Notes on Indeterminancy

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Introduction:

In this class, we discuss three papers on indeterminancy.

- Indeterminacy can yield endogenous business cycles
- Indeterminacy yields similar business cycle moments as RBC model
- The continuos time framework is more convenient for study this problem.

An example :

In this class, we discuss three papers on indeterminancy.

- Suppose the following example

$$x_t = \beta E_t x_{t+1} + A_t \tag{1}$$

$$A_t = \rho A_{t-1} + \varepsilon_t \tag{2}$$

where ε_t is i.i.d shocks, with $E_t \varepsilon_{t+1} = 0$, and $\rho \leq 0$ We are seeking a solution such that $\lim_{j \to \infty} E_t x_{t+j} = 0$.

- The solution for x_t when $-1 < \beta < 1$, is

$$x_t = E_t \sum_{j=0}^{\infty} \beta^j E_t A_{t+j} = E_t \sum_{j=0}^{\infty} \beta^j \rho^j A_t = \frac{1}{1 - \rho\beta} \hat{A}_t \tag{3}$$

An example :

In this class, we discuss three papers on indeterminancy.

- Suppose the following example

$$x_t = \beta E_t x_{t+1} + A_t \tag{4}$$

$$A_t = \rho A_{t-1} + \varepsilon_t \tag{5}$$

where ε_t is i.i.d shocks, with $E_t \varepsilon_{t+1} = 0$, and $\rho \leq 0$ We are seeking a solution such that $\lim_{j \to \infty} E_t x_{t+j} = 0$.

- The solution for x_t when $-1 < \beta < 1$, is

$$\begin{aligned} x_t &= E_t \sum_{j=0}^{\infty} \beta^j E_t A_{t+j} \qquad (6) \\ &= \lim \left[E_t \sum_{j=0}^{T} \beta^j \rho^j A_t + \beta^{T+1} E_t x_{t+T} \right] \\ &= \frac{1}{1-\rho\beta} \hat{A}_t \end{aligned}$$

An example :

- Where $\beta > 1$, and we assume $x_t = \lambda A_t$, and $\beta\rho \neq 1$ as a solution we have

$$\lambda = \frac{1}{1-\beta\rho} \tag{7}$$

or

$$x_t = \frac{1}{1-\beta\rho} A_t \tag{8}$$

is a solution.

- But for any ξ_t, we have

$$x_t = \frac{1}{1-\beta\rho} A_t + \xi_t \tag{9}$$

such that

$$\xi_{t+1} = \beta^{-1}\xi_t + \omega_{t+1} \tag{10}$$

where ω_{t+1} is i.i.d shock is also an solution.

Increasing returns can yield indeterminacy, Multiple local equilibria around the steady-state.

- Suppose the aggregate technology can be written as

$$Y = K^{\alpha^*} N^{\beta} \tag{11}$$

with

$$\alpha^*, \beta > 0; \alpha^* + \beta > 1$$

- However the factor price w, r can be presented by

$$wN = bY \tag{12}$$

$$rK = aY \tag{13}$$

we assume $0 < a \leq \alpha^*; 0 < b \leq \beta$.

- We discuss the cases that yield the above specification.

The model with Externality

- Suppose the aggregate technology can be written as

$$Y = K^a N^b \bar{K}^{a\theta_1} \bar{N}^{b\theta_2} \tag{14}$$

where $\bar{K}$,and $\bar{N}$ represent the average economy-wide levels of capital and the labor.

- The economy is assumed to consist of a large number of identical firms. A representative firm's capital and labor choice would yields

$$aK^{a-1} N^b \bar{K}^{a\theta_1} \bar{N}^{b\theta_2} = r \tag{15}$$

$$bK^a N^{b-1} \bar{K}^{a\theta_1} \bar{N}^{b\theta_2} = w \tag{16}$$

- In equilibrium, we have $K = \bar{K}$, $N = \bar{N}$, so we have

$$wN = bY \tag{17}$$

$$rK = aY \tag{18}$$

- If we assume $a + b = 1$, so we have

$$\alpha^* = a(1 + \theta_1); \beta = b(1 + \theta_2); \alpha^* + \beta > 1 \tag{19}$$

The model with Monopolistic Competition

- Consider an individual firm uses a technology similar to that described by Dixit-Stiglitz. There is a continuum of intermediate goods $Y(i)$, , final output is given by

$$Y = \left[\int_0^1 Y^{\lambda}(i) di \right]^{\frac{1}{\lambda}} \tag{20}$$

  where $\lambda \in (0, 1)$.

- The profit of the final goods sector

$$\Pi = Y - \int_0^1 P(i) Y(i) di \tag{21}$$

- First order conditions for the profit maximization lead to

$$Y(i) = P^{1/(\lambda - 1)}(i) Y \tag{22}$$

The model with Monopolistic Competition

- We assume the technology for producting an intermediate commodity is given by
$$Y(i) = K^{\alpha^*}(i)N^{\beta}(i) \tag{23}$$
- We assume
$$\alpha^* + \beta > 1 \tag{24}$$
- The profit can of the ith intermediate good producer can be expressed as
$$\Pi(i) = \left(\frac{Y(i)}{Y}\right)^{\lambda-1} Y(i) - wN(i) - rK(i).$$

The model with Monopolistic Competition

- We assume the technology for producting an intermediate commodity is given by
$$Y(i) = K^{\alpha^*}(i)N^{\beta}(i) \tag{25}$$
- We assume
$$\alpha^* + \beta > 1 \tag{26}$$
- The profit can of the ith intermediate good producer can be expressed as
$$\Pi(i) = \left(\frac{Y(i)}{Y}\right)^{\lambda-1} Y(i) - wN(i) - rK(i).$$

The model with Monopolistic Competition

- Or we have

$$\Pi(i) = Y^{1-\lambda} K^{\lambda\alpha^*}(i) N^{\beta\lambda}(i) - wN(i) - rK(i). \tag{27}$$

- The first order conditions are

$$\frac{\lambda\alpha^* Y(i)P(i)}{K(i)} = r \tag{28}$$

$$\frac{\lambda\beta Y(i)P(i)}{N(i)} = w \tag{29}$$

- We can set

$$a = \lambda\alpha^*; b = \lambda\beta \tag{30}$$

The model with Monopolistic Competition

- By symmetry,

$$N(i) = N; K(i) = K, \text{ and } P(i) = \bar{P} \tag{31}$$

- The assumption of the final goods sector and constant return to scale is competitive implies

$$\Pi = Y - \int_0^1 P(i)Y(i)di = 0 \tag{32}$$

- By $Y = Y(i)$, so we have $P(i) = \bar{P} = 1$, and the aggregate output is

$$Y = K^{\alpha}N^{\beta} \tag{33}$$

- The instantaneous utility of the representative consumer is

$$U = \log(C_t) - \frac{N_t^{1-\chi}}{1-\chi} \tag{34}$$

where $\chi \leq 0$.

- The objective is

$$\max \int_0^\infty e^{-\rho t}[\log(C_t) - \frac{N_t^{1-\chi}}{1-\chi}]dt \tag{35}$$

- subject to

$$\dot{K}_t = (r_t - \delta)K_t + w_t N_t + \Pi_t^A - C_t \tag{36}$$

Consumer's problem and Market Equilibrium

- Total profit

$$r_t K_t + w_t N_t + \Pi_t^A = Y_t \tag{37}$$

- In the externality model $\Pi_t^A = 0$, by our assumption. In the monopolistic competion model,

$$\begin{aligned} \Pi_t^A &= \int_0^1 \Pi_t(i) di = \int P(i) Y(i) di - wN - rK \\ &= Y - wN - rK \end{aligned} \tag{38}$$

- The focs are

$$\frac{C_t}{N_t^{\chi}} = w_t \tag{39}$$

$$\frac{\dot{C}_t}{C_t} = r_t - \rho - \delta \tag{40}$$

Consumer's problem and Market Equilibrium

- Transversility condition

$$\lim_{t\to\infty} e^{-\rho t}\frac{K_t}{C_t} = 0 \tag{41}$$

- Using the expression of w_t and r_t we have

$$C_t = bY_t N_t^{\chi-1} \tag{42}$$

$$\frac{\dot{C}_t}{C_t} = a\frac{Y_t}{K_t} - \rho - \delta \tag{43}$$

Dynamics

- Define $x_t = \log X_t$, we then translate the two dynamic equations into

$$\dot{k}_t = e^{y_t - k_t} - \delta - e^{c_t - k_t}. \tag{44}$$

$$\dot{c}_t = ae^{y_t - k_t} - \rho - \delta. \tag{45}$$

- where

$$y_t = \alpha^* k_t + \beta n_t \tag{46}$$

- and labor is given by

$$c_t = \log(b) + y_t + (\chi - 1) n_t \tag{47}$$

Dynamics

- We want to solve $y_t - k_t$,

$$y_t - k_t = \lambda_0 + \lambda_1 k_t + \lambda_2 c_t \tag{48}$$

$$\lambda_0 = \frac{-\beta \log(b)}{\beta + \chi - 1} \tag{49}$$

$$\lambda_1 = \frac{(\chi - 1)(\alpha^* - 1) - \beta}{\beta + \chi - 1} \tag{50}$$

$$\lambda_2 = \frac{\beta}{\beta + \chi - 1} \tag{51}$$

Dynamics

- Hence the two dynamics equations become

$$\dot{k}_t = e^{\lambda_0 + \lambda_1 k_t + \lambda_2 c_t} - \delta - e^{c_t - k_t} \tag{52}$$

$$\dot{c}_t = a e^{\lambda_0 + \lambda_1 k_t + \lambda_2 c_t} - \rho - \delta \tag{53}$$

- in the steady-state,

$$e^{\lambda_0 + \lambda_1 k + \lambda_2 c} = \frac{\rho + \delta}{a} \tag{54}$$

$$e^{c-k} = \frac{\rho + \delta}{a} - \delta \tag{55}$$

Dynamics-Linearization

- Consumption equations

$$\dot{c}_t = (\rho + \delta)[\lambda_1 k_t + \lambda_2 c_t] \quad (56)$$

- capital equations

$$\begin{aligned} \dot{k}_t &= \frac{(\rho+\delta)}{a}[\lambda_1 k_t + \lambda_2 c_t] - [\frac{(\rho+\delta)}{a} - \delta][c_t - k_t] \\ &= \left[\frac{(\rho+\delta)}{a}\lambda_1 + \frac{(\rho+\delta)}{a} - \delta\right] k_t \\ &\quad + \left[\frac{(\rho+\delta)}{a}\lambda_2 - \frac{(\rho+\delta)}{a} + \delta\right] c_t \end{aligned} \quad (57)$$

Dynamics-Linearization

- Or we can write in matrix form

$$\begin{bmatrix} \dot{k}_t \\ \dot{c}_t \end{bmatrix} = \begin{bmatrix} \frac{(\rho+\delta)}{a}\lambda_1 + \frac{(\rho+\delta)}{a} - \delta & \frac{(\rho+\delta)}{a}\lambda_2 - \frac{(\rho+\delta)}{\alpha} + \delta \\ (\rho+\delta)\lambda_1 & (\rho+\delta)\lambda_2 \end{bmatrix} \begin{bmatrix} k_t - \bar{k} \\ c_t - \bar{c} \end{bmatrix} \tag{58}$$

- The model's dynamcis are fully discribled by the eigenvalues of the above Jocobian matrix, J.
- Suppose these two eigenvalues are μ_1, μ_2, we then have

$$\mu_1 \mu_2 = \Delta(J) \tag{59}$$

$$\mu_1 + \mu_2 = trace(J) \tag{60}$$

Dynamics-Linearization

- We now want to determine these two

$$\begin{aligned}
\frac{\Delta}{(\rho+\delta)} &= \left(\frac{(\rho+\delta)}{a}\lambda_1\lambda_2 + \frac{(\rho+\delta)}{\alpha}\lambda_2 - \delta\lambda_2\right) \\
&\quad - \left(\frac{(\rho+\delta)}{a}\lambda_2\lambda_1 - \frac{(\rho+\delta)}{a}\lambda_1 + \delta\lambda_1\right) \\
&= \frac{(\rho+\delta)}{a}\lambda_2 - \delta\lambda_2 + \frac{(\rho+\delta)}{a}\lambda_1 - \delta\lambda_1 \\
&= \left[\frac{(\rho+\delta)}{a} - \delta\right][\lambda_1+\lambda_2] \qquad (61)
\end{aligned}$$

- Or we have

$$\Delta = \frac{\rho+\delta}{a}[\rho+\delta(1-a)][\lambda_1+\lambda_2] \qquad (62)$$

Dynamics-Linearization

- And the trace is

$$\begin{aligned} trace &= \frac{(\rho+\delta)}{a}\lambda_1 + \frac{(\rho+\delta)}{a} - \delta + (\rho+\delta)\lambda_2 \\ &= \frac{(\rho+\delta)}{a}[\lambda_1 + a\lambda_2] + \frac{\rho + (1-a)\delta}{a} \end{aligned}$$

- The model is determinate if and only (suppose $\mu_1 > \mu_2$)

$$\mu_1 > 0, \mu_2 < 0 \tag{63}$$

this requires

$$\Delta < 0 \tag{64}$$

- The model is indeterminate if

$$\mu_1, \mu_2 < 0 \tag{65}$$

This requies

$$\Delta > 0; trace < 0 \tag{66}$$

Discussion

In the case $a = \alpha^*$, namely there is no externality in capital,

- in this case

$$\lambda_1 = \frac{(\chi - 1)(a - 1) - \beta}{\beta + \chi - 1} \tag{67}$$

$$\lambda_2 = \frac{\beta}{\beta + \chi - 1} \tag{68}$$

- We need to check the trace and determinate

Discussion

- The trace is

$$
\begin{aligned}
trace &= \frac{(\rho+\delta)}{a}[\frac{(\chi-1)(a-1)-\beta}{\beta+\chi-1}+\frac{a\beta}{\beta+\chi-1}]+\frac{\rho+(1-a)\delta}{a} \\
&= \frac{(\rho+\delta)}{a}[\frac{(\chi-1)(a-1)+\beta(a-1)}{\beta+\chi-1}]+\frac{\rho+(1-a)\delta}{a} \\
&= \frac{(\rho+\delta)}{a}[\frac{(\chi-1+\beta)(a-1)}{\beta+\chi-1}]+\frac{\rho+(1-a)\delta}{a} \\
&= \frac{(\rho+\delta)(a-1)+\rho+(1-a)\delta}{a} \\
&= \frac{\rho(a-1)+\rho}{a}=\rho>0
\end{aligned}
$$

- so one of the the eigenvalues must be postive. Or there is no indeterminacy.

The necessary condition for indeterminancy

- In the indeterminace case. The two roots are negative, so we must have

$$\textit{trace} < 0 \text{ and } \Delta > 0$$

- In the more general case

$$\Delta = \frac{\rho + \delta}{a}[\rho + \delta(1 - a)]\frac{(\chi - 1)(\alpha^* - 1)}{\beta + \chi - 1} > 0$$

- This requires

$$\beta + \chi - 1 > 0.$$

Intuition for indeterminancy

- The labor demand curve

$$\log w = \log(b) + y - n = \log(b) + \alpha^* k + (\beta - 1)n \quad (69)$$

  The slop is

$$\beta - 1 \quad (70)$$

- The labor supply curve

$$\log(w) = c - \chi n \quad (71)$$

  the slop is

$$-\chi > 0 \quad (72)$$

- The condition $\beta + \chi - 1 > 0$, implies the slop of labor demand curve is bigger than the slop of labor supply curve.

This is illstrated by the following figure:

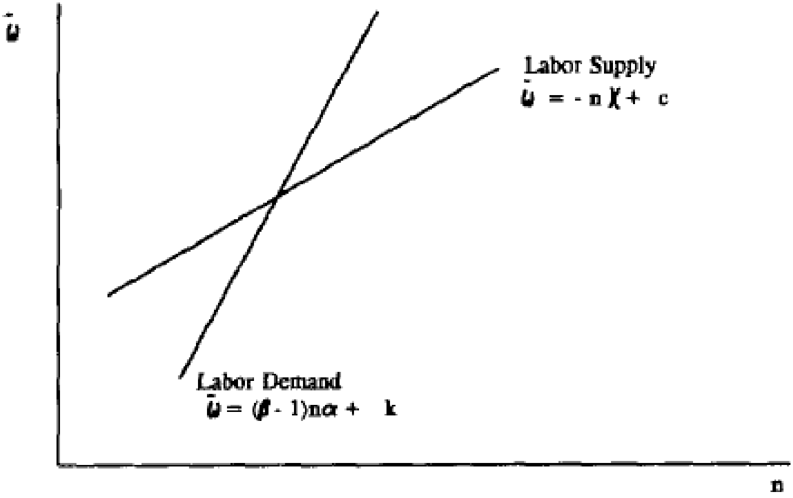

FIGURE 1

Model Setup-Household

- The representative households solve

$$\int_0^\infty e^{-\rho t} \left[\log(C_t) - \frac{N_t^{1+\gamma}}{1+\gamma} \right] dt \tag{73}$$

- with the resource constraint.

$$\dot{K}_t = (e_t K_t)^\alpha N_t^{1-\alpha} \left[(\bar{e}_t \bar{K}_t)^\alpha \bar{N}_t^{1-\alpha} \right] - \delta_t K_t - C_t \tag{74}$$

Where $\delta_t = \delta_0 \frac{e_t^\theta}{\theta}$. $\bar{X}_t$ denotes the aggregate variables.

- It is important to notice that while the households take them as given but in equilibrium we have $\bar{X}_t = X_t$.

First order condition

- The first order condition is

$$\frac{\dot{C}_t}{C_t} = \alpha \frac{Y_t}{K_t} - \rho - \delta_t \tag{75}$$

$$C_t = (1-\alpha)\frac{Y_t}{N_t^{1+\gamma}} \tag{76}$$

$$\alpha \left(e_t K_t\right)^{\alpha-1} N_t^{1-\alpha} \left[\left(\bar{e}_t \bar{K}_t\right)^{\alpha} \bar{N}_t^{1-\alpha}\right] K_t = \delta_0 e_t^{\theta-1} K_t \tag{77}$$

$$\dot{K}_t = \left(e_t K_t\right)^{\alpha} N_t^{1-\alpha} \left[\left(\bar{e}_t \bar{K}_t\right)^{\alpha} \bar{N}_t^{1-\alpha}\right] - \delta_t K_t - C_t \tag{78}$$

First order condition

- In equilibirium, $K_t = \bar{K}_t$, $e_t = \bar{e}_t$, $N_t = \bar{N}_t$, so we have

$$\alpha \frac{Y_t}{K_t} = \delta_0 e_t^{\theta} = \theta \delta_t \tag{79}$$

- Or

$$\alpha \frac{Y_t}{K_t} - \rho - \delta_t = \alpha (\frac{\theta - 1}{\theta}) \frac{Y_t}{K_t} \tag{80}$$

- and

$$\theta \log(e_t) = \log(Y_t) - \log(K_t) + const \tag{81}$$

- implies

$$\begin{aligned} \log(Y_t) &= \alpha(1+\eta)(\log e_t + \log K_t) + (1-\alpha)(1+\eta)\log N_t \qquad (8 \\ &= const + \frac{\alpha(1+\eta)(\theta-1)}{\theta - \alpha(1+\eta)} \log(K_t) + \frac{(1-\alpha)\,(1+\eta)\theta}{\theta - \alpha(1+\eta)} \log(\end{aligned}$$

First order condition

- Notice the sum of two coefficients, for $\eta = 0$

$$
\begin{aligned}
& \frac{\alpha(1+\eta)(\theta-1)}{\theta-\alpha(1+\eta)} + \frac{(1-\alpha)\,(1+\eta)\theta}{\theta-\alpha(1+\eta)} \\
= \;& \frac{\alpha(\theta-1)+(1-\alpha)\theta}{\theta-\alpha} = \frac{\alpha\theta-\alpha+\theta-\alpha\theta}{\theta-\alpha} \\
= \;& 1
\end{aligned} \tag{83}
$$

- So capacity utility itself does not lead to increasing return to scales. For $\eta > 0$

$$
\begin{aligned}
& \frac{\alpha(1+\eta)(\theta-1)}{\theta-\alpha(1+\eta)} + \frac{(1-\alpha)\,(1+\eta)\theta}{\theta-\alpha(1+\eta)} \\
= \;& (1+\eta)\frac{\theta-\alpha}{\theta-\alpha(1+\eta)} > (1+\eta)
\end{aligned} \tag{84}
$$

so capacity can increase the degree of increase return to scales.

- Again suppose that

$$\log \frac{Y_t}{K_t} = \lambda_0 + \lambda_1 k_t + \lambda_2 c_t. \tag{85}$$

- So we have the following two systems

$$\dot{c}_t = \alpha(\frac{\theta - 1}{\theta}) e^{\lambda_0 + \lambda_1 k_t + \lambda_2 c_t} - \rho \tag{86}$$

$$\dot{k}_t = \frac{\theta - \alpha}{\theta} e^{\lambda_0 + \lambda_1 k_t + \lambda_2 c_t} - e^{c_t - k_t} \tag{87}$$

- and in the steady-state

$$\begin{aligned} e^{\lambda_0 + \lambda_1 k + \lambda_2 c} &= \frac{\rho}{\alpha(\frac{\theta-1}{\theta})} \\ e^{c-k} &= \frac{\theta - \alpha}{\theta} \frac{\rho}{\alpha(\frac{\theta-1}{\theta})} = \frac{(\theta - \alpha)\rho}{\alpha(\theta - 1)} \end{aligned} \tag{88}$$

Linearization

- The consumption growth rate equation

$$\begin{aligned} \dot{c}_t &= \alpha(\frac{\theta-1}{\theta}) \times e^{\lambda_0+\lambda_1 k_t+\lambda_2 c_t}[\lambda_1 (k_t-\bar{k})+\lambda_2(c_t-\bar{c})] \\ &= \rho [\lambda_1 (k_t-\bar{k})+\lambda_2(c_t-\bar{c})] \end{aligned} \tag{89}$$

- capital accumulation equation

$$\begin{aligned} \dot{k}_t &= \frac{\theta-\alpha}{\theta}\frac{\rho}{\alpha(\frac{\theta-1}{\theta})}[\lambda_1 (k_t-\bar{k})+\lambda_2(c_t-\bar{c})] \\ &\quad -\frac{(\theta-\alpha)\rho}{\alpha(\theta-1)}[-(k_t-\bar{k})+(c_t-\bar{c})] \end{aligned} \tag{90}$$

Linearization

- The Jacobian matrix is

$$J = \begin{bmatrix} \frac{(\theta-\alpha)\rho}{\alpha(\theta-1)}(\lambda_1+1) & \frac{(\theta-\alpha)\rho}{\alpha(\theta-1)}(\lambda_2-1) \\ \rho\lambda_1 & \rho\lambda_2 \end{bmatrix} \tag{91}$$

- Det is

$$\begin{aligned} \Delta &= \frac{(\theta-\alpha)\rho}{\alpha(\theta-1)}(\lambda_1+1)\rho\lambda_2 - \frac{(\theta-\alpha)\rho}{\alpha(\theta-1)}(\lambda_2-1)\rho\lambda_1 \\ &= \frac{(\theta-\alpha)\rho}{\alpha(\theta-1)}\rho[\lambda_1+\lambda_2] \end{aligned} \tag{92}$$

- trace is

$$trace = \frac{(\theta-\alpha)\rho}{\alpha(\theta-1)}(\lambda_1+1) + \rho\lambda_2 \tag{93}$$

Linearization

- We now turn to the determination of λ_1, λ_2
- By the first order condition

$$c_t = y_t - (1+\gamma)n_t \tag{94}$$

$$\begin{aligned} y_t &= \frac{\alpha(1+\eta)(\theta-1)}{\theta-\alpha(1+\eta)}k_t + \frac{(1-\alpha)(1+\eta)\theta}{\theta-\alpha(1+\eta)}n_t \\ &= \beta_1 k_t + \beta_2 n_t \end{aligned} \tag{95}$$

$$c_t = \beta_1 k_t + \beta_2 n_t - (1+\gamma)n_t \tag{96}$$

- or we have

$$n_t = \frac{c_t - \beta_1 k_t}{\beta_2 - (1+\gamma)} \tag{97}$$

- and we have

$$y_t = \beta_1 k_t + \frac{(c - \beta_1 k_t)\,\beta_2}{\beta_2 - (1+\gamma)} \tag{98}$$

- And

$$\begin{aligned} y_t - k_t &= \beta_1 k_t + \frac{(c - \beta_1 k_t)\,\beta_2}{\beta_2 - (1+\gamma)} - k_t \\ &= \frac{\beta_2 c_t - \beta_1 (1+\gamma) k_t}{\beta_2 - (1+\gamma)} - k_t \end{aligned} \tag{99}$$

Linearization

- so we have

$$
\begin{aligned}
y_t - k_t &= \frac{\beta_2 c_t}{\beta_2 - (1+\gamma)} \\
&\quad + \frac{(1-\beta_1)(1+\gamma) - \beta_2}{\beta_2 - (1+\gamma)} k_t
\end{aligned} \tag{100}
$$

- or we have

$$
\lambda_1 = \frac{-\beta_1(1+\gamma) + (1+\gamma) - \beta_2}{\beta_2 - (1+\gamma)}
$$

$$
\lambda_2 = \frac{\beta_2}{\beta_2 - (1+\gamma)}
$$

Linearization

- Recall

$$\Delta = \frac{(\theta - \alpha)\,\rho}{\alpha\,(\theta - 1)}\rho[\lambda_1 + \lambda_2] \tag{101}$$

- and

$$\begin{aligned} \lambda_1 + \lambda_2 &= \frac{\beta_2 - \beta_1(1+\gamma)}{\beta_2 - (1+\gamma)} - 1 \\ &= \frac{(1+\gamma)(1-\beta_1)}{\beta_2 - (1+\gamma)} \end{aligned} \tag{102}$$

- The necessary for indeterminacy is

$$\lambda_1 + \lambda_2 > 0, \text{ or } \beta_2 > (1+\gamma) \tag{103}$$