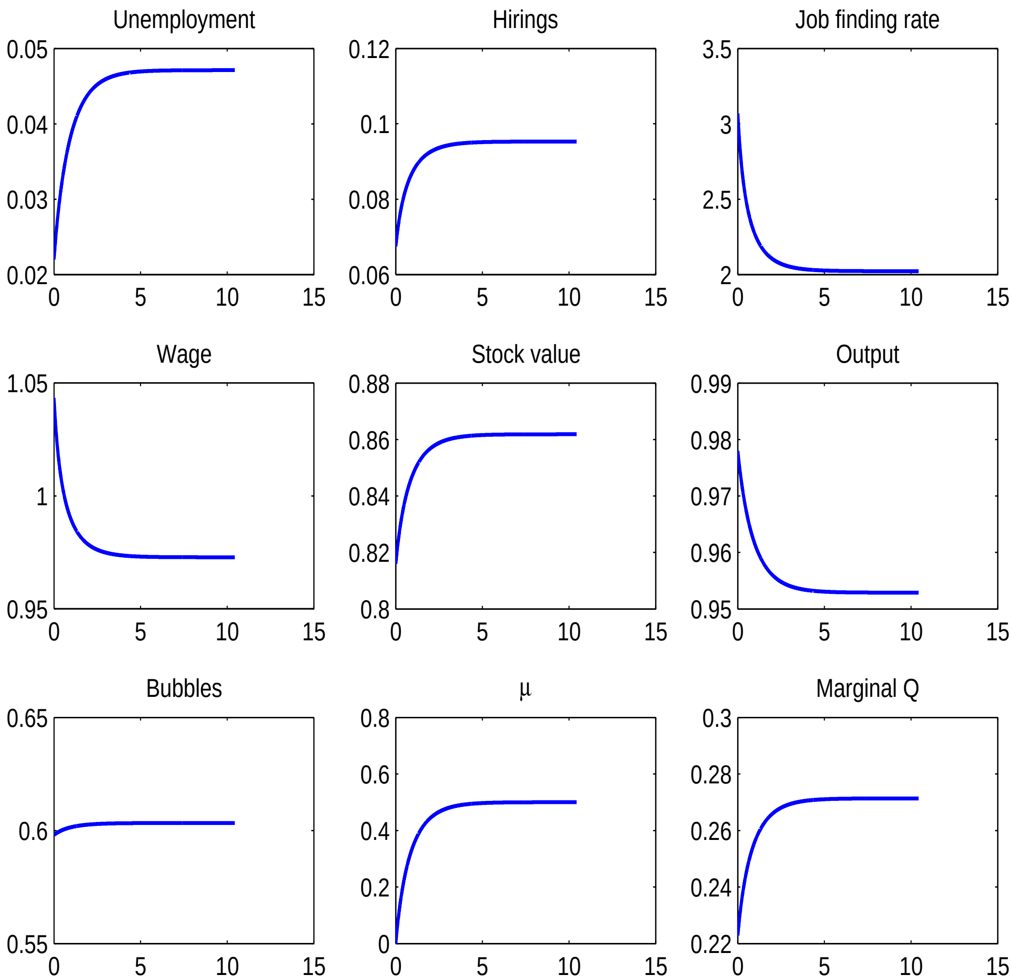

Figure 9: Transition dynamics for the bubbly equilibrium. We set parameter values as: $s = 0.1$, $\pi = 0.02$, $\kappa = 0.1$, $\eta = 0.5$, $\psi = 0.04$, $\alpha = 1$, $c = 0.4$, $A = 1$, and $\xi = 0.9$.

the bubble bursts, it will not reappear in the future by rational expectations. After the burst of the bubble, the economy enters the bubbleless equilibrium studied in Section 4. We use a variable with an asterisk to denote its value in the bubbleless equilibrium. In particular, let $V^*\left(N_t^j, Q_t^*\right)$ denote the value function for firm j with employment N_t^j and the shadow price of capital Q_t^*. As we show in Proposition 1, $V^*\left(N_t^j, Q_t^*\right) = Q_t^* N_t^j$. We can also represent Q_t^* in a feedback form in that $Q_t^* = g\left(N_t\right)$ for some function g.

We denote by $V\left(N_t^j, B_t, Q_t\right)$ the stock market value of firm j at date t before the bubble bursts. This value function satisfies the continuous-time Bellman equation:

$$\begin{aligned} rV\left(N_t^j, B_t, Q_t\right) &= \max_{H_t^j} \; \left(A - w_t\right) N_t^j - sN_t^j V_N\left(N_t^j, B_t, Q_t\right) \\ &\quad + \delta\left[V^*\left(N_t^j, Q_t^*\right) - V\left(N_t^j, B_t, Q_t\right)\right] \\ &\quad + \pi\left[V\left(N_t^j + H_t^j, B_t, Q_t\right) - V\left(N_t^j, B_t, Q_t\right) - \left(\kappa + G_t\right) H_t^j\right] \\ &\quad + \dot{Q}_t V_Q\left(N_t^j, B_t, Q_t\right) + \dot{B}_t V_B\left(N_t^j, B_t, Q_t\right), \end{aligned} \tag{74}$$

subject to the borrowing constraint

$$\left(\kappa + G_t\right) H_t^j \leq \left(A - w_t\right) N_t^j + V(\xi N_t^j, B_t, Q_t). \tag{75}$$

As in Section 3, we conjecture that the value function takes the following form:

$$V\left(N_t^j, B_t, Q_t\right) = Q_t N_t^j + B_t, \tag{76}$$

where Q_t and B_t are to be determined variables. Here B_t represents the stock price bubble. Following a similar analysis in Section 3, we can derive the Nash bargaining wage and characterize the equilibrium with stochastic bubbles in the following:

Proposition 7 *Suppose that $\mu_t > 0$ where μ_t is given by (22). Before the bubble bursts, the equilibrium with stochastic bubbles $\left(B_t, Q_t, N_t, U_t, \theta_t, H_t, w_t\right)$ satisfies the following system of differential equations: (3), (6), (7), (33), (32)*

$$\dot{B}_t = \left(r + \delta\right) B_t - \pi\mu_t B_t, \tag{77}$$

$$\dot{Q}_t = \left(r + s + \delta\right) Q_t - \delta Q_t^* - \left(A - w_t\right) - \pi\mu_t\left(A - w_t + \xi Q_t\right), \tag{78}$$

where G_t satisfies (8) and $Q_t^* = g\left(N_t\right)$ is the shadow price of capital after the bubble bursts.

As Proposition 6 shows, the system for the equilibrium with stochastic bubbles is similar to that for the bubbly equilibrium with two differences. First, the equations for bubbles in (23) and (77) are different. Due to the risk of bubble bursting, the return on bubbles is equal to $r+\delta > r$. Second, the equations for Q in (24) and (78) are different. In particular, immediately after the collapse of bubbles, Q_t jumps to the saddle path in the bubbleless equilibrium, $Q_t^* = g(N_t)$.

Following Weil (1987), Kocherlakota (2009), and Miao and Wang (2011), we focus on a particular type of equilibrium with stochastic bubbles. In this equilibrium, B_t, N_t, Q_t, U_t, H_t, θ_t, and w_t are constant before the bubble bursts. We denote the constant values by B^s, N^s, Q^s, U^s, H^s, θ^s, and w^s. These 7 variables satisfy the system of 7 equations: (44), (45), (46), (47), (63) and

$$0 = (r+\delta) B^s - \pi \mu^s B^s,$$

$$0 = (r+s+\delta) Q^s - \delta g(N^s) - (A - w^s) - \pi \mu^s (A - w^s + \xi Q^s). \tag{79}$$

After the burst of bubbles, the economy enters the bubbleless equilibrium. Immediately after the collapse of bubbles, $B^s > 0$ jumps to zero and Q^s, H^s, θ^s, and w^s jump to the bubbleless equilibrium Q_t^*, H_t^*, θ_t^*, and w_t^*, respectively. But N^s and $U^s = 1 - N^s$ continuously move to N_t^* and $U_t^* = 1 - N_t^*$ because N_t is a predetermined state variable.

Figure 10 plots the transition paths for the equilibrium with stochastic bubbles. We still use the parameter values given in Section 4.2. We suppose that households believe that, with Poisson arrival rate $\delta = 0.01$, the bubble can burst. We also suppose that the bubble bursts at time $t = 10$. Because the unemployment rate is predetermined, it rises continuously to the new higher steady state level. Output falls continuously to the new lower steady state level. Other equilibrium variables jump to the transition paths for the bubbleless equilibrium analyzed in Section 4.2. In particular, the stock market crashes in that the stock market value of the firm falls discontinuously. The hiring rate and the job-finding rate also fall discontinuously.

The wage rate rises immediately after the crash and then gradually falls to the new higher steady state level. The immediate rise in the wage rate reflects three effects. First, the job-finding rate falls on impact, leading to a fall of wage. Second, the external finance premium and marginal Q rise on impact, leading to the rise in the wage rate. Overall the second effect dominates. The rise in wage may seem counterintuitive. Figure 11 plots the data of U.S.

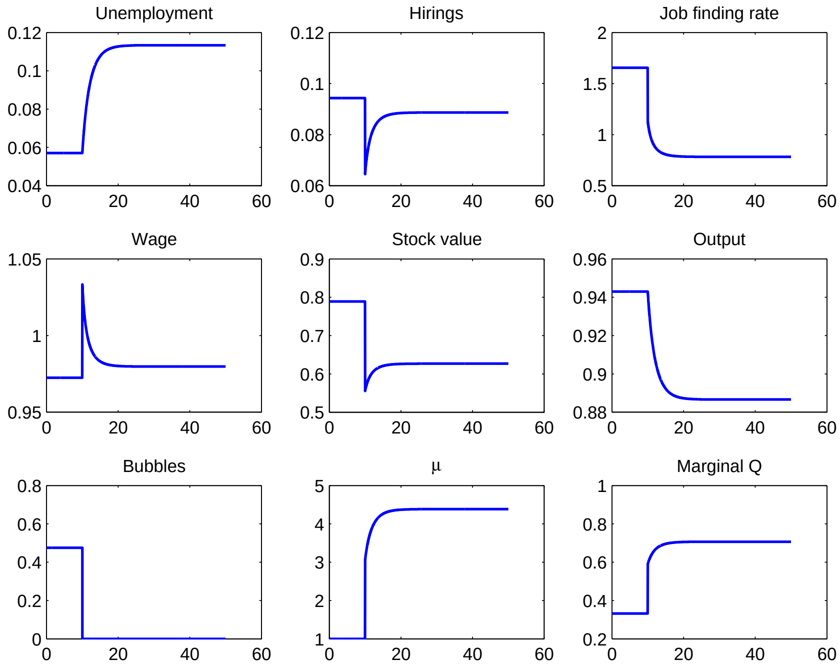

Figure 10: This figure plots transition paths for the equilibrium with stochastic bubbles. We set parameter values as: $s = 0.1$, $\pi = 0.02$, $\kappa = 0.1$, $\eta = 0.5$, $\psi = 0.04$, $\alpha = 1$, $c = 0.4$, $A = 1$, $\xi = 0.9$, and $\delta = 0.01$. We assume that the bubble collapses at $t = 10$.

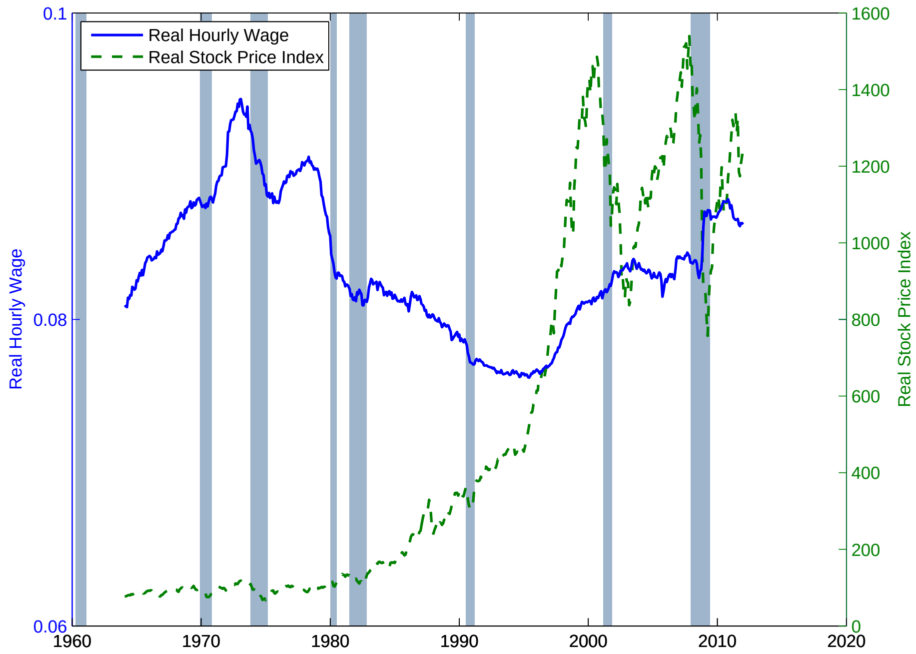

Figure 11: Real hourly wages and the stock market. Shaded areas indicate NBER recessions.

real hourly wages from BLS and the stock price index from Robert Shiller's website. We find that during the recession in the early 2000 and the recent Great Recession, real hourly wages actually rose. However, during other recessions, they fell. Though our model does not intend to explain wage dynamics, it gives an explanation of rising wages during a recession based on the fact that firms that hire during recessions are those that are more profitable and hence can pay workers higher wages.

7 Policy Implications

Our model features two types of inefficiency: credit constraints and search and matching. We have shown that bubbles cannot emerge in an economy with perfect credit markets. They can emerge in the presence of credit constraints. Thus, it is important to improve credit markets in order to prevent the formation of bubbles. Miao and Wang (2011a, 2012a) have discussed credit policy related to the credit market. In this section, we shall focus on policies related to the labor market and study how these policies affect the economy.

7.1 Unemployment Benefits

In response to the Great Recession, the U.S. government has expanded unemployment benefits dramatically. Preexisting law provided for up to 26 weeks of benefits, plus up to 20 additional weeks of "Extended Benefits" in states experiencing high unemployment rates. Starting in June 2008, Congress enacted a series of unemployment benefits extensions that brought statutory benefit durations to as long as 99 weeks. In addition to the moral hazard problem, unemployment insurance extensions can lead recipients to reduce their search effort and raise their reservation wages, slowing the transition into employment.

We now use the model in Section 6 to conduct an experiment in which the unemployment benefit is raised from 0.4 to 0.5 permanently immediately after the burst of the bubble. Figure 12 plots the transition paths. This figure reveals that this policy makes the recession more severe. In particular, the fall of the job-finding rate, hires, and the stock market value is larger on impact and these variables gradually move to their lower steady state values. In addition, the unemployment rate rises and gradually move to a higher steady state level. The new steady state unemployment rate is about 2 percentage point higher than the steady state level without the policy.

7.2 Hiring Subsidies

A potentially powerful policy to bring the labor market back from a recession is to subsidize hiring. In March 2010, Congress enacted the Hiring Incentives to Restore Employment Act, which essentially provided tax credit for private businesses to hire new employees. We now use the model in Section 6 to conduct a policy experiment in which the parameter ψ is reduced from 0.04 to 0.03 permanently immediately after the burst of the bubble. This policy experiment resembles a 25 percent hiring subsidy.

Figure 13 plots the transition paths. This figure shows that hiring subsidies make the recession less severe and help the economy move out of the recession faster. In particular, immediately after the collapse of the bubble, the policy helps firms start hiring more workers. It also helps the job-finding rate rise to a higher level. As a result, the unemployment rate rises to a lower level after the stock market crash, compared to the case without the hiring subsidy

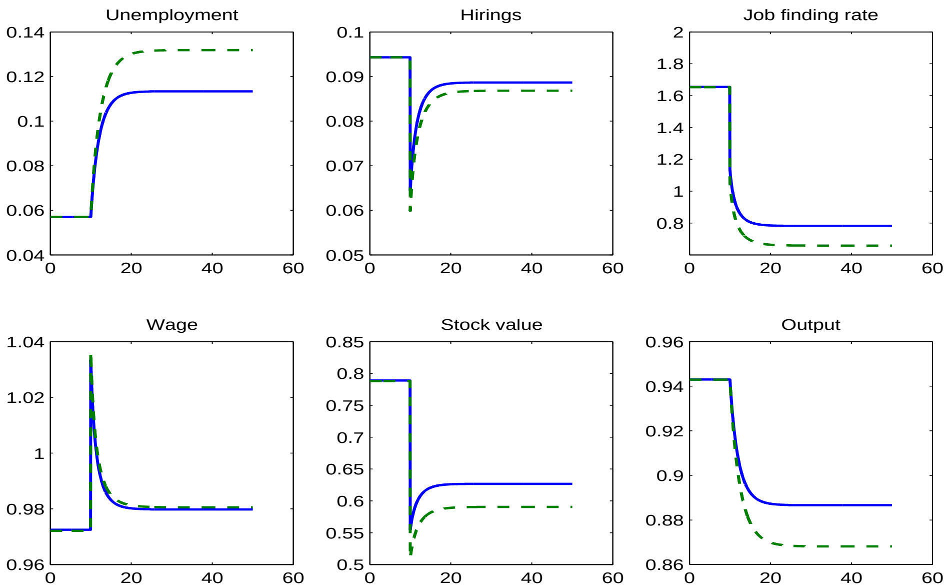

Figure 12: The impact of raising unemployment benefits. This figure plots the transition paths of the equilibria with stochastic bubbles for two economies. The solid lines are for the economy without any policy. The parameter values are $s = 0.1$, $\pi = 0.02$, $\kappa = 0.1$, $\eta = 0.5$, $\psi = 0.04$, $\alpha = 1$, $c = 0.4$, $A = 1$, $\xi = 0.9$, and $\delta = 0.01$. We assume that the bubble collapses at $t = 10$. The dashed lines are for the economy when the government raises unemployment benefits from $c = 0.4$ before time $t = 10$ to $c = 0.5$ after time $t = 10$. Other parameter values are the same as in the first economy.

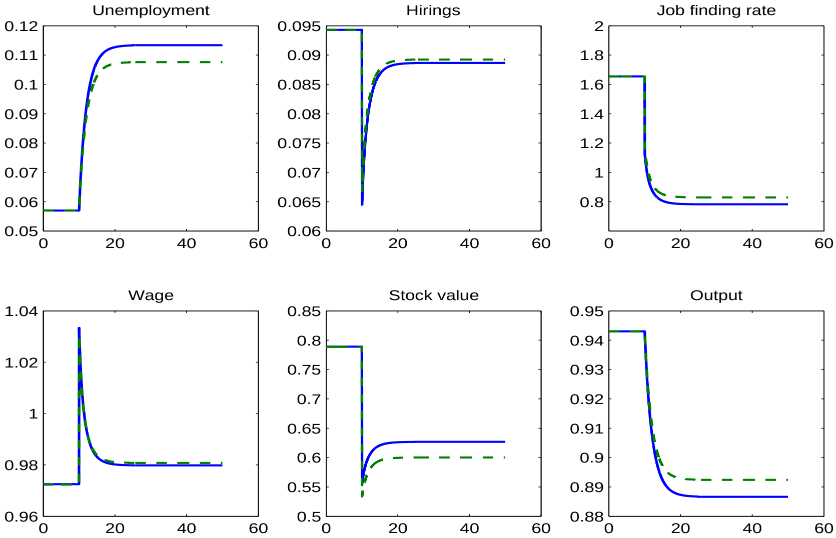

Figure 13: The impact of hiring subsidies. This figure plots the transition paths of the equilibria with stochastic bubbles for two economies. The solid lines are for the economy without any policy. The parameter values are $s = 0.1$, $\pi = 0.02$, $\kappa = 0.1$, $\eta = 0.5$, $\psi = 0.04$, $\alpha = 1$, $c = 0.4$, $A = 1$, $\xi = 0.9$, and $\delta = 0.01$. We assume that the bubble collapses at $t = 10$. The dashed lines are for the economy with hiring subsidies. We set $\psi = 0.04$ before time $t = 10$ and $\psi = 0.03$ after time $t = 10$. Other parameter values are the same as in the first economy.

policy.

8 Conclusion

In this paper, we have introduced endogenous credit constraints in a search model of unemployment. We have shown that the presence of credit constraints can generate multiple equilibria. In one equilibrium, there is a bubble in the stock market value of the firm. The bubble helps relax the credit constraints and allows firms to make more investment and hire more workers. The collapse of the bubble tightens the credit constraints, causing firms to cut investment and reduce hiring. Consequently, workers are harder to find a job, generating high and persistent unemployment. In the model, there is no aggregate shock to the fundamentals. The stock market crash and subsequent recession are generated by shifts in households beliefs.

In terms of policy implications, the policymakers should fix the credit market since it is the root cause of bubbles. Extending unemployment insurance benefits will exacerbate unemployment and recession, while hiring subsidies can help the economy recovers faster. But the economy will converge to a steady state with unemployment still higher than that in the bubbly steady state.

Appendix

A Proofs

Proof of Proposition 1: Let the value function be $V\left(N_t^j, Q_t, B_t\right)$. The continuous-time limit of the Bellman equation (9) is given by

$$\begin{aligned} rV\left(N_t^j, Q_t, B_t\right) &= \max_{H_t^j} \; (A - w_t) N_t^j - sN_t^j V_N\left(N_t^j, Q_t, B_t\right) \\ &\quad +\pi\left(V\left(N_t^j + H_t^j, Q_t, B_t\right) - V\left(N_t^j, Q_t, B_t\right) - (\kappa + G_t) H_t^j\right) \\ &\quad +V_Q\left(N_t^j, Q_t, B_t\right)\dot{Q}_t + V_B\left(N_t^j, Q_t, B_t\right)\dot{B}, \end{aligned} \tag{A.1}$$

subject to (10) and (12). Conjecture that the value function takes the form in (21). Substituting this conjecture into the above Bellman equation yields:

$$\begin{aligned} rQ_tN_t^j + rB_t &= \max_{H_t^j} \; (A - w_t) N_t^j - sN_t^j Q_t \\ &\quad +\pi\left(Q_t - (\kappa + G_t)\right) H_t^j + N_t^j \dot{Q}_t + \dot{B}, \end{aligned}$$

subject to

$$(\kappa + G_t) H_t^j \leq (A - w_t) N_t^j + \xi Q_t N_t^j + B_t. \tag{A.2}$$

Let $\mu_t \pi$ be the Lagrange multiplier associated with the above constraint. Then the first-order condition for H_t^j implies that

$$Q_t = (\kappa + G_t)(1 + \mu_t).$$

Clearly, if $\mu_t > 0$, then the credit constraint (A.2) binds so that H_t^j is given by (26). Matching coefficients of N_t^j and the other terms unrelated to N_t^j yields equations (23) and (24). Q.E.D.

Proof of Proposition 2: We have derive the wage equation in (32). Aggregating H_t^j in equation (26) yields (33). Other equations in the proposition follow from definitions. Q.E.D.

Proof of Proposition 3: Part of the proof is contained in Section 4.1. Equation (53) follows from the substitution of (52) and (49) into (57). Equation (54) follows from (44), (45), and (46). Finally, (55) follows from equations (58) and (59). Q.E.D.

Proof of Proposition 4: The proof uses Figures 6 and 7 and simple algebra. Q.E.D.

Proof of Proposition 5: Part of the proof is contained in Section 5.1 and the rest is similar to that of Proposition 3. Q.E.D.

Proof of Proposition 6: First, we show that the job creation curve shifts to the right in the bubbly equilibrium compared to the bubbleless equilibrium. It follows from equations (58) and (72) that we only need to show that

$$\frac{s(r+\pi\xi)}{r+\xi} > \frac{(r+\pi)(r+s-r\xi)}{1+r}.$$

This inequality is equivalent to

$$\frac{s(r+\pi\xi)}{r+\xi} - \frac{(r+\pi)s}{1+r} > \frac{r(r+\pi)(1-\xi)}{1+r}.$$

Simplifying yields

$$\frac{rs(1-\pi)(1-\xi)}{(r+\xi)(1+r)} > \frac{r(r+\pi)(1-\xi)}{1+r}.$$

This inequality is equivalent to

$$s > \frac{(r+\pi)(r+\xi)}{1-\pi},$$

which is equivalent to condition (64).

Next, we show that the wage curve also shifts to the right in the bubbly equilibrium compared to the bubbleless equilibrium. It follows from equations (59) and (73) that we only need to show that

$$\frac{\xi\pi\mu^* + \theta}{r+s+\xi}\frac{s}{\pi} > \frac{\xi r + \theta}{1+r}\frac{r+\pi}{\pi},$$

for any given θ, where μ^* is given by (52). It is sufficient to show that

$$\frac{\xi\pi\mu^*}{r+s+\xi}\frac{s}{\pi} > \frac{\xi r}{1+r}\frac{r+\pi}{\pi}, \tag{A.3}$$

and

$$\frac{1}{r+s+\xi}\frac{s}{\pi} > \frac{1}{1+r}\frac{r+\pi}{\pi}. \tag{A.4}$$

It is easy to check that (A.4) is equivalent to (64). Turn to (A.3). Substituting μ^* in (52), we can rewrite it as

$$\frac{s}{r}\left(\frac{1-\pi}{\pi}\frac{s}{\xi+r}-1\right) > \frac{(r+\pi)(r+s+\xi)}{\pi(1+r)}. \tag{A.5}$$

To show this inequality, we use (64) to show that

$$\frac{s}{r}\left(\frac{1-\pi}{\pi}\frac{s}{\xi+r}-1\right) > \frac{s}{\pi}.$$

We then show that

$$\frac{s}{\pi} > \frac{(r+\pi)(r+s+\xi)}{\pi(1+r)}.$$

It is easy to check that this inequality is also equivalent to (64).

Now, we compare the bubbly equilibrium and the equilibrium with perfect credit markets. As discussed in the main text, the above method of proof will give an ambiguous result. We then use a different method. It follows from (39) and (40) that $\bar{\theta}$ satisfies the following equation

$$\frac{(1-\eta)(A-c)}{\kappa+\psi\theta^{\alpha}} = \eta\theta + r + s.$$

The expression on the left-hand side of the above equation is a decreasing function of θ, while the expression on the right-side is an increasing function of θ. The solution $\bar{\theta}$ is the intersection of the two curves representing the preceding two functions. Comparing with equation (69), we only need to show that

$$\eta\theta + r + s < \frac{r+\pi}{\pi}\left[\eta\frac{\xi\pi\mu_b+\theta}{1+\pi\mu_b} - \xi + \frac{r+s+\xi}{1+r}\right]$$

for any $\theta > 0$. Substituting $\mu_b = r/\pi$ and simplifying, we can show the above inequality is equivalent to

$$\xi < \frac{s(1-\pi)}{(r+\pi)(1-\eta)} + \frac{(\eta\theta+r)(1-\pi)}{(r+\pi)(1-\eta)}.$$

This inequality holds for any $\theta > 0$ by condition (64). Thus, we deduce that $\theta_b < \bar{\theta}$. Using Figure 6, we deduce that $H_b < \bar{H}$ and $U_b > \bar{U}$. Q.E.D.

Proof of Proposition 7: Substituting the conjecture in (76) into (74) and (75) and matching coefficients, we can derive (77) and (78). The rest of equations follow from a similar argument in the proof of Proposition 2. Q.E.D.

B Isomorphism with a DMP Model

We introduce credit constraints into the large-firm DMP model discussed in Chapter 3 of Pissarides (2000). We shall show that this model is isomorphic to the model studied in Section 2. In the DMP framework, we introduce the matching function $m(u,v) = Bu^{\gamma}v^{1-\gamma}$, where $\gamma \in (0,1)$ and u and v represent aggregate unemployment and vacancy rates, respectively. Define the market tightness as $\vartheta = v/u$, the job-filling rate as $q(\vartheta) = m(u,v)/v = B\vartheta^{-\gamma}$, and the job-finding rate as $q(\vartheta)\vartheta = m(u,v)/u = B\vartheta^{1-\gamma}$. Clearly, the job-filling rate decreases with the market tightness, but the job-finding rate increases with the market tightness.

As in the model in Section 2, there is a continuum of firm of measure one. Each firm j has a Leontief technology and posts vacancies v_t^j when meeting an employment opportunity with Poisson arrival rate πdt. Thus, firm j's employment follows dynamics:

$$N_{t+dt}^j = \begin{cases} (1-sdt)N_t^j + q(\vartheta_t)v_t^j & \text{with probability } \pi dt \\ (1-sdt)N_t^j & \text{with probability } 1-\pi dt \end{cases}, \tag{B.1}$$

Posting each vacancy costs c_e. One filled job requires to buy a new machine at the cost κ. The firm faces the credit constraint:

$$(c_e + \kappa q(\vartheta_t))v_t^j \leq (A - w_t)N_t^j + e^{-rdt}V_{t+dt}(\xi N_t^j). \tag{B.2}$$

Firm j's problem is to choose v_t^j to maximizes its firm value subject to the above two constraints. The discrete-time approximation of the Bellman equation is given by

$$\begin{aligned} V_t(N_t^j) &= \max_{v_t^j} (A - w_t)N_t^j dt - \left(c_e v_t^j + \kappa q(\vartheta_t)v_t^j\right)\pi dt \\ &\quad + e^{-rdt}V_{t+dt}\left((1-sdt)N_t^j + q(\vartheta_t)v_t^j\right)\pi dt \\ &\quad + e^{-rdt}V_{t+dt}\left((1-sdt)N_t^j\right)(1-\pi dt), \end{aligned}$$

subject to (B.2).

The values to the employed and unemployed workers V_t^N and V_t^U are given by (14) and (15), except that the job-finding rate θ is replaced by $q(\vartheta)\vartheta$. The wage rate is defined by the Nash bargaining problem (19).

We now show that our model based on Blanchard and Gali (2010) is isomorphic to the above DMP model. Let

$$H_t^j = q(\vartheta_t)\, v_t^j, \quad \psi = c_e B^{-\frac{1}{1-\gamma}}, \text{ and } \alpha = \frac{\gamma}{1-\gamma}.$$

Then, by letting $H_t = \int H_t^j dj$ and $U_t = u_t$, we can show that the job-finding rate $\theta_t = H_t/U_t$ in the Blanchard-Gali setup is identical to that in the DMP setup $q(\vartheta_t)\,\vartheta_t$. In addition, the vacancy posting costs are equal to the hiring costs:

$$c_e v_t^j = \psi B^{\frac{1}{1-\gamma}} H_t^j / q(\vartheta_t) = G_t H_t^j,$$

where $G_t = \psi \theta_t^{\alpha}$. Thus, (4) is identical to (B.1), and (13) is identical to the continuous time limit of (B.2), and hence the firm's optimization problems in the two setups are identical. Since $\theta_t = q(\vartheta_t)\,\vartheta_t$, the values to the employed and unemployed workers in the two setups are also identical. As a result, the two setups give identical solutions.

In particular, when $\kappa = 0$ and credit markets are perfect, our model is isomorphic to the DMP model without credit constraints analyzed in Chapters 1 and 3 in Pissarides (2000)

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