Credit Constraints and Self-fulfilling Business Cycles

By Zheng Liu and Pengfei Wang*

We argue that credit constraints not only amplify fundamental shocks, they can also lead to self-fulfilling business cycles. We study a model with heterogeneous firms, in which imperfect contract enforcement implies that productive firms face binding credit constraints, with the borrowing capacity limited by expected equity value. A drop in equity value tightens credit constraints and reallocates resources from productive to unproductive firms. Such reallocation reduces aggregate productivity, further depresses equity value, generating a financial multiplier. Aggregate dynamics are isomorphic to those in a representative-agent economy with increasing returns. For sufficiently tight credit constraints, the model generates self-fulfilling business cycles.
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In the presence of credit constraints, financial factors can play an important role in macroeconomic fluctuations. For instance, if it is costly to enforce loan contracts or monitor project outcomes, then borrowing capacity will be limited by the value of the borrower's collateral assets or net worth. When credit constraints are binding, an increase in asset prices eases the constraints and thus helps expand production and investment. Expanded production and investment in turn raise the borrower's collateral value and net worth, further easing the constraints. This financial accelerator can, in principle, amplify macroeconomic fluctuations by transforming small economic shocks into large business cycles (Kiyotaki and Moore, 1997; Bernanke, Gertler and Gilchrist, 1999). Recent studies show that the financial accelerator is empirically important, especially for amplifying and propagating financial shocks (Christiano, Motto and Rostagno, 2010; Liu, Wang and Zha, forthcoming). The goal of this paper is to point out that credit constraints not only amplify fundamental shocks; sufficiently tight credit constraints can also lead to self-fulfilling business cycles.

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A. The key mechanism

To make this point, we study a business cycle model with financial friction. The model features a representative household who consumes a homogeneous good and supplies labor to firms. The household invests the good to accumulate capital, which is rented to firms in a competitive market. Firms have access to a constant returns technology that transforms capital and labor into goods. In each period, firms draw an idiosyncratic productivity that is independently and identically distributed (i.i.d.) across firms and across time. If a firm chooses to produce, it hires workers and rents capital from the household but makes no upfront payments. In this sense, the household is providing working capital loans to finance the firm's operation. After production is completed, firms decide whether or not to pay their workers and capital rents. At the end of the period, a firm needs to pay a fixed cost if it would like to stay in business for the next period.

Imperfect contract enforcement creates incentive for firms to default on loan repayments. Recognizing the possibility of default, the household does not lend freely, so that the amount of loans available to operating firms is bounded above by a fraction of firms' expected equity value. Under optimal incentive-compatible contracts, no firms default in equilibrium. Firms with productivity above a cut-off level operate and face binding credit constraints. Firms with productivity below the cut-off level remain idle in the current period and stay in business expecting that future productivity may improve.[1]

The model generates a financial multiplier that amplifies macroeconomic shocks. Since borrowing capacity is bounded above by firms' expected equity value, an increase in equity value raises credit limits for productive firms. With more credit available, productive firms expand production by hiring more workers and renting more capital. Thus, factor prices rise and low-productivity firms are crowded out. Since resources are reallocated to productive firms, aggregate productivity rises. This leads to further increases in equity value and further expansions in credit, generating a ripple effect.

The reallocation effect stemming from credit constraints leads to procyclical aggregate productivity. In our model, measured total factor productivity (TFP) contains a financial factor, which is a function of aggregate leverage measured by the ratio of working capital loans to aggregate output. An increase in leverage raises firms' borrowing capacity and shifts resources to productive firms. Thus, aggregate productivity increases with leverage.

Since aggregate productivity is procyclical, increases in input factors lead to more than proportional increases in aggregate output. This implies aggregate increasing returns. The degree of increasing returns in the aggregated version of our model corresponds to the size of the financial multiplier. With a sufficiently large financial multiplier, the model generates self-fulfilling, sunspot-driven business

[1]Since firms are ex ante identical, all firms choose to stay in business in equilibrium.

cycles.

To understand our model's mechanism through which equilibrium indeterminacy arises, consider the effect of expectations of higher future output. Such expectations shift the labor supply curve upward through a wealth effect, as in the standard real business cycle (RBC) model. In the standard RBC model, however, the wealth effect raises equilibrium real wage and lowers equilibrium hours and output, which would invalidate the initial expectations. However, in our model with credit frictions, the rise in the real wage makes it unprofitable for low-productivity firms to operate. Thus, labor and capital are reallocated to high-productivity firms and aggregate productivity rises. The improvement in aggregate productivity raises aggregate labor demand. Thus, the labor demand schedule shifts upward, offsetting the negative effects on equilibrium hours from the shift in the labor supply curve. Since a productivity improvement also raises firm value, the credit limit for productive firms expands, reinforcing the reallocation effects through a financial multiplier. The greater the size of the financial multiplier, the larger the rise in labor demand, and the more likely equilibrium hours would rise. If hours do rise, output would rise as well, rendering the initial expectations self-fulfilling.

B. *Empirical evidence*

Our model's implications for the allocations of credit and production over business cycles are consistent with empirical evidence. The model has three key implications. First, credit allocations (leverage) are procyclical, with credit-to-output ratio rising in business cycle booms. Second, productive factors are reallocated from low-productivity to high-productivity firms in business cycle booms, implying procyclical aggregate productivity. Third, the degree of returns to scale increases with the level of aggregation. Each of these implications is consistent with empirical evidence.

Procylical leverage is consistent with empirical evidence. For example, Jermann and Quadrini (2012) present evidence that, in aggregate U.S. data, debt repurchases relative to business sector output is countercyclical. Their debt repurchases are defined as the reductions in total non-financial business debt. Thus, their evidence implies that debt outstanding relative to output is procyclical. Covas and den Haan (2011) study debt and equity issuances by firms of different sizes. They report that, although the cyclical behavior of equity issuance differs across firms (especially for very large firms), debt issuance scaled by total assets is consistently procyclical for firms in all size categories. These studies are consistent with our model's predictions.

Empirical studies also support our model's implication that reallocations help explain procyclical aggregate productivity. For example, Basu and Fernald (2001) report that, in business cycle booms, productive factors are reallocated from low-markup firms to high-markup firms. They further argue that, if factor prices are identical across firms, then high-markup firms are also those with high social marginal productivity. Thus, reallocations from low-productivity to high-productivity firms in business cycle booms raise aggregate productivity, as our model predicts.[2]

In our model, firms operate a constant-returns production technology in variable factors, although the presence of the fixed cost (i.e., the cost for staying in business) implies firm-level increasing returns. We show that, despite a small degree of increasing returns at the firm level calibrated to match microeconomic evidence (Basu and Fernald, 1997), procyclical reallocations of credit and production generate large increasing returns at the aggregate level, with the magnitude of increasing returns corresponding to the size of the financial multiplier. The model's implication that the magnitude of returns to scale increases with the level of aggregation is consistent with the evidence provided by Basu and Fernald (1997).

The financial transmission mechanism in our model is also consistent with some cross-country evidence documented in the literature. For example, Aghion et al. (2010) present evidence from a panel of countries that suggests that tighter credit constraints lead to higher volatility and lower mean growth.[3] This evidence is consistent with our model's prediction. In our model, poorer contract enforcement implies tighter credit constraints, which lead to greater misallocation and thus lower levels of aggregate productivity and output. Poorer contract enforcement also leads to greater volatility of aggregate output because it implies a larger financial multiplier that amplifies fundamental shocks and, with sufficiently tight credit constraints, it may also lead to sunspot-driven business cycle fluctuations.

C. *Relation to literature*

Our work builds on a large strand of literature that examines the possibility of indeterminate equilibria in RBC models. In an influential study, Benhabib and Farmer (1994) first point out that a standard one-sector RBC model with increasing returns to scale can generate indeterminacy. Farmer and Guo (1994) show that, in such an economy, sunspot shocks are quantitatively important for business cycles. The degree of increasing returns required to generate indeterminacy in this class of models, however, is considered too large to be consistent with empirical evidence (Basu and Fernald, 1995, 1997).

Subsequent contributions by Benhabib and Farmer (1996) and Benhabib and Nishmura (1998) show that, in multi-sector RBC models, the required external-

[2]There is also evidence that capital reallocations are procyclical (Eisfeldt and Rampini, 2006; Gilchrist, Sim and Zakrajsek, 2010). To the extent that a higher volume of capital reallocations improve the quality of matching between users and producers of capital, such reallocations can also improve aggregate productivity in business cycle booms and thus lead to procyclical productivity. However, unlike Basu and Fernald (2001), these studies do not explicitly establish the directions of reallocations over the business cycles (e.g., Are resources shifted from low-productivity firms to high-productivity firms in booms?).

[3]See also Ramey and Ramey (1995), who find that countries with lower mean growth experience larger growth volatility.

ity to generate indeterminacy is substantially smaller. Wen (1998) extends the one-sector model in Benhabib and Farmer (1994) by introducing variable capacity utilization and shows that the model can generate indeterminacy with empirically plausible increasing returns. Benhabib and Wen (2004) study a version of the Wen (1998) model and find that, under parameter configurations that allow for indeterminacy, the RBC model driven by demand shocks performs well in matching the business cycle facts along several important dimensions. Schmitt-Grohe (1997) compares four different models and finds that models with countercyclical markups rely on a lower degree of increasing returns to generate indeterminacy than those with constant markups. Galí (1994) and Wang and Wen (2008) show that variations in the composition of aggregate demand help generate countercyclical markups and indeterminacy. Jaimovich (2007) shows that a model with endogenous entry and exit of firms and thus countercyclical markups can generate indeterminacy even without increasing returns.[4]

Our model complements this literature by providing a microeconomics foundation for aggregate increasing returns through financial frictions. We show that, in a model with heterogenous firms, credit constraints not just help amplify fundamental shocks, they may also lead to aggregate increasing returns and self-fulfilling equilibria.

Indeed, aggregate dynamics in our model with credit constraints are isomorphic to those in a representative-agent economy with increasing returns, such as the one studied by Benhabib and Farmer (1994). However, the mechanism through which multiple equilibria can be obtained is quite different. In the representative-agent model, as shown by Benhabib and Farmer (1994), obtaining indeterminacy requires sufficiently large increasing returns so that the labor demand curve is upward sloping and has a slope steeper than the labor supply curve. If this condition is met, then an upward shift in the labor supply curve (through a wealth effect caused by expectations of higher future output) would lead to an increase in equilibrium hours as the real wage rises. The increase in hours and output would make the initial expectation self-fulfilling. A well-known critique of this particular mechanism is that the large increasing returns required to generate indeterminacy in the representative agent model are not supported by empirical evidence (Basu and Fernald, 1997). Our findings suggest that, in an economy with heterogeneous firms and credit constraints, obtaining multiple equilibria does not require implausibly large increasing returns. Thus, our model provides a microeconomic foundation for the standard model with increasing returns through financial frictions.

The idea that financial frictions might lead to multiple equilibria is not new. It has been explored, for example, by Azariadis and Smith (1998). The idea has been revisited in the recent literature on rational asset-price bubbles. Examples of this

[4]The indeterminacy in this class of business cycle models are dynamic examples of the sunspot equilibria initially studied by Azariadis (1981) and Cass and Shell (1983). See Benhabib and Farmer (1999) for a comprehensive survey of the literature.

literature include Farhi and Tirole (forthcoming), Martin and Ventura (2011), Miao and Wang (2011), and Miao and Wang (2012). The bubble literature shows that, in the presence of financial frictions, asset price bubbles help provide liquidity and improve capital allocations. However, the notion of multiple equilibria associated with asset price bubbles is different from that in our model. While bubbly equilibria represents multiple saddle points, we obtain multiple equilibria around a unique steady state, which is a sink instead of a saddle point.

Our mechanism is also related to Woodford (1986), who recognizes the possibility that borrowing constraints might generate stationary sunspot equilibria. Woodford (1986) considers an economy with two classes of representative agents and with no heterogeneity within each class. Thus, the Woodford model does not generate the reallocation effect of credit constraints which, as we show, is a central ingredient in our model's amplification mechanism and crucial for generating indeterminacy.

Our paper adds to the rapidly growing literature on the role of financial friction for macroeconomic fluctuations. A comprehensive survey of that literature is beyond the scope our paper.[5] The credit amplification mechanism in our model is closely related to the financial accelerator studied by Carlstrom and Fuerst (1997), Bernanke, Gertler and Gilchrist (1999), and Kiyotaki and Moore (1997, 2008), Jermann and Quadrini (2012), among others. We follow Kiyotaki and Moore (1997, 2008) and Jermann and Quadrini (2012) and study financial frictions that arise from limited contract enforcement. We build on this literature and introduce firm-level heterogeneity to simplify solution methods for this class of models. Since it is difficult to solve a model with occasionally binding credit constraints, existing literature typically follows Kiyotaki and Moore (1997) and assumes that borrowers are less patient than lenders so that credit constraints are binding in the steady state equilibrium (and also around the steady state). In our model, only high-productivity firms are active and face credit constraints. At the aggregate level, credit frictions are summarized by the cut-off level productivity that depends only on aggregate economic conditions. Thus, when we solve for aggregate dynamics, we do not need to deal with occasionally binding constraints.

I. The model

The model economy is populated by two types of infinitely lived agents— households and entrepreneurs—with a continuum of each. The representative household consumes and invests a homogenous good and supplies labor and capital to the entrepreneur. The entrepreneur family has a large number of managers, each managing a firm with a constant returns technology that transforms labor and capital into consumption goods. To incorporate financial friction, we assume that firms face idiosyncratic productivity shocks. If it is profitable to produce,

[5]For surveys of the literature on financial friction in DSGE models, see Bernanke, Gertler and Gilchrist (1999) and Gertler and Kiyotaki (2010).

active firms need to hire workers and rent capital from the household, but cannot make upfront payments to these input factors until production is completed. Since contract enforcement is costly, firms may default on factor payments. This enforcement problem gives rise to credit constraints.

A. *The representative household and the representative entrepreneur*

The representative household has the utility function

$$\mathrm{E}\sum_{t=0}^{\infty}\beta^t\left\{\ln C_t^h - a_L\frac{N_t^{1+\chi}}{1+\chi}\right\}, \tag{1}$$

where $\beta \in (0,1)$ denotes the household's subjective discount factor, C_t^h denotes the household's consumption, N_t denotes the hours worked, $a_L > 0$ is the utility weight for leisure, $\chi > 0$ is the inverse Frish elasticity of labor supply, and E is the expectation operator.

The household chooses consumption C_t^h, labor supply N_t, and new capital stock K_{t+1}^h to maximize the utility function (1) subject to the sequence of budget constraints

$$C_t^h + K_{t+1}^h \leq w_t N_t + (1 + r_t - \delta)K_t^h \quad \forall t \geq 0, \tag{2}$$

and the non-negativity constraints $C_t^h \geq 0$ and $K_{t+1}^h \geq 0$, taking as given the labor wage rate w_t and the capital rental rate r_t. The parameter $\delta \in (0,1)$ denotes the capital depreciation rate.

The representative entrepreneur has the utility function

$$\mathrm{E}\sum_{t=0}^{\infty}\tilde{\beta}^t \ln C_t^e, \tag{3}$$

where $\tilde{\beta} \in (0,1)$ denotes the entrepreneur's subjective discount factor and C_t^e denotes the entrepreneur's consumption.

The entrepreneur chooses consumption C_t^e and new capital stock K_{t+1}^e to maximize (3) subject to the sequence of budget constraints

$$C_t^e + K_{t+1}^e \leq (1 + r_t - \delta)K_t^e + D_t, \quad \forall t \geq 0, \tag{4}$$

along with the non-negative constraints $C_t^e \geq 0$ and $K_{t+1}^e \geq 0$, taking as given the rental rate r_t. Entrepreneurs are shareholders of firms. The term D_t in the budget constraint denotes the dividend payments that the representative entrepreneur receives from firms.

We assume that the entrepreneur is sufficiently less patient than the household

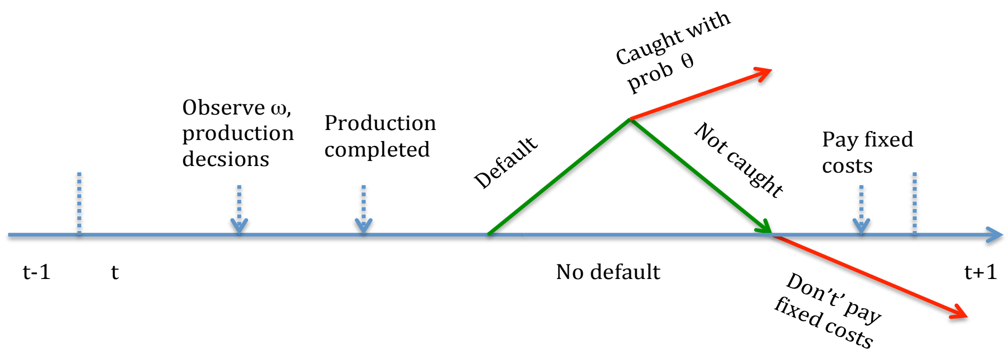

Figure 1. Timing of events and decisions

(i.e., $\tilde{\beta}$ is sufficiently smaller than β) so that, in equilibrium, the household is the only saver and the entrepreneur does not hold any capital (i.e., $K^e_{t+1} = 0$ for all $t \geq 0$). Thus, the entrepreneur's consumption equals dividend payments from firms.[6]

B. *The firms*

The entrepreneur owns a large number of firms with the mass normalized to one. Each firm has access to a constant returns technology that transforms capital and labor into goods.

Timing of events. — Timing is essential for incorporating financial frictions in the model economy. Figure 1 illustrates the timing of events.

In the beginning of period t, each firm observes aggregate shocks and draws an idiosyncratic productivity ω from a Pareto distribution defined over the interval $[1, \infty)$, with the distribution function

$$F(\omega) = 1 - \omega^{-\sigma}, \tag{5}$$

where $\sigma > 1$ is the shape parameter, which determines the dispersion of the distribution.. To simplify analysis, we follow Carlstrom and Fuerst (1997) and Bernanke, Gertler and Gilchrist (1999) by assuming that the idiosyncratic productivity is i.i.d across firm and across time. Thus, firms are ex ante identical.

[6]The assumption that the representative entrepreneur is less patient than the representative household helps simplify the analysis. The derivations of firm-level credit constraints below do not rely on this assumption. In Appendix A.A8, we show that this assumption alone is not sufficient to ensure that firms do not have incentive to accumulate capital. There, we also discuss conditions under which firms choose not to accumulate capital. In solving the model, we focus on equilibria in which firms do not accumulate capital. We later verify that these assumptions are satisfied under our parameter calibration.

After drawing productivities, firms decide whether or not to operate. A firm with productivity ω has the production function

$$y_t(\omega) = \omega A_t k_t(\omega)^{\alpha} n_t(\omega)^{1-\alpha}, \tag{6}$$

where $y_t(\omega)$ denotes output, $k_t(\omega)$ and $n_t(\omega)$ denote capital and labor inputs, $\alpha \in (0,1)$ denotes the elasticity of output with respect to capital input, and A_t denotes aggregate productivity, which follows a stationary stochastic process.[7]

An operating firm hires workers and rent capital from the representative household at the competitive real wage w_t and the rental rate r_t. Since firms do not make upfront payments to the household, the household is effectively providing credit to firms. We interpret this credit as working capital loans.

When production is completed, firms decide whether or not to repay the wages and rents they owe to the household. We assume that contract enforcement is imperfect so that firms have an incentive to default. If a firm defaults, it can be caught with probability θ, in which event the firm would be perpetually excluded from future access to credit. If a defaulting firm is not caught (with probability $1-\theta$), however, it appears no different from a non-defaulting firm and continues to retain access to credit in future periods.[8]

At the end of period t, firms decide whether or not to stay in business for future periods. To stay in business, a firm needs to pay a fixed cost of ϕ. Such fixed costs are deadweight losses to the society. The fixed cost is required not just for firms that are active in period t, but also for inactive firms that choose not to operate after drawing their productivity in the beginning of the period. If a firm has not produced and decides to stay in business for the next period, then it finances the payment of the fixed cost by issuing equity to the representative entrepreneur—the owner of all firms. Or equivalently, the fixed cost represents a lump-sum reduction of dividend payments from firms to the entrepreneur. Since firms are ex ante identical, they all decide to stay in business if the fixed cost does not exceed the expected continuation value.

After making the decision for continuation in business, firms pay dividends to the entrepreneur (if a firm has not produced in period t, then it pays a negative dividend that equals the fixed cost for staying in business). The entrepreneur consumes. The economy then enters period $t+1$, with the same sequence of events repeated.

[7]The specific stochastic process for A_t (e.g., whether it includes a trend or how persistent it is) is not crucial for our analysis. Our goal is to show that credit frictions at the microeconomic level can generate increasing returns at the aggregate level and they can thus potentially generate equilibrium indeterminacy. For this purpose, we do not need to take a stand on the stochastic processes of fundamental shocks. We allow A_t to be time varying mainly to illustrate how credit constraints amply fundamental shocks (such as productivity shocks).

[8]The parameter θ can also be interpreted as the fraction of firm value that the lender can recover in the event of a default (e.g., Jermann and Quadrini (2012)).

The credit constraints. — Since contract enforcement is imperfect, firms have an incentive to default on their loans (i.e., wages and rents). Optimal contracts solve the firm's problem subject to an incentive constraint, so that no default occurs in equilibrium. We now show that such incentive constraints give rise to a credit constraint for all firms, under which the amount of loans cannot exceed a credit limit determined by the expected continuation value of the firms.

If a firm with productivity ω does not default at the end of the period, it pays wages and rents to the household and keeps the remaining revenue. Further, by paying the fixed cost, the firm can stay in business in the next period and obtain the continuation value. The non-default value for the firm is thus given by

$$V_t^N(\omega) = y_t(\omega) - w_t n_t(\omega) - r_t k_t(\omega) + \max\left\{\tilde{\beta} E_t \frac{\Lambda_{t+1}^e}{\Lambda_t^e} V_{t+1} - \phi, 0\right\}, \tag{7}$$

where Λ_t^e denotes the marginal utility of the entrepreneur (who owns the firm) and $V_t \equiv \int V_t(\omega) f(\omega) d\omega$ denotes the ex ante value of the firm, with idiosyncratic productivity integrated out. The max operator on the right hand side of equation (7) represents the firm's entry and exit decision. If the expected continuation value exceeds the fixed cost, then the firm decides to pay the fixed cost and stay in business. Otherwise, it exits. Since the idiosyncratic shocks are i.i.d., firms are ex ante identical so that they face identical entry and exit decisions. We impose parameter restrictions such that all firms choose to stay in business. We later verify that such parameter restrictions are plausible.[9]

If the firm chooses to default, then it can keep all the revenue $y_t(\omega)$. If it is caught (with probability θ), then the firm would be excluded from future access to credit and it would lose the continuation value. If it is not caught (with probability $1-\theta$), however, the firm can stay in business and enjoy the continuation value provided that it pays the fixed cost at the end of the period. Thus, the expected value of default is given by

$$V_t^D(\omega) = y_t(\omega) + (1-\theta)\max\left\{\tilde{\beta} E_t \frac{\Lambda_{t+1}^e}{\Lambda_t^e} V_{t+1} - \phi, 0\right\}. \tag{8}$$

The firm chooses not to default if and only if $V_t^D(\omega) \le V_t^N(\omega)$. Therefore, by comparing (7) and (8), we obtain the incentive constraint

$$w_t n_t(\omega) + r_t k_t(\omega) \le \theta \max\left\{\tilde{\beta} E_t \frac{\Lambda_{t+1}^e}{\Lambda_t^e} V_{t+1} - \phi, 0\right\}. \tag{9}$$

Since we focus on the case in which the continuation value net of fixed costs is

[9]We abstract from more interesting entry and exit decisions in this model, not because we view these decisions as unimportant but we would like to gain analytical tractability to illustrate the role of financial frictions in generating indeterminacy.

non-negative, the incentive constraint can be rewritten as

$$w_t n_t(\omega) + r_t k_t(\omega) \le \theta \left(E_t \tilde{\beta} \frac{\Lambda^e_{t+1}}{\Lambda^e_t} V_{t+1} - \phi \right) \equiv b_t, \tag{10}$$

where b_t denotes the credit limit. Thus, imperfect contract enforcement gives rise to a credit constraint, under which the available credit (and thus the factor payments that a firm can afford) is limited by a fraction of the expected continuation value net of fixed costs. Since all firms are ex ante identical, the expected continuation value is also identical, so that the credit limit b_t is independent of firms' productivity.

C. *Allocations of credit and production*

We now examine the allocations of credit and production across firms with different productivity draws.

Optimal contracts maximize the firm's value subject to the incentive constraint (10). Formally, the firm with productivity ω has the value function

$$V_t(\omega) = y_t(\omega) - C(y_t, \omega) - \phi + \tilde{\beta} E_t \frac{\Lambda^e_{t+1}}{\Lambda^e_t} V_{t+1}, \tag{11}$$

where the term $C(y, \omega)$ denotes the variable cost function. In particular,

$$\begin{aligned} C(y, \omega) \quad \equiv \quad & \min_{n,k} \quad w_t n + r_t k, \\ & s.t. \quad \omega A_t k^{\alpha} n^{1-\alpha} \ge y. \end{aligned} \tag{12}$$

Cost-minimizing implies that the variable cost function is given by

$$C(y, \omega) = y \frac{\omega^*_t}{\omega}, \tag{13}$$

where the term ω^*_t is given by

$$\omega^*_t \equiv \frac{1}{A_t} \left(\frac{w_t}{1-\alpha} \right)^{1-\alpha} \left(\frac{r_t}{\alpha} \right)^{\alpha}. \tag{14}$$

Thus, we can rewrite the incentive constraint (10) as

$$y_t(\omega) \frac{\omega^*_t}{\omega} \le b_t. \tag{15}$$

The firm chooses $y_t(\omega)$ to maximize the value $V_t(\omega)$ in (11) subject to (13), (14), the incentive constraint (15), and a non-negativity constraint $y_t(\omega) \ge 0$.

Solving the optimal contract problem gives the allocations of production and credit, as we summarize in the following proposition:

PROPOSITION 1: *There exists a cut-off level of productivity* $\omega_t^* \in [1, \infty)$ *such that the production allocation across firms is given by*

$$y_t(\omega) = \begin{cases} \frac{b_t \omega}{\omega_t^*}, & \text{if} \omega \geq \omega_t^*, \\ 0, & \text{otherwise}, \end{cases} \tag{16}$$

and the credit allocation across firms is given by

$$b_t(\omega) = \begin{cases} b_t, & \text{if} \omega \geq \omega_{\mathrm{t}}^* \\ 0 & \text{otherwise.} \end{cases} \tag{17}$$

The credit limit b_t *and the cut-off productivity* ω_t^* *are defined in* (10) *and* (14), *respectively.*

Proposition 1 shows that firms with sufficiently high levels of productivity choose to operate and less productive firms remain inactive. The marginal firm with productivity ω_t^* is indifferent in operating or not. Without loss of generality, we assume that the marginal firm operates.

Under optimal contracts, high-productivity firms are credit-constrained, with the credit limit given by a fraction θ of the firms' expected equity value net of fixed costs (see (10)). Low-productivity firms stay inactive and they do not borrow.[10] The parameter θ captures the strength or effectiveness of contract enforcement. Stronger contract enforcement (i.e., larger θ) implies a higher credit limit.

D. *Competitive equilibrium and aggregation*

A competitive equilibrium consists of sequences of prices $\{w_t, r_t\}$, allocations for the household $\{C_t^h, K_{t+1}^h, N_t\}$, allocations for the entrepreneur $\{C_t^e, K_{t+1}^e\}$, and allocations for firms $\{k_t(\omega), n_t(\omega), y_t(\omega)\}$ such that, taking the prices as given, the allocations for each type of agents solve their optimizing problems and all markets clear.

Factor market clearing implies that

$$\int k_t(\omega) dF(\omega) = K_t^h + K_t^e = K_t, \quad \int n_t(\omega) dF(\omega) = N_t. \tag{18}$$

[10] As we have discussed in Section I.B, these unproductive firms do not have any revenue to cover the fixed cost and they finance the fixed cost by paying negative dividends to the entrepreneur, who owns all firms. Thus, the fixed cost is required for all firms and is ultimately borne by the entrepreneur in the form of a lump-sum reduction in dividend income.

Goods market clearing implies that

$$C_t^h + C_t^e + K_{t+1} = (1-\delta)K_t + Y_t - \phi, \tag{19}$$

where $Y_t = \int y_t(\omega)dF(\omega)$ denotes aggregate output.[11]

Integrating the production allocation in equation (16) across firms, we obtain aggregate output

$$Y_t = \int_{\omega \geq \omega_t^*} \frac{b_t}{\omega_t^*} \omega dF(\omega) = \frac{\sigma}{\sigma-1} b_t \omega_t^{*-\sigma}. \tag{20}$$

With constant returns technology and perfect mobility of factors, the capital-labor ratio is independent of firms' idiosyncratic productivity. In particular, cost-minimizing implies that

$$\frac{w_t}{r_t} = \frac{1-\alpha}{\alpha}\frac{k_t(\omega)}{n_t(\omega)} = \frac{1-\alpha}{\alpha}\frac{K_t}{N_t}. \tag{21}$$

Cost minimizing also implies that aggregate factor payments are given by

$$w_t N_t + r_t K_t = \omega_t^* A_t K_t^\alpha N_t^{1-\alpha}. \tag{22}$$

As the capital-labor ratio for each firm is identical, integrating the production function (6) across firms leads to

$$A_t K_t^\alpha N_t^{1-\alpha} = \int \frac{y_t(\omega)}{\omega} dF(\omega) = \int_{\omega \geq \omega_t^*} \frac{b_t}{\omega_t^*} dF(\omega) = b_t \omega_t^{*-\sigma-1}, \tag{23}$$

where the second equality follows from the production allocation (16) and the final equality from the Pareto distribution function.

Combining (20) and (23), we obtain the aggregate production function

$$Y_t = \frac{\sigma}{\sigma-1}\omega_t^* A_t K_t^\alpha N_t^{1-\alpha}. \tag{24}$$

II. Credit constraints and aggregate productivity

Since productive firms face binding credit constraints and cannot operate at full capacity, the presence of credit constraints leads to misallocation and depressed total factor productivity (TFP). We now draw a formal connection between credit constraints and aggregate productivity in our model. We show how credit con-

[11] Since all firms—not just active firms—need to pay the fixed cost for staying in business (in equilibrium, they all choose to pay), aggregate output available for consumption and investment is reduced by the amount of the fixed cost ϕ.

straints amplify macroeconomic fluctuations through reallocation of resources.

A. Credit constraints and misallocation

The aggregate production function (24) reveals that measured TFP is given by

$$TFP_t \equiv \frac{Y_t}{K_t^{\alpha} N_t^{1-\alpha}} = \frac{\sigma}{\sigma - 1}\omega_t^* A_t. \tag{25}$$

Thus, measured TFP reflects the joint effects of true technology changes (A_t) and endogenous variations in the cutoff level of productivity (ω_t^*) that determines which firms are active.

Absent credit constraints, the only firms that operate would be those with the highest productivity $\omega^{\max}$.[12] In the presence of credit constraints, however, less productive firms (i.e., those with productivity between ω_t^* and $\omega^{\max}$) become active. In this sense, credit constraints create misallocation of resources, the magnitude of which is captured by the cutoff level of productivity ω_t^*.

The following proposition establishes the relation between the endogenous component of TFP (the misallocation effect) and the average tightness of credit constraints.

PROPOSITION II.1: *There is a one-to-one and monotonic mapping between ω_t^* and aggregate loan-to-output ratio. In particular,*

$$\omega_t^* = \left[\frac{b_t}{Y_t}\frac{\sigma}{\sigma - 1}\right]^{1/\sigma}. \tag{26}$$

The loan-to-output ratio in turn depends on the tightness of the credit constraints:

$$\frac{b_t}{Y_t} = \theta\left[\frac{\tilde{\beta}}{1-\tilde{\beta}}(\frac{1}{\sigma} - \bar{\phi}_t) - \bar{\phi}_t\right], \tag{27}$$

where $\bar{\phi}_t \equiv \phi/Y_t$.

PROOF:

The equality in (26) follows immediately from (20). To obtain the relation in (27), we first note that (10) implies that the credit limit b_t is determined by expected future equity value. The equity value V_t can be solved out from the recursive relation $V_t = C_t^e + \tilde{\beta}\mathrm{E}_t\frac{C_t^e}{C_{t+1}^e}V_{t+1}$, which yields

$$V_t = \frac{1}{1-\tilde{\beta}}C_t^e. \tag{28}$$

[12]Strictly speaking, the notion of the "most productive firms" is a limiting concept since the support of the Pareto distribution is not bounded from above (i.e., with $\omega^{\max}$ approaching infinity).

In equilibrium, C_t^e is the aggregate dividend payments from all firms, which is given by

$$C_t^e = Y_t - w_t N_t - r_t K_t - \phi = \frac{1}{\sigma} Y_t - \phi, \tag{29}$$

where the last equality follows from (22) and (24). We then have the credit limit

$$b_t = \theta \left[\tilde{\beta} E_t \frac{C_t^e}{C_{t+1}^e} V_{t+1} - \phi \right] = \theta \left[\frac{\tilde{\beta}}{1-\tilde{\beta}} (\frac{1}{\sigma} Y_t - \phi) - \phi \right], \tag{30}$$

where the second equality uses (28) to substitute out V_{t+1} and (29) to substitute out C_t^e. Dividing (30) through by Y_t, we obtain the desired equality in (27).

Proposition II.1 implies that more credit would be available for productive firms if contract enforcement is stronger (i.e., θ is larger). With more available credit, resources are more concentrated in high-productivity firms, so the economy should have a higher level of TFP (see (26)). Empirical studies suggest that misallocation accounts for a large fraction of cross-country differences in TFP (Hsieh and Klenow, 2009). Our theory suggests that financial friction can be a source of misallocation that depresses TFP.

Since aggregate output increases with the level of TFP, which in turn increases with θ, we have

COROLLARY II.2: *Steady-state output is an increasing function of θ.*

B. *Amplification through reallocation*

Credit constraints not only lead to misallocation that depresses steady-state TFP, they also amplify technology shocks through reallocation of resources between firms with different levels of productivity. The following proposition establishes the conditions under which a financial multiplier arises.

PROPOSITION II.3: *Holding input factors constant, a 1 percent change in technology shock leads to $\mu > 1$ percent change in aggregate output, where μ is given by*

$$\mu \equiv \frac{d \log Y_t}{d \log A_t} = \frac{\sigma}{\sigma + 1 - \xi}, \quad \xi \equiv \left[1 - \frac{\bar{\phi}\sigma}{\tilde{\beta}} \right]^{-1} > 1, \tag{31}$$

where $\bar{\phi} = \phi/Y$ measures the steady-state cost of financial intermediation as a fraction of GDP.

PROOF:

Substituting for ω_t^* in (24) by the relations in (26) and (27) and rearranging terms, we obtain

$$Y_t^{1+\sigma} = \left(\frac{\sigma}{\sigma-1}\right)^{1+\sigma} \frac{\tilde{\beta}\theta}{1-\tilde{\beta}} \left[\frac{1}{\sigma}Y_t - \frac{\phi}{\tilde{\beta}}\right] \left[A_t K_t^{\alpha} N_t^{1-\alpha}\right]^{\sigma}. \tag{32}$$

Taking logarithms on both sides and applying total differentiation, we obtain (31).

Proposition II.3 shows that credit constraints help amplify technology shocks. Weaker contract enforcement (i.e., a smaller value of θ) implies stronger amplification (i.e., a greater value of μ). This is because a lower θ implies a lower steady-state output (see Corollary II.2) and thus a higher average fixed cost for firms to stay in business (i.e., a higher $\bar{\phi}$). From equation (31), it follows that a lower θ implies a greater μ.

To understand how the credit amplification mechanism works, consider a positive technology shock. The shock raises output and therefore firms' equity value and the credit limit (see (30)). In the presence of the fixed cost, the credit limit rises more than proportionately than does aggregate output because the boom in output reduces the average fixed cost for firms to stay in business (see (27)). In other words, the loan-to-output ratio is procyclical. The increased credit limit enables high-productivity firms to expand production and forces low-productivity firms to become inactive. As a consequence, the cutoff level of productivity shifts up as the loan-to-output ratio rises (see equation (26)), which raises measured TFP (equation (25)) and thereby reinforcing the initial technology shock. Following a negative technology shock, the same logic applies with the direction reversed. Credit constraints thus generate a reallocation effect that leads to procyclical aggregate productivity and amplification of macroeconomic fluctuations.

III. Aggregate Increasing Returns and Indeterminacy

The reallocation effect represents a source of inefficiency that stems from credit constraints. In an unconstrained economy, resources are concentrated in the most productive firms and changes in equity value do not have direct impact on production allocation. In the presence of credit constraints, however, changes in equity value directly affect firms' borrowing capacity and lead to procyclical reallocation.

We have established that the reallocation effect of credit constraints is crucial for amplifying fundamental shocks (such as technology shocks). We now show that credit constraints at the firm level are equivalent to increasing returns at the aggregate level, despite that all firms operate a constant returns technology with respect to variable factors. Further, the economy with sufficiently tight credit constraints is prone to indeterminate equilibria and sunspot driven fluctuations.

A. Credit constraints and aggregate increasing returns

To see that aggregate technology in the model exhibits increasing returns, we log-linearize the reduced-form aggregate production function (32) around the (unique) deterministic steady state to get

$$\hat{y}_t = \mu[\hat{a}_t + \alpha\hat{k}_t + (1-\alpha)\hat{n}_t], \tag{33}$$

where a hatted variable denotes the log-deviations of the corresponding variable from its steady-state value (e.g., $\hat{y}_t \equiv \log(Y_t/Y)$, where Y is the steady-state level of output). The term μ is the financial multiplier given by (31). Equation (33) shows that μ also measures aggregate returns to scale. This result is formally stated in the following proposition.

PROPOSITION III.1: *The reduced-form aggregate technology in our model exhibits increasing returns if and only if there is a positive financial multiplier (i.e., $\mu > 1$).*

B. The possibility of self-fulfilling equilibria

It is well known that an economy with increasing returns can be prone to self-fulfilling, sunspot driven business cycles (Benhabib and Farmer, 1994). Since credit constraints in our model are observationally equivalent to aggregate increasing returns, we now examine the conditions under which sunspot-driven fluctuations can occur.

To help exposition, we follow the literature by focusing on local dynamics around the deterministic steady state and we abstract from aggregate shocks. We first establish analytical conditions for indeterminacy to arise. We then evaluate the empirical plausibility for credit frictions to generate indeterminacy under calibrated parameter values based on numerical simulations.

The following proposition summarizes the conditions for equilibrium indeterminacy.

PROPOSITION III.2: *Assume that the output elasticity of capital in the aggregate production function is less than one (i.e.,$\mu\alpha < 1$). Assume also that $2 - \beta < \frac{1+\chi}{1-\alpha}$. The necessary and sufficient condition for equilibrium indeterminacy in the benchmark economy is given by*

$$\mu > \max\{\mu_1^*, \mu_2^*\} \equiv \mu^*, \tag{34}$$

where

$$\mu_1^* = \frac{\beta(2-\delta)(1+\chi)}{(1+\beta)(1-\alpha)\beta(1-\delta) - [1-\beta(1-\delta)](1+\chi)}, \tag{35}$$

and

$$\mu_2^* = \frac{\left\{\beta(2-\delta) + [1-\beta(1-\delta)]\,\frac{1-\beta+\beta\delta(1-\alpha)}{2\alpha}\right\}(1+\chi)}{(1+\beta)(1-\alpha)\beta(1-\delta) - [1-\beta(1-\delta)]\,(1+\chi)\left[1 - \frac{1-\beta+\beta\delta(1-\alpha)}{2}\right]}. \tag{36}$$

PROOF:

See Appendix A.A2.

Proposition III.2 implies that, all else equal, equilibrium indeterminacy arises if the size of the financial multiplier (μ) is sufficiently large.

To understand how indeterminacy arises in our model, consider a hypothetical increase in expected future equity value without any fundamental shocks. If such expectations can be validated in an equilibrium, then indeterminacy and self-fulfilling equilibria would arise. If such expectations are not validated, however, there is a unique equilibrium.

[INSERT FIGURE 2 HERE]

We illustrate here with a labor market diagram the mechanism through which credit constraints can generate indeterminacy. Figure 2 (the top panel) shows the adjustments in the labor market in our benchmark model with credit constraints following a rise in expected future equity value. We assume that the labor supply curve is flat, or equivalently, labor is indivisible (Hansen, 1985; Rogerson, 1988). Suppose that the initial equilibrium is at point A. The expectation of higher future equity value creates a positive wealth effect that shifts the labor supply curve upward. In a model without credit frictions, such as the standard real business cycle (RBC) model, labor demand does not shift because the capital stock is predetermined and there is no fundamental technology shock. Thus, the upward shift in the labor supply curve leads to a rise in equilibrium real wage and a fall in equilibrium employment (from point A to point B). The decline in employment leads to a recession, which invalidates the initial expectations of a higher equity value. Thus, the standard RBC model without credit constraints implies a unique equilibrium.

In the presence of credit constraints as in our benchmark model, however, the expectation of a rise in equity value enables high-productivity firms to borrow more and produce more and thus leads to a reallocation that implies higher aggregate productivity. With higher aggregate productivity, the aggregate labor demand curve shifts upward. The tighter the credit constraints, the greater the financial multiplier, and the larger the increase in aggregate productivity following an expectation shock. Thus, with a sufficiently large financial multiplier (μ), the upward shift in the labor demand curve can more than offset the recessionary effects of the upward shift in the labor supply curve; it can potentially lead to an expansion in equilibrium employment and output (from point B to point C). Such an expansion confirms the initial expectations and thus leads to self-fulfilling equilibria.

C. Relation to Benhabib and Farmer (1994)

In the continuous-time limit with $\delta \to 0$ and $\beta \to 1$, we have $\mu_1^* = \mu_2^* = \frac{1+\chi}{1-\alpha}$. In this case, the indeterminacy condition in equation (34) reduces to

$$\mu(1-\alpha) - 1 > \chi, \tag{37}$$

which is apparently identical to that obtained by Benhabib and Farmer (1994) in the real business cycle model with increasing returns, where μ corresponds to the degree of returns to scale in their aggregate production function.[13] The mechanism through which indeterminacy arises, however, is quite different. In their model, indeterminacy requires a large enough increasing returns so that the labor demand curve is upward-sloping, with a slope steeper than the labor supply curve (see the bottom panel of Figure 2). Under such conditions, the upward shift in the labor supply curve following a change in expectations of future output raises the real wage and moves along the (upward-sloping) labor demand curve, resulting in an increase in equilibrium hours and an expansion in output. This expansionary effects confirm the initial expectations, rendering the expectations self-fulfilling.

In contrast, our model does not require labor demand curve to be upward-sloping. Instead, the presence of credit constraints introduces a financial factor in total factor productivity, so that aggregate productivity increases when equity value is expected to rise; the increase in aggregate productivity shifts up the downward-sloping labor demand curve, which can potentially lead to an expansion in equilibrium hours and output.

IV. Empirical plausibility of self-fulfilling equilibria

We have established that, to a first-order approximation, the model with financial friction is observationally equivalent to a representative-agent economy with aggregate increasing returns (Proposition III.1). We have also shown that it is possible, at least in theory, to generate self-fulfilling equilibria in our model, provided that the financial multiplier is sufficiently large (Proposition III.2). We now examine the empirical plausibility of self-fulfilling business cycles under calibrated parameters in an extended version of the model with variable capacity utilization. In an important contribution, Wen (1998) shows that incorporating variable capacity utilization in the Benhabib and Farmer (1994) model helps to

[13] In a working paper version (Liu and Wang, 2010), we provide a formal proof that the indeterminacy conditions in our model are identical to those in the real business cycle model with increasing returns studied by Benhabib and Farmer (1994).

lower substantially the required degree of increasing returns to generate multiple equilibria. We now show that our model with credit constraints generalized to incorporate variable capacity utilization can generate multiple equilibria under plausible parameter calibration.

A. *The model with variable capacity utilization*

Following Wen (1998), we assume that increases in the capacity utilization rate accelerate capital depreciation. In particular, we assume that the household's budget constraint (2) is replaced by

$$C_t^h + K_{t+1} \leq w_t N_t + \left[1 + r_t u_t - \delta(u_t)\right] K_t, \quad \forall t \geq 0, \tag{38}$$

where u_t denotes the capacity utilization. As in the benchmark model, all capital accumulation is done by the household. The capital depreciation rate varies with capacity utilization according to

$$\delta(u_t) = \delta_0 \frac{u_t^{1+\eta}}{1+\eta}, \tag{39}$$

where $\delta_0 \in (0,1)$ is a constant and $\eta > 0$ measures the elasticity of the depreciation rate with respect to capacity utilization. The household's optimizing choices now include an additional endogenous variable—the capacity utilization rate u_t.

With variable capacity, the aggregate production function (32) becomes

$$Y_t = \frac{\sigma}{\sigma - 1}\left[\frac{\tilde{\beta}\theta}{1-\tilde{\beta}}\left(\frac{1}{\sigma}Y_t - \tilde{\phi}\right)\right]^{\frac{1}{\sigma+1}} \left(A_t (u_t K_t)^{\alpha} N_t^{1-\alpha}\right)^{\frac{\sigma}{\sigma+1}}. \tag{40}$$

where $\tilde{\phi} = \frac{\phi}{\tilde{\beta}}$. Here, unlike in the benchmark model, the quantity produced depends on the effective capital services $u_t K_t$ rather than the physical units of capital K_t.

We first note that introducing variable capacity utilization raises the magnitude of the financial multiplier. In particular, we have

PROPOSITION IV.1: *In the extended model with variable capacity utilization, a* 1 *percent change in TFP holding input factors constant results in* $\tilde{\mu} > 1$ *percent change in aggregate output, where*

$$\tilde{\mu} \equiv \frac{d\log Y_t}{d\log A_t} = \frac{\mu(1+\eta)}{1+\eta-\alpha\mu} > \mu, \tag{41}$$

where μ *is the financial multiplier in the benchmark model given by equation* (31).

PROOF:

See Appendix A.A4.

Accordingly, introducing variable capacity utilization makes indeterminacy more likely, as we show in the Proposition below.

PROPOSITION IV.2: *Assume that* $\alpha \frac{\eta}{1+\eta}\tilde{\mu} < 1$. *The necessary and sufficient condition for equilibrium indeterminacy in the model with variable capacity utilization is given by*

$$\tilde{\mu} > \max(\tilde{\mu}_1^*, \tilde{\mu}_2^*) \equiv \tilde{\mu}^*, \tag{42}$$

where

$$\tilde{\mu}_1^* = \frac{2(1+\chi)}{(1+\beta)(1-\alpha) - \frac{1+\eta-\alpha}{(1+\eta)}(1+\chi)(\beta^{-1}-1)}, \tag{43}$$

$$\tilde{\mu}_2^* = \frac{2(1+\chi) + \frac{1}{2}(1-\beta)\frac{1+\eta-\alpha}{\alpha}(1+\chi)\delta}{(1+\beta)(1-\alpha) - \frac{\eta(1+\eta-\alpha)}{(1+\eta)}(1+\chi)\delta[1-\frac{1}{2}(1-\beta)]}. \tag{44}$$

PROOF:

The proof is similar to that of Proposition III.2, with the financial multiplier μ replaced by $\tilde{\mu}$.

In the continuous-time limit with $\delta \to 0$ and $\beta \to 1$, we again obtain $\tilde{\mu}_1^* = \tilde{\mu}_2^* = \frac{1+\chi}{1-\alpha}$. In this case, the necessary and sufficient conditions for indeterminacy are simplified to $\tilde{\mu} > \frac{1+\chi}{1-\alpha}$.

B. *Parameter calibration*

The parameters to be calibrated include the subjective discount factors β for the household and $\tilde{\beta}$ for the entrepreneur, the capital depreciation rate δ, the capital income share α, the inverse Frisch elasticity of labor supply χ, the elasticity η of depreciation with respect to capacity utilization rate, the scale parameter of the Pareto distribution for the idiosyncratic productivity shocks σ, the parameter measuring the strength of contract enforcement θ, and the steady-state ratio of fixed costs to aggregate output $\bar{\phi}$. Table 1 summarizes the calibrated parameter values.

We set $\beta = 0.99$, implying an annual risk-free interest rate of 4 percent. We set $\delta = 0.025$, corresponding to an annual capital depreciation rate of 10 percent. We set $\alpha = 0.3$ to match the labor income share of 70 percent in the U.S. data. We assume that labor is indivisible (Hansen, 1985; Rogerson, 1988), which implies that $\chi = 0$. We follow Wen (1998) and set $\eta = 0.4$.

The remaining parameters are those related to financial friction. We set the subjective discount factor for the entrepreneur to $\tilde{\beta} = 0.98$, implying an average

Table 1—Calibrated Parameters

Parameter	Description	Value
β	Household discount factor	0.99
$\tilde{\beta}$	Entrepreneur discount factor	0.98
α	Capital income share	0.30
δ	Capital depreciation rate	0.025
η	Utilization elasticity of depreciation	0.40
χ	Inverse Frisch elasticity of labor supply	0
σ	Shape parameter of productivity distribution	6.00
θ	Contract enforcement	0.41
$\bar{\phi}$	Steady-state share of fixed cost in aggregate output	0.10

excess return of about 4 percent per year, in line with the estimates obtained by Liu, Wang and Zha (forthcoming).

Since active firms have higher productivity than the cut-off level, these firms earn economic profits, with the steady-state profit rate given by $\frac{\sigma}{\sigma-1}$. We set $\sigma = 6$, which implies a steady-state profit rate of about 17 percent for active firms.[14] We set $\bar{\phi} = 0.1$, which, together with the calibration of $\sigma = 6$, implies an average economic profit (net of fixed costs) of about 7 percent, in line with the empirical evidence provided by Basu and Fernald (1997).

Given the calibrated values of $\tilde{\beta}$, σ, and $\bar{\phi}$, we set $\theta = 0.41$ so that the model implies a steady-state ratio of private credit to quarterly GDP of 2.08, as in the U.S. data.[15]

These calibrated parameters imply a financial multiplier of $\mu = 1.11$ in the benchmark model and $\tilde{\mu} = 1.45$ in the extended model with variable capacity utilization. The size of the financial multiplier μ implies that, under the calibrated labor share and Frisch elasticity of labor supply, the condition (34) for multiple equilibria is *not* satisfied and thus the benchmark model has a unique local equi-

[14]The economic profit in our model is different from the standard monopolistic markup because firms in our model are competitive.

[15]We measure nominal private credit by the non-farm, nonfinancial business liabilities based on credit market instruments. The data are taken from the Flow of Funds Tables of the Federal Reserve through Haver Analytics (private credit is the sum of OL10TCR5@FFUNDS and OL11TCR5@FFUNDS). The sample mean of the ratios of nominal private credit to annualized nominal GDP is about 0.52 for the period from 1960:Q1 to 2011:Q1, implying a quarterly average of 2.08. This is a broad measure of non-financial business debt, which includes both long-term and short-term debts. It thus overstates the importance of working capital loans (which are short-term) in the economy. With a narrower measure of credit that corresponds more closely to working capital loans in our model, the implied value of θ would be smaller. Since a smaller θ implies a greater financial multiplier and thus a larger degree of increasing returns, our calibration with a high value of θ biases the numerical results against finding indeterminacy (see Section IV.C).

This calibration also verifies that all firms choose to stay in business at the end of the period because the expected continuation value exceeds the size of the fixed cost. To see this, note that equation (30) implies that the expected continuation value exceeds the fixed cost if and only if $b_t > 0$. With our calibration, $\frac{b}{Y} = 2.08 > 0$ in the steady state, so that all firms choose to stay in business.

librium. However, the size of $\tilde{\mu}$ in the extended model with capacity utilization does meet the condition (A45), where the threshold value is $\tilde{\mu}^* = 1.44$. Thus, under our calibration, the model does generate self-fulfilling equilibria.

C. Sensitivity of the indeterminacy results

To examine the sensitivity of the indeterminacy results, we consider the combinations of the parameters θ and ϕ that lead to multiplicity of equilibria in the model with variable capacity utilization, while fixing all other parameters at their calibrated values. We focus on admissible values of θ and ϕ so that the model has an interior solution. The parameter restrictions are derived in Appendix A.A5.

[INSERT FIGURE 3 HERE]

Figure 3 plots the admissible region and indeterminacy region in the space of θ and ϕ. The figure shows that there is a sizable set of combinations of financial friction parameters that leads to multiple equilibria. For any given value of θ, a large enough value of ϕ within the admissible region leads to multiplicity; for any given value of ϕ, a small enough value of θ within the admissible region implies indeterminacy.

D. The importance of procyclical leverage

We have shown that credit constraints not only amplify fundamental shocks, but can also generate self-fulfilling sunspot-driven fluctuations. The main driving mechanism works through procyclical leverage, which arises from a fixed cost in our benchmark model. We now show that, while having fixed costs in the model is one way of getting procyclical leverage, it is not the only way. We do this by presenting an example economy without fixed costs, which nonetheless implies multiple local equilibria that arise from credit constraints.

The model is a variation of our benchmark model presented in Section I with endogenous leverage. In particular, we assume that firms are not required to pay any fixed costs to stay in business. Instead, we assume that, in the event that a firm defaults on repayment of a loan, the lender can increase the probability of recouping the loan through a litigation process. Thus, the strength of contract enforcement is no longer represented by a constant parameter θ, but it becomes an endogenous variable.

Denote by $\varphi(\theta_t)$ such litigation costs. We assume that the function $\varphi(\theta)$ has a support $\theta \in [0, 1]$, with $\varphi'(\cdot) > 0$ and $\varphi''(\cdot) > 0$. These assumptions imply increasing costs for raising the marginal effectiveness of contract enforcement.[16]

[16]These litigation costs are incurred only in the event of a default. In equilibrium, with incentive compatible loan contracts, firms choose not to default and thus the litigation issue is moot.

By committing to paying a cost of $\varphi(\theta)$, the household is able to recoup the value of the firm with probability θ.[17]

Litigation reduces the lender's payoff. The lender chooses θ_t to solve

$$\max \quad \tilde{\beta}\theta_{\mathrm{t}}\mathrm{E}_{\mathrm{t}}\frac{\Lambda^{\mathrm{e}}_{\mathrm{t}+1}}{\Lambda^{\mathrm{e}}_{\mathrm{t}}}\mathrm{V}_{\mathrm{t}+1} - \varphi(\theta_{\mathrm{t}}). \tag{45}$$

The optimal θ_t satisfies the equation

$$\tilde{\beta}E_t\frac{\Lambda^{e}_{t+1}}{\Lambda^{e}_{t}}V_{t+1} - \varphi'(\theta_t) = 0. \tag{46}$$

Since $\varphi(\cdot)$ is convex, the optimal strength of enforcement (measured by θ) increases with firms' expected equity value.

To obtain further insight about the mechanism, we assume that $\varphi(\theta_t)$ takes the functional form

$$\varphi(\theta_t) = \theta_0\frac{\theta_t^{1+\gamma}}{1+\gamma}, \tag{47}$$

where $\theta_0 > 0$ is a constant scale parameter and $\gamma > 0$ is a curvature parameter.

With this litigation cost function, the optimal θ is given by

$$\theta_t = \left[\frac{\tilde{\beta}}{\theta_0}E_t\frac{\Lambda^{e}_{t+1}}{\Lambda^{e}_{t}}V_{t+1}\right]^{\frac{1}{\gamma}}. \tag{48}$$

The lender provides credit to firms only to the extent that the loans can be recouped upon litigation. Thus, the credit limit is given by

$$b_t = \tilde{\beta}\theta_t E_t\frac{\Lambda^{e}_{t+1}}{\Lambda^{e}_{t}}V_{t+1} - \varphi(\theta_t) = \frac{\gamma}{1+\gamma}\theta_t\tilde{\beta}E_t\frac{\Lambda^{e}_{t+1}}{\Lambda^{e}_{t}}V_{t+1}. \tag{49}$$

From equation (49), one can interpret θ_t as the endogenous loan-to-value ratio or endogenous leverage, which is procyclical as shown in equation (48). The reason why leverage is procyclical in this model is that, in a business cycle boom, firms' value rises, so that the benefit of strengthening contract enforcement also rises. The lender responds by increasing enforcement efforts, which raise the probability of recouping loans from defaulting firms. With a stronger contract enforcement, the credit limit for firms expands.

In Appendix A.A7, we derive the financial multiplier in this model (with vari-

[17] We assume that the household can sell captured defaulting firms to the entrepreneur. Thus, the firm value is discounted with the entrepreneur's marginal utility.

able capacity utilization), which is given by

$$\tilde{\mu}^e = \frac{(1+\eta)\mu^e}{1+\eta-\alpha\mu^e}, \quad \mu^e \equiv \frac{\sigma}{\sigma - 1/\gamma}, \tag{50}$$

where $\tilde{\mu}^e > 1$ denotes the financial multiplier or equivalently, the aggregate degree of returns to scale, in the model with endogenous enforcement and variable capacity utilization. The term $\mu^e > 1$ is the counterpart in the model without variable capacity.

Comparing equations (41) and (50) reveals a direct mapping between our benchmark model with fixed costs and this alternative model without fixed costs. The two models are equivalent if $\xi = 1 + \frac{1}{\gamma}$, where ξ is defined in equation (31).

Given other calibrated parameters, the alternative model with endogenous enforcement efforts implies that indeterminacy can arise if γ is small. For example, if $\gamma = 1$, then we have $\mu^e = 1.2$ and $\tilde{\mu}^e = 1.62$. From Proposition IV.2, the threshold value of the financial multiplier for generating indeterminacy is 1.44 under our calibration. Thus, the model with endogenous enforcement and variable capacity utilization is able to generate indeterminacy. If $\gamma = 0.5$, then we have $\mu^e = 1.5$ and $\tilde{\mu}^e = 2.21$, implying that both models, with or without variable capacity utilization, is able to generate indeterminacy.[18]

V. Conclusion

We have studied the possibility of self-fulfilling, sunspot-driven fluctuations in an economy with credit constraints. We find that financial friction in this economy leads to misallocation of resources since productive firms face binding credit constraints. Interactions between firms' equity value and credit limits under credit constraints generate a financial multiplier that amplifies the effects of fundamental shocks on macroeconomic fluctuations. At the aggregate level, the financial multiplier manifests in the form of increasing returns and can lead to multiple local equilibria under plausible parameter values. Our finding suggests that an economy with poor contract enforcement suffers not just from lower levels of TFP and larger volatility following fundamental shocks, it is also prone to sunspot-driven business cycle fluctuations.

To keep our analysis tractable, we have focused on a stylized model that abstracts from some realistic features in the actual economy. For example, the financial friction is represented by within-period contracts in our model. Generalizing our model to incorporate intertemporal debt contracts such as those studied by Bernanke, Gertler and Gilchrist (1999) and Kiyotaki and Moore (1997) is clearly desirable. This class of models with intertemporal debts is empirically relevant for studying the role of leverage in propagating macroeconomic fluctuations, as shown by Christiano, Motto and Rostagno (2010) and Liu, Wang and Zha (forth-

[18] These values of γ are arbitrary and presented here only for illustrative purposes.

coming). The key mechanism of our model, however, is likely to carry over to a model with more general financial contracts. For example, in our model, the key to generating aggregate increasing returns and possible indeterminacy is the model's ability to generate procyclical leverage. The recent study by Jermann and Quadrini (2012), for example, shows that a model with intertemporal debt contracts also implies procylical leverage.

For our purpose, we have focused on understanding the positive implications of credit constraints for business cycle fluctuations. An important direction for future research is to study optimal fiscal and monetary policy interventions in the presence of credit constraints and nominal rigidities. Our model implies that credit constraints lead to inefficiency in resource allocations because productive firms face binding borrowing constraints. This inefficiency calls for policy interventions. For example, in a recession, adopting an expansionary fiscal or monetary policy helps alleviate credit constraints facing productive firms and thus mitigate the declines in leverage and aggregate productivity. Appropriate policy interventions may also help stave off sunspot driven fluctuations. Future research along these lines should be both promising and fruitful. Our work represents a small step toward this direction.

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Appendix: Derivations and proofs

In this section, we derive the log-linearized equilibrium system around the deterministic steady state in the benchmark model and provide proofs for the propositions in the text.

A1. Equilibrium dynamics in the benchmark model

Define $X_t \equiv \frac{\sigma-1}{\sigma} Y_t$. With no fundamental shocks, the perfect foresight equilibrium in the benchmark model is summarized by the following system of equations.

$$\text{(A1)} \qquad X_t = \left[\frac{\tilde{\beta}\theta}{1-\tilde{\beta}} \left(\frac{1}{\sigma-1} X_t - \frac{\phi}{\tilde{\beta}} \right) \right]^{\frac{1}{\sigma+1}} \left(A K_t^{\alpha} N_t^{1-\alpha} \right)^{\frac{\sigma}{\sigma+1}},$$

$$\text{(A2)} \qquad \frac{1}{C_t^h} = \beta \frac{1}{C_{t+1}^h} \left(\alpha \frac{X_{t+1}}{K_{t+1}} + 1 - \delta \right),$$

$$\text{(A3)} \qquad a_L N_t^{\chi} C_t^h = (1-\alpha) \frac{X_t}{N_t},$$

$$\text{(A4)} \qquad K_{t+1} = (1-\delta) K_t + X_t - C_t^h,$$

where (A1) is the aggregate production function (with no technology shocks, so that A is a constant), (A2) is the intertemporal Euler equation, (A3) is the household's labor-leisure decision, and (A4) is the aggregate resource constraint.

We log-linearized these equilibrium conditions around the deterministic steady state to characterize indeterminacy. Denote by $\rho = \beta^{-1} - 1$. The log-linearized system of equations are summarized by

$$\text{(A5)} \qquad \hat{X}_t = \mu \left[\alpha \hat{K}_t + (1-\alpha) \hat{N}_t \right],$$

$$\text{(A6)} \qquad \hat{C}_t^h = \hat{C}_{t+1}^h - \beta(\rho+\delta)(\hat{X}_{t+1} - \hat{K}_{t+1}),$$

$$\text{(A7)} \qquad \hat{X}_t = (1+\chi) \hat{N}_t + \hat{C}_t^h,$$

$$\text{(A8)} \qquad \hat{K}_{t+1} = (1-\delta) \hat{K}_t + \left(\frac{\rho+\delta}{\alpha} \right) \hat{X}_t - \left(\frac{\rho+(1-\alpha)\delta}{\alpha} \right) \hat{C}_t^h.$$

In these equations, the hatted variables denote log-deviations of the level variables from their steady-state values.

A2. Proof of Proposition III.2

In Proposition III.2, we establish the condition for indeterminacy in the benchmark model. We now provide a formal proof.

PROOF:

We reduce the system of log-linearized equilibrium conditions (A5) -(A8) into two equations in $\hat{C}_t^h$ and $\hat{K}_t$. The reduced system of equations are summarized in

$$M_1 \begin{bmatrix} \hat{C}_t^h \\ \hat{K}_t \end{bmatrix} = M_2 \begin{bmatrix} \hat{C}_{t+1}^h \\ \hat{K}_{t+1} \end{bmatrix}, \tag{A9}$$

where

$$M_1 = \begin{bmatrix} 1 & 0 \\ \frac{\rho+\delta}{\alpha}\Omega_2 - \frac{\rho+(1-\alpha)\delta}{\alpha} & 1-\delta+\frac{\rho+\delta}{\alpha}\Omega_1 \end{bmatrix} \tag{A10}$$

$$M_2 = \begin{bmatrix} 1-\beta(\rho+\delta)\Omega_2 & \beta(\rho+\delta)(1-\Omega_1) \\ 0 & 1 \end{bmatrix}, \tag{A11}$$

with $\Omega_1 \equiv -\frac{\mu\alpha(1+\chi)}{\mu(1-\alpha)-(1+\chi)}$ and $\Omega_2 \equiv \frac{\mu(1-\alpha)}{\mu(1-\alpha)-(1+\chi)}$.

Denote by $J = M_2^{-1}M_1$ the Jacobian matrix of this equilibrium system. Since $\hat{K}_t$ is the only endogenous state variable, indeterminacy of equilibria obtains if and only if both eigenvalues of the matrix J are less than one in modulus. Equivalently, the necessary and sufficient conditions for indeterminacy are given by

$$-1 < \det(J) < 1, \quad -(1+\det(J)) < tr(J) < 1+\det(J), \tag{A12}$$

where $\det(J)$ and $tr(J)$ denote the determinant and the trace of J. The determinant of J is given by

$$\det(J) = \frac{\det(M_1)}{\det(M_2)} = \frac{\beta^{-1} - (\rho+\delta)\frac{(1+\chi)(\mu-1)+\mu(1-\alpha)}{\mu(1-\alpha)-(1+\chi)}}{1-\beta(\rho+\delta)\frac{\mu(1-\alpha)}{\mu(1-\alpha)-(1+\chi)}}. \tag{A13}$$

The trace of J is given by
(A14)
$$tr(J) = 1+\det(J)+\beta(\rho+\delta)\frac{\rho+(1-\alpha)\delta}{\alpha}\frac{1}{1-\beta(\rho+\delta)\frac{\mu(1-\alpha)}{\mu(1-\alpha)-(1+\chi)}}\frac{(\mu\alpha-1)(1+\chi)}{\mu(1-\alpha)-(1+\chi)}.$$

We follow Benhabib and Farmer (1994) and assume that $\mu\alpha < 1$ so that equilibria with explosive growth are excluded. We further make the mild technical parameter restriction such that $2-\beta < \frac{1+\chi}{1-\alpha}$. Under these assumptions, we check the necessary and sufficient conditions for indeterminacy in (A12) in several steps.

First, the necessary condition for indeterminacy $tr(J) < 1+\det(J)$ implies that

$$\frac{1}{1-\beta(\rho+\delta)\frac{\mu(1-\alpha)}{\mu(1-\alpha)-(1+\chi)}}\frac{(\mu\alpha-1)(1+\chi)}{\mu(1-\alpha)-(1+\chi)} < 0, \tag{A15}$$

which requires

$$\mu > \frac{1+\chi}{(1-\alpha)\left[1-\beta(\rho+\delta)\right]}. \tag{A16}$$

Second, given the inequality in (A16), the necessary condition for indeterminacy $\det(J) < 1$ is satisfied if $2-\beta < \frac{1+\chi}{1-\alpha}$, which holds under our parameter restrictions.

Third, the condition $-1 < \det(J)$ is satisfied if

$$\mu > \frac{[(\beta^{-1}+1)-\rho-\delta](1+\chi)}{(\beta^{-1}+1)(1-\alpha)-(\rho+\delta)[(1+\chi)+(\beta+1)(1-\alpha)]} \equiv \mu_1^*. \tag{A17}$$

Finally, the condition $-[1+det(J)] < tr(J)$ is satisfied if
(A18)
$$\mu > \frac{[(\beta^{-1}+1)-(\rho+\delta)(1-\frac{\beta}{2}\frac{\rho+(1-\alpha)\delta}{\alpha})](1+\chi)}{(\beta^{-1}+1)(1-\alpha)-(\rho+\delta)[(\beta+1)(1-\alpha)+(1+\chi)(1-\frac{\beta}{2}(\rho+(1-\alpha)\delta))]} \equiv \mu_2^*.$$

Since $\rho = \beta^{-1}-1$, it is clear that μ_1^* and μ_2^* here correspond to those in equations (35) and (36) in the text.

A3. *Equilibrium dynamics in the model with variable capacity utilization*

We now derive the equilibrium conditions for the model with variable capacity utilization. To help exposition, we follow Wen (1998) by assuming that the household decides the capacity utilization rate. Since the entrepreneur is less patient, only the household holds capital in equilibrium. We derive the optimizing conditions that characterize the aggregate dynamics.

The household maximizes the utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(C_t^h) - a_L \frac{N_t^{1+\chi}}{1+\chi} \right], \tag{A19}$$

subject to the budget constraint

$$C_t^h + K_{t+1} \leq [1 + r_t u_t - \delta(u_t)] K_t + w_t N_t. \tag{A20}$$

The term u_t denotes the fraction of capital stock that the household rents out to the firms. Heavier capacity utilization accelerates capital depreciation. The depreciation function is given by

$$\delta(u_t) = \delta_0 \frac{u_t^{1+\eta}}{1+\eta} \tag{A21}$$

where $\eta > 0$.

The household's optimizing choices of capital stock, the utilization rate, and labor supply lead to the first-order conditions

$$\frac{1}{C_t^h} = \beta E_t \frac{1}{C_{t+1}^h} [1 + r_{t+1} u_{t+1} - \delta(u_{t+1})], \tag{A22}$$

$$r_t = \delta_0 u_t^{\eta}, \tag{A23}$$

$$\frac{1}{C_t^h} w_t = a_L N_t^{\chi}. \tag{A24}$$

Using the depreciation function (A21) and the optimizing condition for the utilization rate (A23), we can rewrite the capital Euler equation (A22) as

$$\frac{1}{C_t^h} = \beta E_t \frac{1}{C_{t+1}^h} [1 + \frac{\eta}{1+\eta} r_{t+1} u_{t+1}]. \tag{A25}$$

The financial contracts are the same as in the benchmark model. The factor market clearing conditions now need to take into account of the utilization rate and become

$$\int k_t(\omega) f(\omega) d\omega = u_t K_t, \quad \int n_t(\omega) f(\omega) d\omega = N_t. \tag{A26}$$

The goods market clearing condition is the same as in the benchmark model:

$$C_t^h + K_{t+1} + C_t^e = (1 - \delta_t) K_t + Y_t - \phi, \tag{A27}$$

where Y_t denotes aggregate output and is given by

$$Y_t = \frac{\sigma}{\sigma - 1}\left[\frac{\tilde{\beta}\theta}{1-\tilde{\beta}}\left(\frac{1}{\sigma}Y_t - \frac{\phi}{\tilde{\beta}}\right)\right]^{\frac{1}{\sigma+1}}\left(A_t u_t^{\alpha} K_t^{\alpha} N_t^{1-\alpha}\right)^{\frac{\sigma}{\sigma+1}}, \tag{A28}$$

and C_t^e denotes aggregate dividend (or the entrepreneur's consumption) and is related to aggregate output by

$$C_t^e = \frac{1}{\sigma}Y_t - \phi. \tag{A29}$$

Aggregating the firms' cost-minimizing conditions results in

$$w_t N_t = (1-\alpha)\varphi_t A_t u_t^{\alpha} K_t^{\alpha} N_t^{1-\alpha} = (1-\alpha)\frac{\sigma-1}{\sigma}Y_t, \tag{A30}$$

and

$$r_t u_t K_t = \alpha\varphi_t A_t u_t^{\alpha} K_t^{\alpha} N_t^{1-\alpha} = \alpha\frac{\sigma-1}{\sigma}Y_t. \tag{A31}$$

Using (A30) and (A31), we rewrite the household's optimizing conditions as

$$\frac{1}{C_t^h}(1-\alpha)\frac{\sigma-1}{\sigma}\frac{Y_t}{N_t} = a_L N_t^{\chi}, \tag{A32}$$

$$\frac{1}{C_t^h} = \beta E_t \frac{1}{C_{t+1}^h}[1 + \alpha\frac{\eta}{1+\eta}\frac{\sigma-1}{\sigma}\frac{Y_{t+1}}{K_{t+1}}]. \tag{A33}$$

Aggregate dynamics in the model with variable capacity utilization are fully characterized by the following system of equations:

$$X_t = \left[\frac{\tilde{\beta}\theta}{1-\tilde{\beta}}\left(\frac{1}{\sigma-1}X_t - \frac{\phi}{\tilde{\beta}}\right)\right]^{\frac{1}{\sigma+1}}\left(A_t u_t^{\alpha} K_t^{\alpha} N_t^{1-\alpha}\right)^{\frac{\sigma}{\sigma+1}}, \tag{A34}$$

$$X_t = C_t^h + K_{t+1} - (1-\delta_t)K_t, \tag{A35}$$

$$\frac{1}{C_t^h}(1-\alpha)\frac{X_t}{N_t} = a_L N_t^{\chi}, \tag{A36}$$

$$\frac{1}{C_t^h} = \beta E_t \frac{1}{C_{t+1}^h}[1 - \delta_{t+1} + \alpha\frac{X_{t+1}}{K_{t+1}}], \tag{A37}$$

$$\alpha\frac{X_t}{K_t} = \delta_0 u_t^{\eta}, \tag{A38}$$

where $X_t \equiv \frac{\sigma-1}{\sigma} Y_t$ and $\tilde{\phi} = \phi/\beta$.

A4. Proof of Proposition IV.1

We are now ready to derive the financial multiplier in the model with variable capacity utilization as stated in Proposition IV.1 in the text.

PROOF:

Log-linearizing the aggregate production function (A34) around the deterministic steady state, we obtain

$$\hat{X}_t = \left(\alpha \hat{u}_t + \alpha \hat{K}_t + (1-\alpha)\hat{N}_t + \hat{A}_t\right) \frac{\sigma}{\sigma + 1 - \xi}.$$

Using the log-linearized version of the optimizing conditions for the utilization rate in (A38), we substitute out $\hat{u}_t$ and obtain

$$\hat{X}_t = \tilde{\mu}\hat{A}_t + \tilde{\mu}\alpha \frac{\eta}{1+\eta} \hat{K}_t + \tilde{\mu}(1-\alpha)\hat{N}_t, \tag{A39}$$

where $\tilde{\mu} \equiv \frac{\mu(1+\eta)}{1+\eta-\alpha\mu} > \mu$, with μ being the financial multiplier in the benchmark model. Thus, holding input factors constant, a 1 percent change in TFP leads to a $\tilde{\mu}$ percent change in aggregate output, as stated in Proposition IV.1.

A5. Parameter restrictions and the indeterminacy region

We now derive the restrictions on the financial friction parameters θ and ϕ that permit interior solutions, that is, the admissible region. We also derive the combinations of these parameters in the admissible region that lead to indeterminacy.

Admissible parameters. — We begin by deriving the combinations of (ϕ, θ) that permit interior solutions.

In the steady-state equilibrium, equations (A35) and (A37) show that the output-capital ratio $X_k \equiv \frac{X}{K}$ and the consumption-capital ratio $C_k \equiv \frac{C^h}{K}$ are independent of financial friction parameters. It follows from the labor supply decision (A36) and the optimal capacity utilization decision (A38) that the steady-state levels of employment N and utilization u are also independent of the financial friction parameters. Without loss of generality, we normalize the steady-state level of technology A such that $Au^{\alpha}N^{1-\alpha} = 1$. The aggregate production function (A34) then implies that, in the steady state,

$$X = \left[\frac{\tilde{\beta}\theta}{1-\tilde{\beta}} \left(\frac{1}{\sigma-1} X - \frac{\phi}{\tilde{\beta}}\right)\right]^{\frac{1}{\sigma+1}} (K^{\alpha})^{\frac{\sigma}{\sigma+1}}. \tag{A40}$$

Since aggregate output is given by $X = \omega^* A(uK)^\alpha N^{1-\alpha} = \omega^* K^\alpha$, where $\omega^* \geq 1$ is the cutoff level of productivity, interior solution requires that

$$X \geq K^\alpha \equiv X_{\min}. \tag{A41}$$

The minimum output $X_{\min}$ can be solved in terms of (nonfinancial) parameters using the fact that the output-capital ratio is invariant to financial friction parameters. In particular, $X_{\min} = K^\alpha = (X_{\min}/X_k)^\alpha$, so that

$$X_{\min} = X_k^{\frac{\alpha}{\alpha-1}}. \tag{A42}$$

The inequality in (A41) puts restrictions on the admissible values of ϕ and θ. All else equal, aggregate output decreases with ϕ and increases with θ. Taking the constraint in (A41) as given, we first derive the upper bound for ϕ by holding $\theta = 1$ and then derive the lower bound for θ as a function of ϕ. Denote the upper bound for ϕ by $\phi^{\max}$. If $\phi = \phi^{\max}$, then aggregate output reaches its lowest level $X_{\min}$. Thus, from (A40) with $\theta = 1$ imposed, we have

$$X_{\min} = \left[\frac{\tilde{\beta}}{1-\tilde{\beta}}\left(\frac{1}{\sigma-1}X_{\min} - \frac{\phi}{\tilde{\beta}}\right)\right]^{\frac{1}{\sigma+1}} (X_{\min})^{\frac{\sigma}{\sigma+1}}.$$

Rearranging terms, we obtain

$$\phi^{\max} = \tilde{\beta}\left[\frac{1}{\sigma-1} - \frac{1-\tilde{\beta}}{\tilde{\beta}}\right] X_k^{\frac{\alpha}{\alpha-1}}. \tag{A43}$$

We now derive the lower bound of θ as a function of ϕ such that (A41) is satisfied. Denote this lower bound by $\theta_{\min}(\phi)$. We use the production function (A40) to solve for $\theta_{\min}(\phi)$ by setting $X = X_{\min}$. This procedure yields

$$\theta_{\min}(\phi) = \frac{\frac{1-\tilde{\beta}}{\tilde{\beta}} X_{\min}}{\frac{1}{\sigma-1}X_{\min} - \frac{\phi}{\tilde{\beta}}}. \tag{A44}$$

Thus, the set of admissible financial friction parameters are given by

$$\mathcal{A} = \{(\phi, \theta) | 0 \leq \phi \leq \phi^{\max}, \theta_{\min}(\phi) \leq \theta \leq 1\}.$$

A6. Indeterminacy region

We now derive the combinations of ϕ and θ that lead to indeterminacy.

Proposition IV.2 establishes that the necessary and sufficient condition for indeterminacy is given by

$$\tilde{\mu} \geq \tilde{\mu}^*, \tag{A45}$$

where $\tilde{\mu}^*$ is the threshold value of $\tilde{\mu}$ for indeterminacy and is defined in equations (42)-(44).

From Proposition IV.1, we have $\tilde{\mu} = \frac{(1+\eta)\mu}{1+\eta-\alpha\mu}$, where $\mu = \frac{\sigma}{\sigma+1-\xi}$ and $\xi = \frac{1}{1-\phi(\sigma-1)/(\tilde{\beta}X)}$, as shown in Proposition II.3.

The indeterminacy condition in (A45) puts restrictions on the parameters. We now derive these restrictions.

Define the term

$$\xi^* \equiv \sigma + 1 - \frac{\sigma}{\mu^*}, \tag{A46}$$

where $\mu^* \equiv \frac{(1+\eta)\tilde{\mu}^*}{1+\eta+\alpha\tilde{\mu}^*}$. Since $\tilde{\mu}$ is a monotone function of ξ, we have $\tilde{\mu} \geq \tilde{\mu}^*$ if and only if $\xi \geq \xi^*$. Thus, from the definition of ξ, indeterminacy obtains if and only if

$$1 - \frac{\phi(\sigma-1)}{\tilde{\beta}X} \leq \frac{1}{\xi^*}.$$

Define the term

$$X^* \equiv \frac{\phi}{\tilde{\beta}} \frac{(\sigma-1)\xi^*}{\xi^*-1}. \tag{A47}$$

Indeterminacy obtains if and only if $X \leq X^*$. Thus, we can solve the maximum value of θ as a function of ϕ such that, for all θ below this maximum value, we have $X \leq X^*$ and indeterminacy obtains. Denote by $\theta^{\max}(\phi)$ the upper bound of θ for the indeterminacy region. At this value of θ, we have $X = X^*$. Thus,

$$X^* = \left[\frac{\tilde{\beta}\theta^{\max}(\phi)}{1-\tilde{\beta}} \left(\frac{1}{\sigma-1}X^* - \frac{\phi}{\tilde{\beta}}\right)\right]^{\frac{1}{\sigma+1}} \left(\frac{X^*}{X_k}\right)^{\frac{\alpha\sigma}{\sigma+1}}, \tag{A48}$$

where we have used the equilibrium condition that $K = \frac{X}{X_k}$ for all admissible values of X. Substituting out X^* using (A48) and rearranging terms, we obtain

$$\theta^{\max}(\phi) = \left[\frac{\xi^*(\sigma-1)}{\xi^*-1}\frac{\phi}{\tilde{\beta}}\right]^{\sigma+1-\alpha\sigma} \frac{X_k^{\alpha\sigma}}{\phi}(\xi^*-1)(1-\tilde{\beta}). \tag{A49}$$

where ξ^* is given by (A46).

Thus, the region of indeterminacy is summarized by the set

$$\mathcal{I} \equiv \left\{(\phi, \theta)|0 \leq \phi \leq \phi^{\max}, \theta_{\min}(\phi) \leq \theta \leq \min(\theta^{\max}(\phi), 1)\right\}. \tag{A50}$$

A7. Endogenous leverage

We now derive the equilibrium dynamics in the model with endogenous contract enforcement discussed in Section IV.D. We focus on the version of the model with variable capacity utilization and derive the financial multiplier in equation (50).

Substituting out the cut-off level of productivity ω_t^* in the aggregate production function (24) (with variable capacity utilization), we obtain

$$Y_t = \frac{\sigma}{\sigma - 1} (b_t)^{\frac{1}{\sigma+1}} \left(A_t (u_t K_t)^{\alpha} N_t^{1-\alpha}\right)^{\frac{\sigma}{\sigma+1}}, \tag{A51}$$

where b_t is the credit limit given by equation (49) and we rewrite it here for convenience of references

$$b_t = \frac{\gamma}{1+\gamma} \theta_t \tilde{\beta} E_t \frac{\Lambda_{t+1}^e}{\Lambda_t^e} V_{t+1}.$$

With log-utility, the entrepreneur's marginal utility is given by $\Lambda_t^e = \frac{1}{C_t^e}$. It follows that the firm's equity value is given by $V_t = \frac{1}{1-\tilde{\beta}} C_t^e$. Thus, the credit limit can be simplified into

$$b_t = \frac{\gamma}{1+\gamma} \theta_t \frac{\tilde{\beta}}{1-\tilde{\beta}} C_t^e, \tag{A52}$$

where, from equation (48) in the text, the endogenous leverage ratio θ_t is given by

$$\theta_t = \left[\frac{\tilde{\beta}}{1-\tilde{\beta}} \frac{1}{\theta_0} C_t^e\right]^{\frac{1}{\gamma}}. \tag{A53}$$

Using the relations (22) and (24) in the text, we can express the entrepreneur's consumption (i.e., aggregate dividend) in terms of aggregate output. Specifically, we have

$$C_t^e = Y_t - W_t N_t - R_t K_t = \frac{1}{\sigma} Y_t. \tag{A54}$$

Substituting out the credit limit b_t in the aggregate production relation (A51) using equations (A52), (A53), and (A54), we obtain

$$Y_t = \kappa \left[A_t (u_t K_t)^{\alpha} N_t^{1-\alpha}\right]^{\mu^e}, \tag{A55}$$

where κ is a constant and $\mu^e = \frac{\sigma}{\sigma - 1/\gamma}$.

The rest of the equilibrium conditions are identical to those summarized in equations (A35)-(A38).

Log-linearizing equation (A55) around the deterministic steady state, we obtain

$$\hat{Y}_t = \mu^e \left[\hat{A}_t + \alpha \hat{u}_t + \alpha \hat{K}_t + (1-\alpha)\hat{N}_t\right]. \tag{A56}$$

Substituting out the utilization rate $\hat{u}_t$ using the log-linearized version of equation (A38), we obtain

$$\hat{Y}_t = \tilde{\mu}^e \hat{A}_t + \tilde{\mu}^e \alpha \frac{\eta}{1+\eta} \hat{K}_t + \tilde{\mu}^e (1-\alpha)\hat{N}_t, \tag{A57}$$

where

$$\tilde{\mu}^e = \frac{(1+\eta)\mu^e}{1+\eta-\alpha\mu^e}, \quad \mu^e \equiv \frac{\sigma}{\sigma - 1/\gamma},$$

as in equation (50) in the text.

A8. *Discussion: Do firms and the entrepreneur have an incentive to save?*

We focus on the steady state behavior of the entrepreneurs and firms. In the steady state. The interest rate from saving is

$$r = \frac{1}{\beta} - 1 + \delta \tag{A58}$$

Since we have

$$\tilde{\beta}(1 + r - \delta) < 1 \tag{A59}$$

so the entrepreneur does not have incentive to save.

The firms however may have incentive to accumulate capital despite $\tilde{\beta} < \beta$. To exclude such possibility, we now explicitly consider the firm decision. Suppose a firm decides to operate to the next period in the end of period. If the firm saves one unit of capital, it costs one unit of good is period t. In the next period, the return however depends on its idiosyncratic shock.

If the firm's $\omega^j_{t+1} \leq \omega^*$, then the firm will not produce and in that event, the firm rents out the unit of capital and earns a return r_{t+1}. Cost-minimizing and the assumption of perfect factor mobility imply that

$$r_{t+1} = \omega_t^* A_t \left(\frac{K_{t+1}}{N_{t+1}}\right)^{\alpha-1}. \tag{A60}$$

If $\omega_{t+1}^j > \omega^*$, then the firm will produce by its own. In this event, the return for the firm from holding a unit of capital is given by

$$\alpha \frac{Y_{t+1}^j}{K_{t+1}^j} = \alpha \omega_{t+1}^j K_{t+1}^{j\alpha-1} N_t^{j1-\alpha} = \alpha \omega_{t+1}^j A_{t+1} \left(\frac{K_{t+1}}{N_{t+1}}\right)^{\alpha-1}, \tag{A61}$$

where the last equality follows from the assumption of perfect factor mobility across firms.

Thus, the return for the firm to accumulate one unit of capital is given by

$$\tilde{r}_{t+1}(\omega) = \max(1, \frac{\omega}{\omega^*}) r_{t+1}.$$

A firm will not have an incentive to accumulate capital if and only if

$$\Lambda_t^e > \tilde{\beta} \Lambda_{t+1}^e E_t \left[\max(1, \frac{\omega}{\omega^*}) r_{t+1} + 1 - \delta\right], \tag{A62}$$

where $\frac{\omega}{\omega^*}$ can be considered the external finance premium. In the steady state, this condition becomes

$$\begin{aligned} 1 &> \tilde{\beta}(1 - \delta + r(1 + \frac{1}{\sigma - 1}\omega^{*-\sigma}) \\ &= \tilde{\beta}(1 - \delta + r(1 + \frac{1}{\sigma}\frac{Y}{b}). \end{aligned} \tag{A63}$$

Under our parameter calibration, we have $\frac{b}{Y} = 2.08$, $\sigma = 6$, $\delta = 0.025$, $\beta = 0.99$, $\tilde{\beta} = 0.98$, and $r = 0.0351$. These parameter values imply that the right hand side of the equation equals 0.9927, which is less than the left hand side. Thus, under our parameterization, firms do not have an incentive to accumulate capital in the steady state.

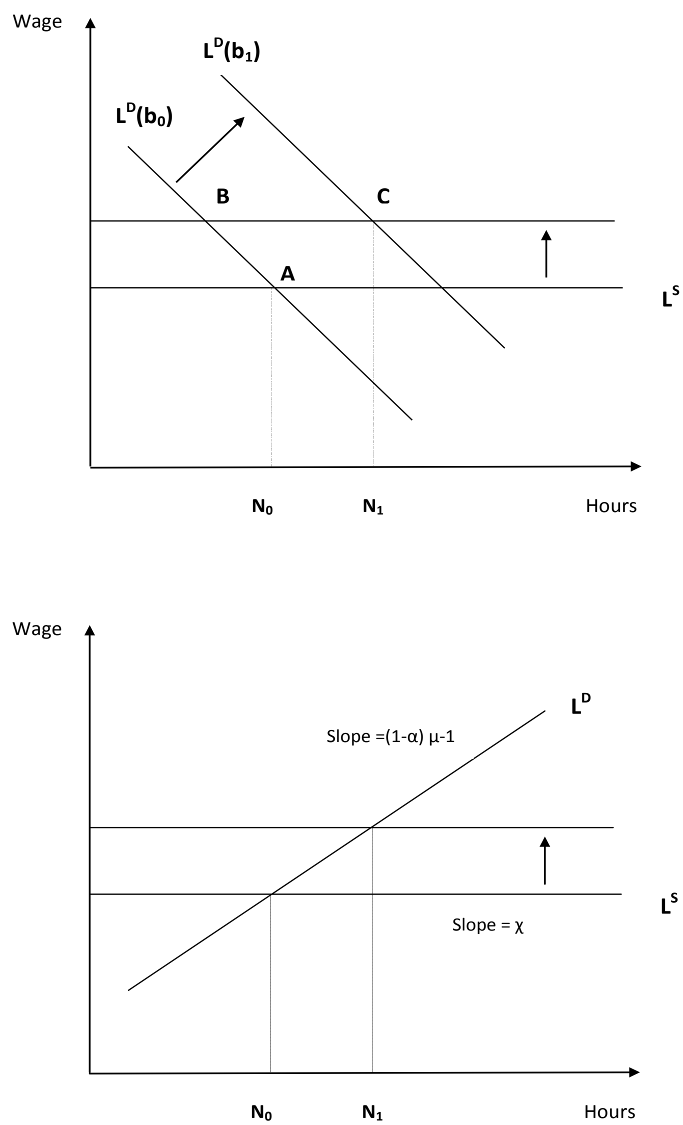

Figure 2. Labor market adjustment to expectations of increases in wealth under conditions for indeterminacy.

Note: Top panel: Benchmark model with financial friction. Bottom panel: Representative-agent model with increasing returns (Benhabib and Farmer, 1994).

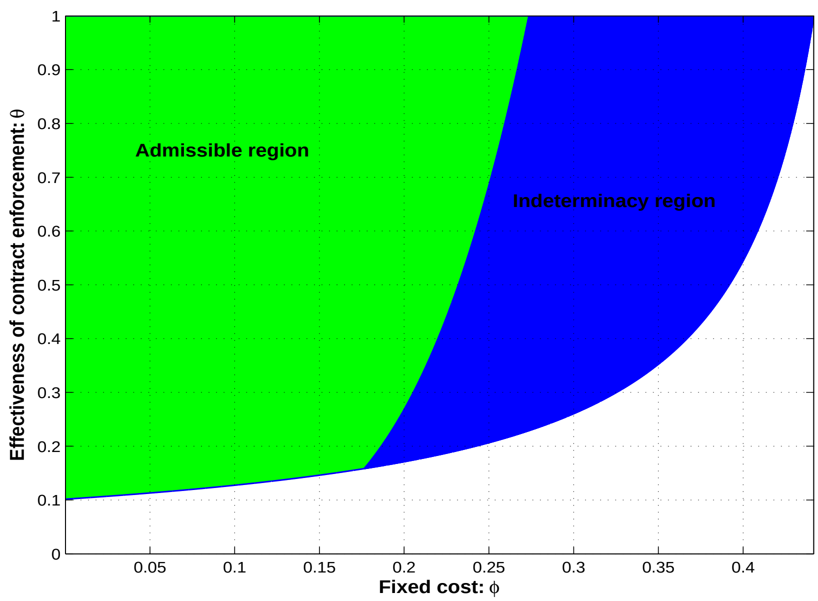

Figure 3. Region of parameters for indeterminacy

Note: The figures displays the parameter regions for indeterminacy in the model with variable capacity utilization. The horizontal axis shows the fixed cost parameter (ϕ) and the vertical axis shows the strength of contract enforcement parameter (θ). The admissible region is the sum of the two colored areas. The indeterminacy region is the blue area..